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THE FLASH AND BURNING POINTS OF KEROSENE-GASOLENE MIXTURES

BY
JAMES T. ROBSON
AND
JAMES R. WITHROW

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INTRODUCTION

The heavy loss of life and damage to property both urban and rural from the domestic and industrial use of kerosene contaminated with light-boiling or gasoline fractions, long ago compelled enactment of laws for kerosene or burning-oil inspection. Ohio has such a state oil inspection department which is entrusted with the execution of the law setting a minimum flash temperature (120° Fahr.) below which burning oils are declared dangerous and illegal merchandise. In recent years there have been newspaper mention of efforts to have the legislature abolish these minimum requirements. Many citizens and even scientific men have erroneously concluded that such requirements and inspection are now made unnecessary by the fact that it obviously no longer pays an oil refiner to add gasoline to his kerosene to increase its bulk since the latter sells for less than the gasoline. The fallacy lies, however, in the fact that nevertheless, it may cost the refiner more to take a relatively small amount of gasoline out of a dangerously contaminated kerosene than the enhanced sale value of the gasoline. This latter fact together with the constant factor of human error in the mixing of gasoline with kerosene in manufacturing or handling as merchandise makes such a minimum flash point law a constant necessity for the public welfare.

Within the last two years the State Oil Inspector has detected among others, one such manufacturing error which sent over the state upwards of twenty cars (8-10,000 gallons each) which were dangerously contaminated and were detected and rejected by his office. Refiners are glad to be protected against errors of their own employees by this state inspection.

This bulletin gives the results of an effort to find out just how much gasoline was necessary to make modern kerosene dangerous and fall below the legal flash limit in Ohio.

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THE FLASH AND BURNING POINTS OF GASOLINE-KEROSENE MIXTURES

By **JAMES T. ROBSON** and **JAMES R. WITHROW**

Read at the Boston Meeting, June 1919

The results of this work indicate:

1. The profound effect upon the flash and burning point of kerosene and therefore the fire and explosion hazard of the smallest percentages of gasoline or similar volatile petroleum fractions added to the kerosene.

2. Gasoline cannot safely be permitted in kerosene.

3. The early records of the influence of light petroleum distillates upon burning oils were based upon fractions radically different in properties from similarly named products of modern refining.

4. The main original need for State Flash Point laws has ceased. Nevertheless, public safety still requires their enforcement.

6. The Foster closed cup flash determination permits the presence of much lighter petroleum distillates than the Cleveland open cup method.

6. The gasoline and other light petroleum fractions are found to be miscible instantaneously with kerosene if intimately mixed. It is concluded that stratification in large tanks is merely due to insufficient mixing or lack of time for diffusion.

7. While ordinary kerosene of commerce extinguishes a burning match, the addition of gasoline to the extent of 1% may give a flash. 3 to 5% may give violent flashing and 7½% may inflame at once. Any of these would have given explosions if confined.

8. The relation is established between flash and burning point and percentage of volatile petroleum fractions admixed with kerosene. Working conditions are given minutely, to establish the comparative dependability of the final values for future cases of comparison. Distillation and gravity characteristics of the fractions used are recorded.

Because of our own inability to obtain material assistance from the literature on the influence of the addition of gasoline to kerosene with repect to the explosion hazard after the kerosene was marketed, it seemed desirable to publish portions of the work made necessary by this scanty literature.

Fatal Results from Gasoline Contaminated Kerosene.

One or several deaths appear to have resulted from explosions in the use of kerosene admitted to have been marketed in Ohio by an oil refiner. It is also admitted that an employee of the refiner inadvertently pumped gasoline into a kerosene storage tank and also that kerosene was subsequently sold from this tank batch. It was claimed by the plaintiff that the kerosene causing fatal explosions, could be directly traced to this contaminated kerosene storage. It is claimed by employees of the refiner that a gallon sample was drawn off of the contaminated kerosene tank and its flashing properties tested and found "safe."

It would probably have to be admitted that a draw off of one gallon would not clear the uncontaminated kerosene from the draw-off line and as a result the "safe" flash test would not reveal the true character of the tank contents.

In this same connection also it was claimed by the plaintiff that stratification probably existed in the tank anyway and therefore a sample withdrawn from the bottom of the tank would inevitably test "safe" in flash point. It was also claimed that dispensing from said tank would soon reach a highly dangerous zone because richer in gasoline.

No exact evidence was available as to the amount of kerosene originally in the tank or the amount of gasoline pumped into it or the amount of the mixture resulting. It would probably have to be admitted, however, that there was more than 2% gasoline as a minimum admixed with the kerosene or any lot sold therefrom if admixture were complete. No actual sample of the mixture appears to have been preserved by either side. The samples causing the explosions were destroyed thereby.

Early Work on Explosion Hazards.

In 1861 Allen (Smithsonian Reports 1861, p. 330) reported on the "Explosibility of Coal Oil" an investigation instigated by the

Rhode Island Mutual Fire Insurance Company to study various "coal oils." Both petroleum and coal tar products were included under "coal oil." That gasoline was then a drug on the market was pointed out as was the artificial production of kerosene with a kerosene gravity by judicious mixtures of higher and lower fractions than the kerosene fraction proper.

Allen made flash-point determinations in open cups floating in water at various temperatures. This seems to us a cumbersome manipulation as compared with our simple direct heated or water-bath heated devices. He did not know the amount of "adulteration," however.

K. v. Weise in "Wagner's Chemische Technologie," 1871, p. 865, with the Open Cup Method found:

	Sp. Gr.	Flash Point	Burning Point
Petroleum	0.805	30°	43°
Petroleum + ½% Naphtha..	0.710	25°	40°
Petroleum + 1% Naphtha...	0.710	22°	32°
Petroleum + 3% Naphtha...	0.710	20°	29°

This work is by far the most complete along this line in the literature. The oils used, however, are quite different from those with which we are concerned. The closest to our conditions are these:

	Flash Point	Burning Point
Petroleum 0.806	55°	70°
Petroleum + 2% Naphtha............	45°	60°
Petroleum + 4% Naphtha............	24°	55°
Petroleum + 10% Naphtha...........	20°	32°

There was no closed cup work. The author states that for the determination of the naphtha content, neither the flash point method or fractional distillation is satisfactory.

In 1887 Crew (Treatise on Petroleum 1887, Philadelphia, Henry C. Baird and Co., p. 395) pointed out adulteration of kerosene with dangerous low-boiling fractions and presented a simple "saucer" test to show whether or not a given oil was dangerous for use in lamps.

Crew quotes experiments of Dr. C. B. White of New Orleans resulting as follows:

	Flash Point	Burning Point
Oil alone........................	118° F.	135° F.
Oil with 1% "Benzene," 65° Bé....	112° F.	129° F.
Oil with 3% "Benzene," 65° Bé....	103° F.	123° F.
Oil with 5% "Benzene," 65° Bé....	96° F.	116° F.
Oil with 10% "Benzene," 65° Bé....	83° F.	102° F.
Oil with 1% "Benzene," 72° Bé....	107° F.	133° F.
Oil with 5% "Benzene," 72° Bé....	70° F.	105° F.

Allen's Commercial Organic Analysis, 4th Ed., Blakiston, Philadelphia, 1914, p. 120, states that "Dr. B. W. White" using the "open test" found that with

Oil which originally flashes at 113° F.
1% naphtha reduced flash point to 103° F.
2% naphtha reduced flash point to 92° F.
5% naphtha reduced flash point to 83° F.
10% naphtha reduced flash point to 59° F.
20% naphtha reduced flash point to 40° F.

No further information is given.

It is not evident whether this is the same Dr. White as referred to by Crew.

These results correspond only approximately with ours. They are upon the older type of low flash kerosene and with uncertain procedure. They are interesting, however, as tending to show the influence of definite admixture on an oil of know flash point. No modern case has been found by us where there is a clear correlation between definite admixtures of characterized products and the flash and burning test with prescription of conditions of test.

Steingraber, Chem. Zeit. Oesterr, 1900, p. 589-99, Jr. Soc. Chem. Ind., 1901, p. 352, tried to determine how far the properties of "illuminating" fractions of oil were affected by benzine mixtures. He used 21° C. flash oil as standard and added various amounts of benzine and higher fractions. His results showed that only benzines boiling below 100° C. have strong influence on flash. Additions of higher boiling benzines have little effect. 21° C. (or

70° F.) is too low as a flash oil to take as a standard to determine effect of benzine on flash. With 110°-120° F. flash oils as standards,

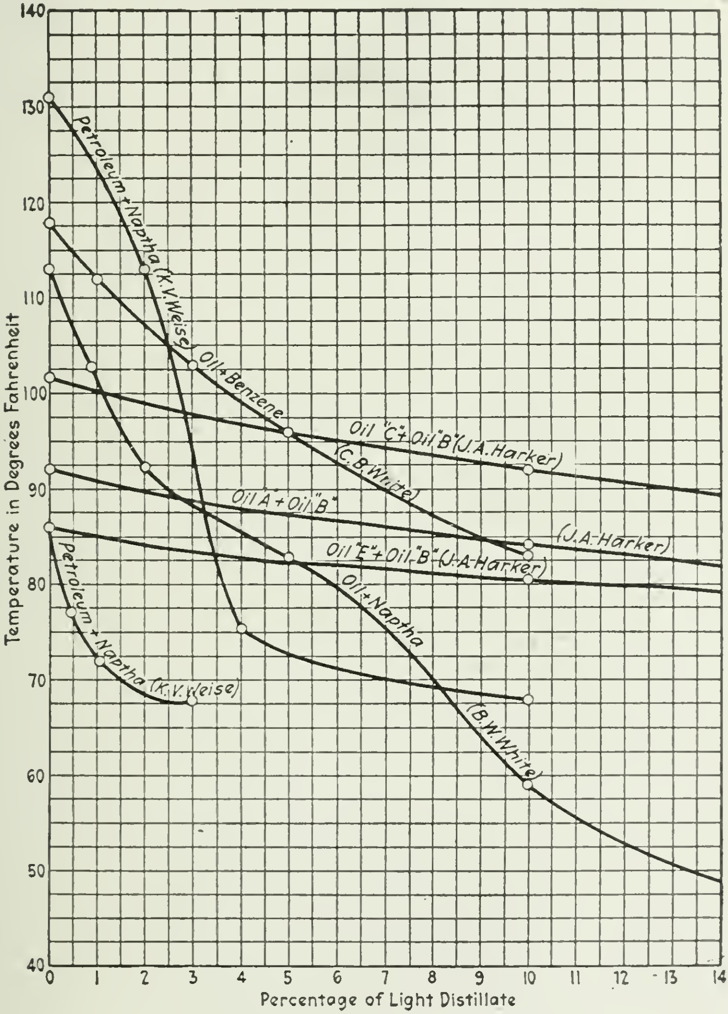

Fig. 1.—Flash Point Curve Early Workers.

effect of higher boiling benzines are somewhat different than found by Steingraber.

H. C. Sherman, T. T. Gray and H. A. Hammerschlag, Jour.

Ind. and Eng. Chem., vol. 1, 1909, p. 13, compared the calculated and determined viscosities and flashing and burning points of oil mixtures. This work shows that the calculated flash point is not the same as the actual. The work was not on kerosene, however.

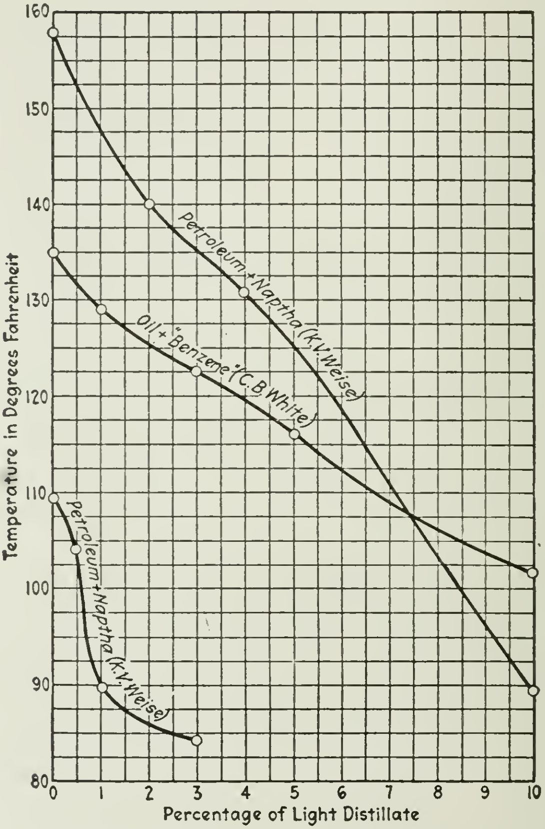

FIG. 2.—Burning Point Curves Early Workers.

J. A. Harker and W. F. Higgins on the Methods and Apparatus Used in Petroleum Testing, Collected Researches, Nat. Phys. Lab. vol. 8, 1912, p. 38, give a method for calculating flash points of different mixtures of oils. No method is given of identifying their oils.

Intervals (10%) of mixture used are so large that is not justifiable to determine flash point for any intervening mixture (say 92% or 97%).

Harker's work intends to give a method of determining the flash point when the composition is known; our work will give approximately the composition of the mixture from a determination of the flash point and interpreting on the basis of the distillation curves which identify our oils. Curves representing the values recorded by these earlier workers appear on Plates No. 1 and 2.

FACTORS INFLUENCING FIRE HAZARDS OF GASOLINE AND KEROSENE.

Strictly speaking, the flash point by itself does not determine the fire hazard of any substance. The factors which also influence fire hazard are:

(*a*) Volatility.
(*b*) Boiling Point.
(*c*) Vapor Pressure.
(*d*) Vapor Density.
(*e*) Diffusibility and tendency of vapors to travel.
(*f*) Explosive limits in air.
(*g*) Tendency to chemical change.
(*h*) Quantity of heat liberated per unit of volume.
(*i*) Temperature of flame.
(*j*) Corrosive action.
(*k*) Behavior toward water before and after ignition.
(*l*) Tendency of substance to leak.
(*m*) Prolonged heating effect.

Kerosene in tropical climate gives lower "flash" than same oil in temperate climate. The trouble is due primarily to accumulation of vapors caused by continued high temperature and secondarily in discharging or moving oil by vapors disengaged on pouring and shaking.

It is well known that cold kerosene of good legal quality will not take fire when light is applied, nor will the supernatant vapor inflame. The "burning test," fire test—i.e., temperature at which oil permanently inflames—is sometimes taken as a test of burning or firing value but is not strictly reliable since *oils, spilled in a thin* *flow will ignite instantly* on approach of a flame, even when the burning point is considerably higher than the flash point.

It is also well known that with refined burning oil no inflammable or explosive mixture could be formed, even in the absence of ventilation, up to 70° F. At temperatures much above 80° F. and with no ventilation, such an atmosphere could be produced with most oil.

EARLY FLASH POINT LAWS AND PRESENT REVERSED STATUS OF GASOLINE AND KEROSENE PRODUCTION.

It would have to be admitted that the presence of lighter or gasoline fractions in kerosene has long ceased to be profitable to the refiner. Hence, as an evil it has needed little attention in recent times.

If commercial gasoline of any gravity is accidently pumped into or, by faulty valving or otherwise, gets into a kerosene fractions storage its detection is quite simple by the flash test. Simple consideration indicate, however, that this can be no quantitive indication of the amount of such contamination except the "Flash Point" determination be calibrated with reference to this particular kind of occurrence. Individual oil companies or chemists doubtless have felt the need for such a correlation and perhaps have made it. Few of these results have gotten into the literature. From year to year we have had cases arise which raised the question: how much gasoline does this particular contamination represent? Recently the question arose again under the circumstances already mentioned which made a partial answer to the general question necessary and the results obtained with our set of conditions are here recorded.

In the earlier period of petroleum refining, kerosene or "burning oil" was the valuable light product or early distillate. At that time it was a temptation to producers to put into kerosene all the traffic would stand of lower boiling (gasoline) fractions, together with the maximum of higher boiling fractions next above the burning oil, to balance the lighter gravity of the gasoline fraction. This virtually amounted to a physical synthesis of kerosene by mixing with each other the next lower and higher fractions. This was a perfectly justifiable procedure, if a safe one, to meet commercial demands in the days that developed "crocking" for burning oil formation. "Gravity," however, was not the sole consideration.

There was a safety limit which was important because the consumer in the homes of the land might be made the victim of unnecessary fire hazard, through refiners' carelessness or unscrupulousness in this connection. Many tragedies resulted and legislation put a curb to syntheses of this kind by setting legal limits in a modified practical safety-test in the Flash Point Laws.

Of the law requirements (Gill's "Oil Analysis," 3rd Ed., J. B. Lippencott, Philadelphia, p. 131; Stillman's "Engineering Chemistry," Chemical Publishing Co., Easton, Pa., 1916, p. 450) in 32 states, 19 fix a minimum Flash Point of 100° F. to 126° F. The other 12 states make a minimum "Burning Point" instead of Flash Point requirement. This Burning Point runs from 110° to 150° F. in the states making the requirements. Four states set both a minimum Flash and Burning Test and 16 no Burning Point test at all. Those state laws specified in all except 8 cases the form of test whether in "open" or "closed" apparatus. In only four cases was "open" testing permitted. In all other cases a closed tester was prescribed, thus closely simulating lamp or confined conditions, and the specific instrument was named such as the Tagliabue or Foster. From the results herein reported it will be seen that the closed cup of the Foster type may permit the presence of much more gasoline in a burning oil than does the open cup test for the same flash point.

In that early period of the petroleum refining industry gasoline was a drug on the market. All the art of sales was directed to the creation of an outlet. The public had to be protected, therefore, against unscrupulous addition of gasoline to kerosene to increase production volume of the latter. Now the situation is reversed. The advent of the hydrocarbon or internal combustion engine about 1898 has created ever increasing demands for the gasoline fraction so that the kerosene demand by comparison stands still. Now as a result, the refiner puts everything into gasoline that the traffic will stand. This is as it should be, if understood. Consequently, the cheaper kerosene fraction is stripped as lean as possible of volatiles to raise the volume of the gasoline fraction. This puts so much of the front end of kerosene into gasoline that, of course, the gasoline is made heavier in gravity as is the kerosene. This greatly raises the flash and burning point of the kerosene because the more volatile portion was cut off to add to gasoline. This is so effectively done in wideawake refineries that the State Flash Point laws are almost a dead letter. They, nevertheless, are necessary, however, because there are still a few poorly equipped or carelessly operated refineries. Occasionally errors or accidental contamination of kerosene storage will occur and the plant manager refuses to re-strip the mixture to save expense and instead works it off on the trade. We seem to meet occasionally just this situation in Ohio State Oil Inspection and the protection to the public in those cases justifies the continuance of this state activity. In addition, there are the mistakes of delivery and petty storage-filling between wholesaler, retailer and eventual delivery to consumer. These, while not common, are much more common than refiner's mistakes, but are depressed in number by vigorous legal inspection and law enforcement particularly in rural sections.

When the State Oil Inspector's office picks up offenders under the Ohio law it at times can trace a somewhat connected series of infractions indicating a bad lot of kerosene, and at times traceable to the refinery. In referring such matters to this laboratory the question is sometimes asked about the extent of gasoline contamination. It is not necessary to cite cases except to state that confirmation of the field inspectors' test have at times been so good as to have the product pop-off on lighting the taper in the Foster closed cup. (The legal test instrument in Ohio.) The taper is lighted customarily at 90° F., and the Ohio minimum flash point for kerosene is 120° F. Such a product is highly dangerous as a kerosene.

For this portion of the public welfare and safety, the State Oil Inspection laws are still an important factor. Careful, honest dealers and refiners have no quarrel with these laws.

The Flash and Burning Point Tests.

There are some factors which must be clearly understood when dealing with flash-point determination. The substances—kerosene and gasoline—themselves are not chemical compounds but mixtures of hydrocarbons. "Kerosene" is a trade name. It is also called "coal-oil" and "Illuminating oil." Industrially it is understood to be a mixture of hydrocarbons "entirely" free from gasoline and naphtha on the one hand and from heavy hydrocarbons belonging to gas oil and lubricating oil on the other. This (kerosene) mixture is only supposed to be moderately soluble in alcohol but miscible in all proportions with ether, chloroform, "benzene," petroleum spirit, volatile and fixed oils except castor oil.

Gasoline is a mixture of the lower series of hydrocarbons. It is lower in boiling point and lighter in gravity than kerosene. It includes all of them that are not "wild" or which will stay in a reasonably corked container and running as heavy in gravity as motor vehicles can start and navigate under. It usually includes all the old materials called variously petroleum spirit, petrol, benzine, etc. Formerly, that which distilled below 150° C. was "spirit" or gasoline, 150-270° C. was kerosene and 270° C. up was heavy oils. This was only conventional. Refineries ran largely on gravity. The consumer became acquainted with a specific gravity test. It was easy to perform and the refiner cut his fractions at various gravities in order to have his cuts average some other gravity required in the trade.

The flash point is the lowest temperature to which the oil need be heated under a fixed set of conditions to give off vapors in sufficient quantity when mixed with air to explode upon the approach of a flame.

The burning point or "fire test" is the lowest temperature at which oil will give off sufficient vapors to burn continuously when ignited.

The Flash Point and Burning Point Tests are arbitrary. They are not measures of rigorous constants yet they are capable of much refinement. They are accurate determinations of the practical effects of complex factors. For these reasons the values as determined are susceptible to influence from many sources among which may be mentioned:

1. Barometric Pressure.
2. Nature of bath as a source of heat.
3. Design of oil cup.
4. Features of thermometer.
5. Rate of heating.
6. Effect of prolonged heating or cooling.
7. Manipulation of test. (Manner flame is applied for flashing, etc.)
8. Room temperature and conditions.
9. Initial temperature of oil being tested.
10. Influence of water presence.
11. Personality of operator.

Even the determination of the flash point of a pure hydrocarbon (from the mixture whose vapor pressure effect is really the matter at issue) would be influenced by most of these factors.

The aim was a practical test and experience has forced precautions and developed refinements. Burrell and Boyd (U. S. Bureau of Mines Tech. Paper 117) have pointed out the relation between vapor pressure and explosiveness of air gasoline mixtures as related to sewer atmosphere. No one appears to have correlated vapor pressures and explosion and ignition studies—of vapor—air mixtures in connection with flash point determinations. The machines in use are accurate enough for short cut results and are certainly practical in that they simulate burning lamp conditions.

EXPERIMENTAL.

CHARACTER OF THE MISCIBILITY OF GASOLINE-KEROSENE FRACTIONS.

Among the questions raised in connection with the litigation referred to were (1) the nature or degree of miscibility of gasoline and kerosene fractions in each other and (2) the influence upon the flashing properties of a kerosene in the event of miscibility proceeding at a practically instantaneous rate.

It is well known that petroleum fractions are miscible in one another. It cannot, therefore, be assumed that this has no limit in the case of any two restricted fractions in the absence of whole series of hydrocarbons of the original petroleum. Experiment upon various "gasolines" and a sample of kerosene using every precaution to prevent mechanical mixing, practically always resulted, in the laboratory, in a product with the same flash and burning point as a thoroughly mixed sample. This appeared to be independent of the time of contact, position of admission to the kerosene, depth of kerosene layer (up to one foot); portion of layer from which test sample was withdrawn, and the relative proportions used. This completely answers the ultimate degree of miscibility of the gasoline and kerosene fractions available for our experiments. The results also appear to answer the question of the nature of miscibility, i.e., it appears to be instantaneous. As these were laboratory experiments, they cannot be offered as conclusive, for they are subject to the criticism that an apparently instantaneous time factor in the laboratory may be quite a different matter in the plant where great volumes and depths of tank are factors modifying the situation. Also it is well known to refiners of light boiling oils such as petroleum and coal tar hydrocarbons that mixing or blending in tanks even as small as 10′ x 12′ does not proceed automatically upon mere addition, but air agitation is necessary to insure completeness of mixing. We possess no exact knowledge on this subject and carried out no experiments in view of the trouble and expense attached thereto and because of the existence of definite experience that miscibility has a real time factor in large scale operations. As an illustration, petroleum distillate tanks have been known to test 45° Bé. at the bottom, 47° at middle and 49° at top. Again, pumping a light gravity oil into a 250-barrel tank containing heavier oil, inlet at bottom, showed when 240 barrels were in tank, 58° Bé. at bottom, 61° at middle and 68° at top; quantities used not known.

THE INFLAMMABILITY OF GASOLINE-KEROSENE MIXTURES.

The following results indicate the effect upon the flash and burning behavior of kerosene used by admixtures of gasoline at ordinary temperature. These experiments were carried out in an ordinary graniteware cup, using 100 cc. of oil each time. In these experiments, "low test" gasoline was used. The kerosene was measured in a 100-cc. graduate, and the gasoline up through 3% was measured in pipettes. From 5% on the gasoline was measured in the cylinder. With 100% kerosene a lighted match when applied quickly or very slowly was extinguished. With 99% kerosene and 1% gasoline the match flashed when added, but was extinguished when immersed in oil. With 3% gasoline and 97% kerosene, there was a flash more noticeable than the 2% mixture but the match was also extinguished when immersed in the oil. With 5% gasoline and 95% kerosene the oil flashed violently but did not burn. With 7½% gasoline and 92½% kerosene the oil flashed with a puff and readily burned when the lighted match *touched* it. With 12½% gasoline and 87½% kerosene the oil flashed with a loud puff and readily burned when the lighted match *approached* it.

Flash and Burning Points of Kerosene-Gasoline Mixtures.

Since so little is said in the literature about the effects of highly volatile petroleum fractions upon the flash and burning point constants of ordinary kerosene, we have determined the influence upon them in the case of a kerosene mixed with "low test" gasoline, "high test" gasoline and "petroleum ether."

Both the open and closed cup methods were used for flash point determinations.

The open cup test depends not only upon the skill of the operator but also the condition of the atmosphere and is affected by the slightest current of air. It is a reliable test, however, for such cases where an oil will be heated in open vessels to determine to what degree it can safely be heated without ignition by flame. With such a case the open test is an actual imitation of the real conditions and therefore fully warranted.

The flash point of kerosene should be high enough so that it will insure safety when the kerosene is burned in a lamp. Danger from a petroleum lamp can occur if the flame flashes back into the petroleum reservoir by accident or carelessness and ignites the vapors which had accumulated in the closed basin (Am. Soc. Test. Mat. 10, p. 464, 1910).

The Foster Cup Method is the standard method for determining the point of kerosene prescribed by law in Ohio, which states that kerosene shall not flash below 120° F. by this method. (Allen's Commercial Organic Analysis III, p. 127.)

Method of Using Open Cup Procedure.—For determining the open cup flash and burning points an apparatus similar to the Cleveland Open Cup was used with the addition of a casing to more efficiently prevent drafts of air. A sketch of this apparatus follows (Fig. 3). It consisted of a 40 cc. nickel crucible "A" whose upper surface measured about 4 cm. in diameter. This was placed inside a large 120 cc. nickle crucible "B" which contained sand to a depth of about 2 cm. This sand bath was then supported in the flame guard "C" by means of the asbestos board "D" which had a hole cut in its center to admit "B" as shown. The flame protector was supported on the tripod stand "E." Heat was supplied by the Bunsen burner "F" as shown. The whole apparatus was protected from air current by the casing of thin sheet copper

"G." A thermometer was held suspended in the oil by means of a burette clamp on a ring stand. An ordinary mouth blow pipe connected to the gas cock by rubber tubing was used as a tip for the flame to be passed over the oil.

After thoroughly shaking the oil sample, the small crucible was filled to within ½ cm. of the brim, this required thirty-two cubic centimeters. The thermometer was then lowered by sliding the burrete clamp on the ring stand. The thermometer was held at such a height by the clamp that when the clamp was lowered to the top of the copper shield the bulb was totally immersed in the center of the oil to the same depth each time. Heat was then applied by means of the Bunsen burner, so that the temperature rose at the rate of 5° C. per minute. The test torch was then lighted and the flame made ½ cm. in length. As the flash point was approached, for every degree rise in temperature the flame was passed slowly over the cup horizontally about 1 cm. above the surface of the oil and near the thermometer.

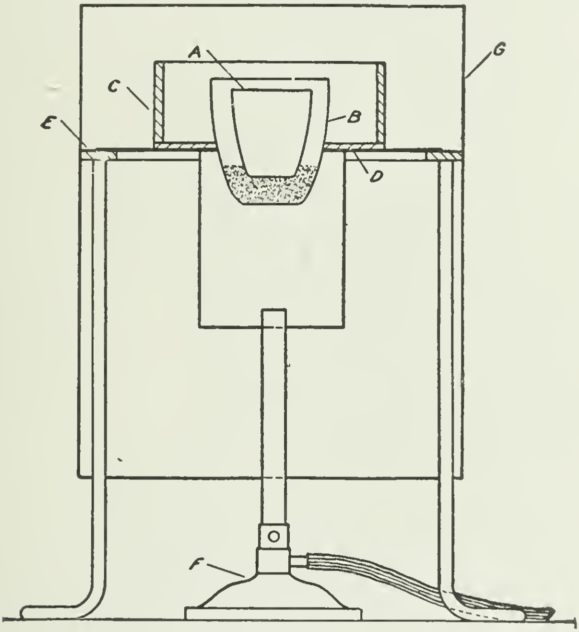

Fig. 3.—Open Cup Oil Tester.

After obtaining the flash point, the test flame was still passed over the oil in a similar manner until the oil permanently ignited. The thermometer was then quickly raised from the oil and the flame in the cup extinguished by excluding the air by means of a solid piece of asbestos board placed over the cup.

All open cup temperatures were read on a 400° Centrigrade thermometer. The calibration of this thermometer was checked up by comparing its two fixed points 0° and 100° C. with a standard thermometer. To fix the zero point, the thermometer was surrounded by melting ice and the reading compared with that of the standard thermometer under the same conditions. The 100° point was then determined by placing the bulb of the thermometer used in the neck of an Erlenmeyer flask containing boiling water so that it was entirely surrounded by steam and the reading on the thermometer was noted, in comparison with that of the standard thermometer under like conditions. The barometer reading was also taken and corrections applied. The uniformity of the bore was checked up by detaching a thread of mercury and measuring its length throughout different portions of the stem between 0° and 300°. Required corrections were applied throughout all of the work.

For each succeeding test the warm sand from the test preceding was emptied from the cup and fresh sand put in so that all tests were made under exactly the same conditions. The cup was then cooled by immersion in cold water and thoroughly dried.

In testing mixtures where the flash point was below room temperature, the sand was omitted from the bath and cracked ice, or ice plus calcium chloride, when necessary, was substituted. Cases where this was required are marked on the table by an asterisk (*).

Method of Using Closed Cup Procedure.—A standard Foster Cup apparatus was used. Its thermometer was checked against U. S. Bureau of Standards Thermometer No. 4576. The method of procedure in determining the flash point by the Foster Cup was as described in Stillman's Engineering Chemistry, pp. 448-449. This was as follows: The thermometer with its mounting was removed from the oil cup. The oil cup containing the flashing taper was removed and the open water bath half filled with water. The flashing taper was then removed from the cup and, after thoroughly shaking the oil in its container, it was carefully poured into the cup at the place of the flashing taper wick holder until it just reached the gauge mark at the thermometer hole. The cup was then tipped so that the oil flowed away from the gauge, it was then gradually restored to the horizontal position. If the surface again adhered to the mark, it was all right; in case it did not, a little more oil was added. The flashing taper was then adjusted to give a flame "¼ inch" in height. The oil cup was now set in the water bath, the flashing taper returned to its place, the conical thimble inverted around it and the thermometer returned to its place upon the cup, care being taken that the latter was pushed down upon the cup as far as possible. The lamp beneath was half filled with alcohol, lighted, and placed beneath the water bath. The wick was so adjusted that the temperature of the oil was raised at the rate of "2° F." per minute. When the thermometer read 90° the flashing taper was lighted and closely observed. In tests where the flash point was 90° or lower, the flashing taper was placed in the cup before applying heat from below. In several instances as previously mentioned in the open cup method, the mixtures flashed below

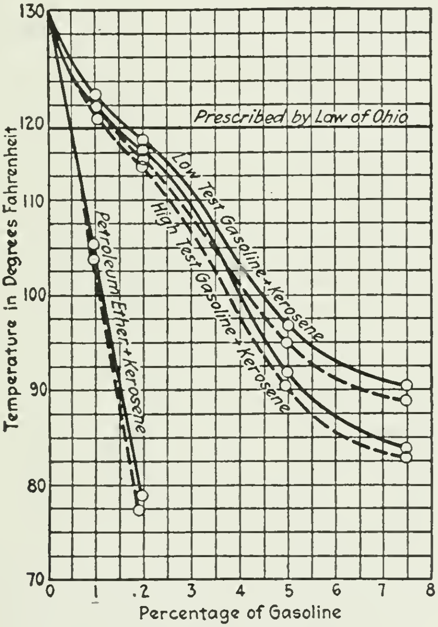

FIG. 4.—Open Cup Flash Point Curves.

room temperature. In these cases cracked ice was placed in the water bath and the test repeated. These cases are shown by an asterisk (*) in the tables.

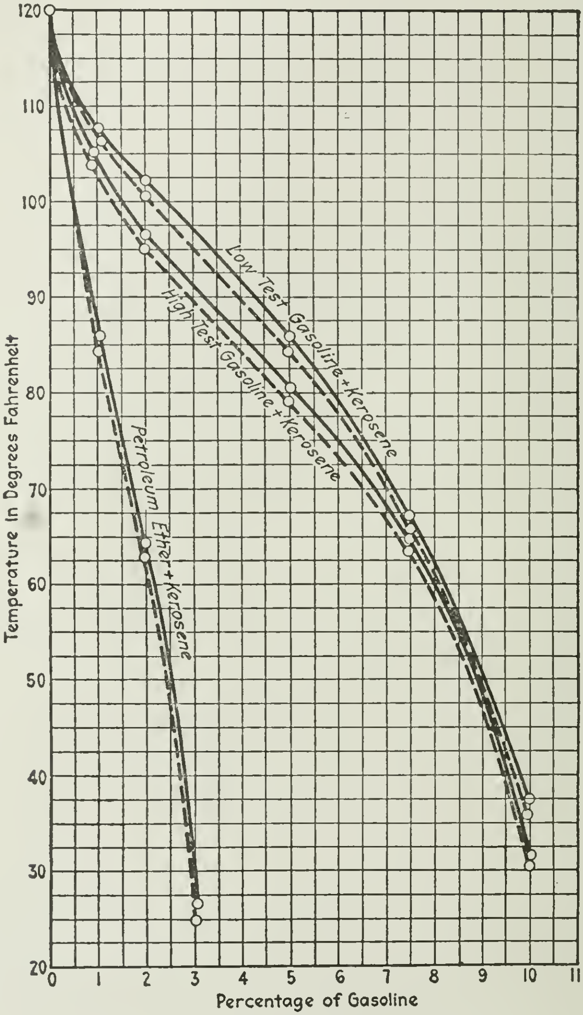

Fig. 5.—Foster Cup Flash Point Curves.

A rather peculiar phenomena took place in the closed cup flash test. When the oil was previously cooled by ice, it flashed some 5 or 6 degrees higher than when not previously cooled. Thus, in the case of the 7½% mixture of high test gasoline and kerosene, when ice was not used in the bath it flashed at 75° F. (as soon as the pilot was lighted), however, when previously cooled by ice, the mixture had to be heated to 84° F. before flashing, the pilot burning constantly. This may be due to the design of the cup. The flashing taper when in place forms a draft so that when lighted while the oil was below 75° the volatile gases liberated would pass out through the opening and escape, whereas when no light was present the draft was not so great and consequently the concentration of highly volatile vapors in the cup would be greater so that when a flame was admitted at 75° the concentration of the gases was sufficient to extinguish it.

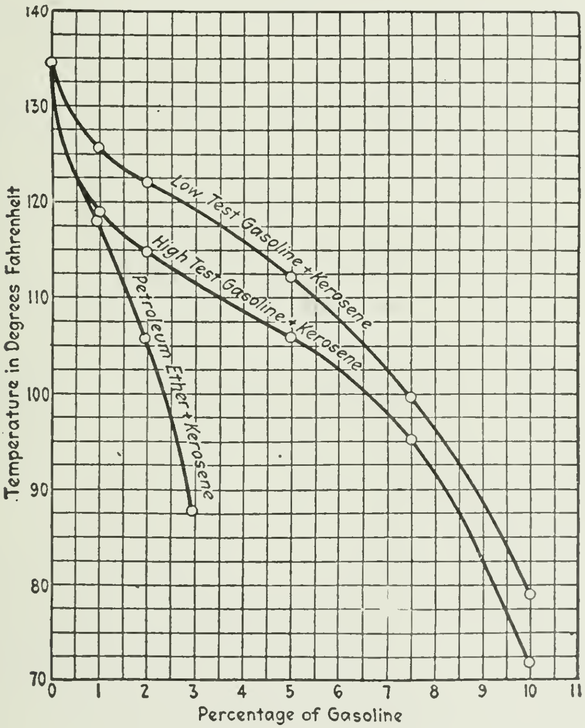

FIG. 6.—Burning Point Curves.

All flash points in both open and closed cup have been corrected for normal barometric pressure, 760 mm., by Table I. (U. S.

Bureau of Mines Technical Paper No. 49). In order to show to what extent the barometric pressure affects the flash point, the solid curves have been drawn which represent the flash points when corrected for barometric pressure and the dotted curves which do not include barometric pressure correction.

No corrections for pressure have been applied to the burning point results as no correction table was available.

The kerosene before being used in these tests was filtered through filter paper to remove all traces of moisture fog.

Discussion of Results.—The tables and accompanying curves (Figs. 4, 5 and 6) shows the results obtained by these tests. Blank spaces in the tables show that the tests were either unable to be carried out because of the low temperature required or the test was considered unsafe.

FLASH AND BURNING POINTS OF "' LOW TEST " GASOLINE AND KEROSENE.

Gasoline by volume	0%	1%	2%	5%	7.5%	10%
Kerosene by volume	100%	99%	98%	95%	92.5%	90%
Open Cup Flash Point, ° F.:						
Without barometric correction	118.4	105.8	100.4	84.2	65.3*	35.6*
With barometric correction	120.0	107.4	102.0	85.8	67.0*	37.2*
Foster Cup Flash Point, ° F.:						
Without barometric correction	128.9	119.3	114.5	95.0	88.6*	....
With barometric correction	130.5	120.9	116.1	96.6	90.2*	....
Burning point ° F	134.6	125.6	122.0	112.1	99.5*	78.8*

*Shows that freezing mixture was used in bath.

GASOLINE USED—" LOW TEST."

Specific Gravity = 0.738 = 59.7° Bé. at 15.5° C.

KEROSENE USED.

Specific Gravity = 0.815 = 41.8° Bé. at 15.5° C.

FLASH AND BURNING POINTS OF "HIGH TEST" GASOLINE AND KEROSENE

Gasoline by volume	0%	1%	2%	5%	7.5%	10%
Kerosene by volume	100%	99%	98%	95%	92.5%	90%
Open Cup Flash Point, ° F.:						
Without barometric correction	118.4	104.0	95.0	78.8	63.5*	30.2*
With barometric correction	120.0	105.3	96.3	80.1	64.8*	31.5*
Foster Cup Flash Point, ° F.:						
Without barometric correction	128.9	118.3	114.0	90.5	82.6*	....
With barometric correction	130.5	119.6	115.3	91.8	83.9*	.....
Burning point ° F	134.6	118.8	114.8	105.8	95.0*	71.6*

*Shows that freezing mixture was used in bath.

GASOLINE USED—" HIGH TEST."

Specific Gravity = 0.703 = 69.15° Bé. at 15.5° C.

KEROSENE USED.

Specific Gravity = 0.815 = 41.8° Bé. at 15.5° C.

FLASH AND BURNING POINTS OF PETROLEUM ETHER AND KEROSENE.

Petroleum ether by volume	0%	1%	2%	3%
Kerosene by volume	100%	99%	98%	97%
Open Cup Flash Point, ° F.:				
Without barometric correction	118.4	84.2	62.6*	24.8*
With barometric correction	120.0	85.8	64.2*	26.4*
Foster Cup Flash Point, ° F.:				
Without barometric correction	128.9	103.7	77.3*	.....
With barometric correction	130.5	105.3	78.9*	.....
Burning point, ° F	134.6	118.4	105.8*	87.8*

* Shows that freezing mixture was used in bath.

PETROLEUM ETHER USED.

Specific Gravity = 0.635 = 89.0° Bé. at 15.5° C.

KEROSENE USED.

Specific Gravity = 0.815 = 41.8° Bé. at 15.5° C.

In the open cup method the flash points of the "high test" gasoline-kerosene mixtures were naturally lower than the corresponding mixtures of the "low-test gasoline," being a minimum of 2.1° F. less at 1% and 7.5% gasoline and a maximum of 5.7° F. less at 10% gasoline. The fact that a minimum difference occurred at both 1% and 7½% shows that the open cup flash point of the "low" and "high" test gasoline-kerosene mixtures did not fall at the same rate with increase in percentage of gasoline.

In the case of the closed cup the flash points for mixtures above 2% high and low test gasoline regularly fall at different rates with increase in concentration of gasoline as shown by the curves falling away from one another. This shows that the open cup method does not bear a constant relation to the closed cup method or, graphically speaking, their curves are not the same shape.

The flash point of the petroleum ether-kerosene mixtures decreased regularly with increase in concentration of petroleum ether so that in the case of the open cup method the curve was almost a straight line and in the case of the closed cup method it was a straight line.

The lowering in the flash point of kerosene by adding petroleum ether was remarkable. In the open cup method 2.9% petroleum ether lowered the flash point of the kerosene about 88.5° F. or as much as for 10% high test gasoline. In the case of the closed cup method 1.8% petroleum ether lowered the flash point about 47° or the same as for 7.5% of high test gasoline.

The decrease in the open cup flash points for the gasoline-kerosene mixtures was somewhat over 1.3 times as much as the decrease in the burning point for the same concentration of gasoline. For example, 10% low test gasoline lowered the flash point 82.8° whereas it lowered the burning point but 55.8°. Ten per cent high test gasoline lowered the flash point 88.2°, and the burning point 63.0°.

In the case of the petroleum ether-kerosene mixtures, the flash point was lowered just about twice as much as the burning point, e.g., 3% petroleum ether lowered the flash point 93.6° and the burning point only 46.8°.

The burning point curves are of about the same shape as the open cup point curves, showing that in the case of the high and low test gasoline-kerosene mixtures the burning points did not decrease at the same rate as each other. In making up mixtures, the order of addition did not affect results, though such a difference is possible for a time at least on a larger scale.

It is evident that the flash and burning points of kerosene are greatly reduced by small admixtures of more volatile petroleum fractions such as gasoline so that grave danger can easily result therefrom.

Distillation Tests of Fractions Used.—In order that the oils used in this work may be identified the specific gravity was determined by the hydrometer method and distillation cuts of the oils obtained.

The distillations were carried out as prescribed in Technical Paper No. 166 of the Bureau of Mines, with the addition of reading the number of cubic centimeters distilled every 5° rise in temperature as recorded by the thermometer as well as readings at every 10 per cent by volume.

The 250 cc. flask connected with the condenser was filled with 100 cc. of the oil measured in a 100 cc. cylinder. The same cylinder was then used as the receiving vessel. In the kerosene and gasoline distillation the flask was covered with an asbestos box; this being omitted in the distillation of the petroleum ether. Heat was applied so that the distillation proceeded at the rate of about 5 cc. per minute; the entire distillation requiring about 25 minutes. The thermometer used in these distillations was the same as that previously described as being used for the open cup flash and burning point determinations. Besides applying the calibration correction, a correction was added for the emergent stem of the thermometer, varying up to 2.5° C. Formula for corrections was obtained from Physico Chemical Tables, Castell Evans, vol. 1, p. 126, J. B. Lippincott Co., Philadelphia, Pa. The condenser trough was filled with cracked ice to insure complete condensation. There were from two to three cubic centimeters of residue left in the flask at the end of each distillation. The results obtained by the distillations are shown in the tables and curves following. Fig. 7 contains all of the oils in order that a comparison may be made between them.

No stem temperature correction was necessary in the case of the distillation of the petroleum ether since the last fraction boiled off at 103° C. as recorded by the thermometer and the emergent stem began at 105° C. in all cases.

The solid curve represents the distillation when stem temperature corrections have been applied. The dotted curves represent the distillation without stem temperature correction.

E. W. Dean in his report on gasoline distillation, Bureau of Mines Technical Paper No. 214, p. 29, plots the dry point on the

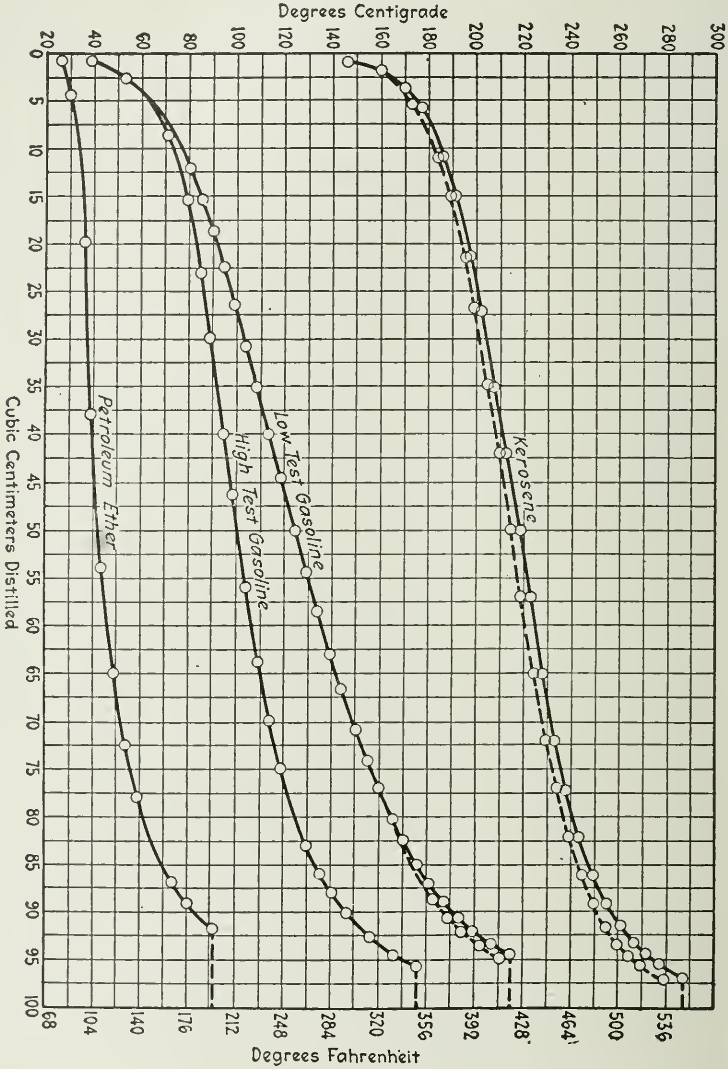

Fig. 7.—Distillation Curves.

100 cc. ordinate. This is not exactly correct since the dry point is always obtained before 100 cc., the initial volume of gasoline have

Temp. ° C.	Temp. ° F.	Kerosene.	Low Test Gasoline.	High Test Gasoline.	Petroleum Ether.
24	75.2	....	....	....	1.0
29	84.2	....	....	....	4.5
34	93.2	....	....	....	20.0
38	100.4	....	1.0 cc.	....	....
39	102.2	....	....	....	38.0
41	105.8	....	....	1.0 cc.	....
44	111.2	....	....	....	54.0
49	120.2	....	1.5	....	65.0
54	129.2	....	2.0	....	72.5
59	138.2	....	3.0	....	78.0
64	147.2	....	5.0	4.0	....
69	156.2	....	9.0	5.5	....
71	159.8	....	....	10.0	....
74	165.2	....	10.5	....	87.0
79	174.2	....	12.5	15.5	89.0
84	183.2	....	15.5	23.0	90.0
89	192.2	....	19.0	30.0	91.0
92	197.6	....	....	....	91.5
94	201.2	....	22.5	40.0	....
99	210.2	....	26.5	46.5	....
104	219.2	....	30.5	56.5	....
109.2	228.6	....	35.0	64.0	....
114.2	237.6	....	40.0	70.0	....
119.3	246.8	....	44.5	75.0	....
124.5	256.1	....	50.0	81.0	....
129.6	265.3	....	54.5	83.0	....
134.7	274.5	....	58.5	86.0	....
139.8	283.7	....	63.0	88.0	....
145.0	293.0	1.0	66.5	90.0	....
149.1	300.4	....	71.0	92.0	....
155.3	311.5	1.4	74.0	93.0	....
160.4	320.7	1.8	77.0	94.0	....
165.6	330.1	2.4	80.0	94.5	....
170.8	339.4	3.3	82.0	95.0	....
175.9	348.7	5.5	85.0	96.0	....
181.2	358.2	8.5	87.0	....	....
186.3	367.3	11.0	89.0	....	....
191.5	376.7	15.0	91.0	....	....
196.7	386.1	21.5	92.0	....	....
201.9	395.4	27.0	92.5	....	....
207.1	404.8	35.0	93.0	....	....
212.4	414.3	42.0	93.5	....	....
217.7	423.7	50.0	95.0	....	....

Temp .° C.	Temp. ° F.	Kerosene.	Low Test Gasoline.	High Test Gasoline.	Petroleum Ether.
222.8	433.2	57.0	....	....	....
228.1	442.5	65.0	....	....	....
233.4	452.1	72.0	....	....	....
238.7	461.7	77.0	....	....	....
244.0	471.2	82.0	....	....	....
249.3	480.7	86.0	....	....	....
254.6	490.3	89.0	....	....	....
260.0	500.0	91.5	....	....	....
265.3	509.5	93.5	....	....	....
270.3	518.5	94.5	....	....	....
275.7	528.2	95.5	....	....	....
281.3	538.3	96.0	....	....	....
286.9	548.4	97.0	....	....	....

DISTILLATION TESTS IN TEN PER CENT. CUTS.

Volume in Percentage.	Petroleum Ether.		High Test Gasoline.		Low Test Gasoline.		Kerosene.	
	° C.	° F.	° C.	° F.	° C.	° F.	° C.	° F.
10	31.0	87.8	71.0	159.8	73.0	163.4	185.0	365.0
20	34.0	93.2	83.0	181.4	90.0	194.0	195.6	384.1
30	36.5	97.7	89.0	192.2	103.7	218.6	204.2	399.5
40	39.5	103.1	94.0	201.2	114.2	237.6	210.3	410.4
50	43.0	109.4	101.0	213.8	124.4	256.1	217.6	423.6
60	47.0	116.6	106.0	222.8	136.7	278.0	225.1	437.0
70	52.0	125.6	114.2	237.6	149.0	300.2	232.3	450.1
80	61.0	141.8	123.9	255.0	165.6	330.1	241.5	466.6
90	84.0	183.2	145.0	293.0	190.0	374.0	256.9	494.5

been received. Plotting this, results in the curve ending in a sharper bend than it should. To obviate this error, we have plotted the actual dry point on the curve where it is observed and extended a straight horizontal dotted line to the 100 cc. ordinate in order to show where the curve would come had Dean's method of plotting been used. This actually represents approximate loss in distillation.

For convenience, the ordinates are labeled both in Centigrade and Fahrenheit readings.

We are indebted to Mr. Frank C. Vilbrandt, Instructor of Industrial Chemistry in this laboratory, for material assistance in the search of the literature for data covering the ground of this report and kindred topics.

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Laboratory of Industrial Chemistry,
The Ohio State University.
Columbus, Ohio.
July 2, 1919.

THE OHIO STATE UNIVERSITY
July, 1921

THE ECONOMICAL UTILIZATION OF LIQUID FUEL

BY
CARL A. NORMAN

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BULLETIN No. 19
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A

THE ECONOMICAL UTILIZATION
OF
LIQUID FUEL

BY

CARL A. NORMAN
Professor of Machine Design
The Ohio State University

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FOREWORD AND ACKNOWLEDGMENTS.

The people of the state of Ohio (to judge by the registration of passenger cars and trucks,) must now spend from 80 to 100 million dollars annually on automotive fuel. By the application of the principles of fuel economy to engine design and by some interest on the part of the general public in the demands of fuel economy, it is safe to say that over one-half of this sum could be saved.

No apology need, therefore, be offered for the attempt of the Engineering Experiment Station of The Ohio State University to publish a bulletin dealing in a fairly broad way, with the question of the economical utilization of liquid fuel. An apology may be offered for the small amount of the experimental information here given which has been produced at the university. The author of the bulletin has, however, for many years, made the scientific principles underlying prime-mover economy his special study. He has also had an opportunity to take part in and interpret a broadly conceived series of experiments on the fuel economy of an automotive engine undertaken by the International Harvester Company and extending over a period of more than a year. For the authorization of these experiments, as well as of their subsequent publication in the technical press, great thanks are due to the International Harvester Company and to the manager of its experimental department, Mr. E. A. Johnston. For coöperation in transforming the scientific studies of the author into a practical tool to be used by engineers for prime-mover analysis, thanks are due also to a number of the young engineers of the above mentioned concern, some of whom are mentioned by name in the text.

Of the members of the teaching staff of the university who have collaborated in the preparation of the bulletin, Mr. Karl

W. Stinson and Miss Sada A. Harbarger should be especially mentioned for detailed verification and proof-reading throughout; Messrs. Wm. T. Magruder, James R. Withrow, and Dana J. Demorest, members of the Experiment Station Council, for general perusal of the manuscript and valuable suggestions regarding arrangement and improvements.

THE DIRECTOR OF THE ENGINEERING
EXPERIMENT STATION.

Columbus, June 15, 1921.

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PURPOSE OF THE BULLETIN

The purpose of this bulletin is to review the liquid fuel situation in the United States, as it has developed in the last few years; and to ascertain as far as can be done, (1) what the available petroleum resources of the country are according to best obtainable information; (2) what substitutes for petroleum have been proposed; (3) what the main reasons are for the shortages experienced; (4) what possibilities there are for relieving these shortages by economy and care in production and use of petroleum and by the systematic development of substitutes or new resources. The possibilities of economy, as discussed herein, are described almost entirely from the fuel users' point of view, especially from the point of view of possible improvements in the economy of the engines used. The results of comparatively recent experiments and investigations on this latter point are summarized as simply and briefly as possible, and explanations are given of the methods of operation and designs leading to improved economy. In all this discussion little original research by the author of the bulletin is presented. The bulletin is rather a compilation of references and information scattered over a number of years, and in a variety of periodicals.

In addition, there are set forth also the results of a few investigations, mainly of a somewhat theoretically scientific nature, which have been undertaken by the author in order to clear up certain points in connection with engine performance, a knowledge of which would appear useful to systematic development and understanding. No great originality is claimed for this part either, although the matter may not in all cases have been presented to American engineers in exactly the form given here.

An entropy chart, to be used for analysis and computation of combustion engine processes, is an addition to the bulletin.

PART I.

THE OIL-FUEL SITUATION

SUMMARY

In this part the extent of the shortage in domestic petroleum production in recent years is first given; then the total petroleum resources of the world are considered. It is found that if the oil fuel requirements of the United States are going to continue increasing, or simply even maintaining themselves at the present rate, then the country will have to depend to an increasing extent on sources of supply hitherto little drawn upon. The strong tendency at present seems to be to rely wholly on importation from the still very great oil resources of other countries. Possibilities of political complications arising out of the domination of the oil supply of these countries by foreign capital, however, make the dependence of this country for its oil supply on the outside world dangerous.

The shortage is found to be due mainly to the demand for gasoline created by the automobile. Possible substitutes for gasoline are discussed. Alcohol and benzol are mainly considered, and various sources and methods of production for these two fuels are scrutinized. The conclusion is that, while these fuels may play a perhaps not unimportant part in relieving the gasoline shortage, yet, there would seem to be slight possibility of producing them in quantities and at prices which would enable them to replace gasoline more or less completely.

The necessity for avoiding the use of petroleum fuel in stationary plants and in steam plants generally is emphasized; the convenience and the economies resulting from the use of oil fuel in connection with combustion engines operating on the injection principle on board ship, however, are admitted.

Finally, the shale oil resources of the country are discussed. The general conclusion seems to be (1) that these resources are sufficient to cover the liquid fuel demand of the United States for many generations; (2) that the cost of production is such as to render hardly justifiable, for a long time to come, prices hugely

in excess of those at present prevalent for petroleum; (3) that a good deal of the shale oil seems to lend itself even more readily to gasoline production than most of the present day petroleum. The necessity for developing a mining industry as large as the present coal mining industry for the purpose of supplying the country's liquid fuel need entirely by shale oil is, however, also accentuated; hence, the demand for economy in the use of oil fuel, if temporary stringencies and difficulties are to be avoided. Given, however, this economy, and systematic and rational development of the shale oil industry, it should be possible gradually to pass from petroleum fuel to shale oil fuel without very many strictures, in the course of the next 25 or 30 years.

ARTICLE 1.

RESOURCES AND CONSUMPTION OF PETROLEUM

SUMMARY: *In 1916 and 1917 a shortage of petroleum fuels first made itself felt in America. Efforts to educate the people to fuel economy were immediately started and the question of the improvement of engine performance and of the development of new fuels and fuel resources became of vital importance. Very wasteful methods of production were noted. The main source of fuel waste was found perhaps in the poor economy of the ever-increasing number of automobile engines.*

The main method of relief has, however, so far been only increased importation, especially from Mexico. The rest of the world now possesses untouched oil resources many times larger than those of the United States, but in most of these oil fields of the future British capital appears to have established control. The danger of a complete dependence of the United States on South American countries for its oil supply is pointed out, and the need for increased activity in the development of economy and new domestic liquid fuel resources is emphasized.

In 1916 and still more in 1917, a serious situation for the first time presented itself to the people of the United States: They were consuming more oil fuel than they were producing. In 1917 the country was at war, and its dependence on oil fuel was greater than ever. To remedy this shortage, energetic measures were resorted to under the direction of the National Government. The gasolineless Sundays introduced for fuel saving are still a recent recollection. An energetic drilling campaign was started. As a result, stocks actually accumulated in the first half of 1919. During the latter half of 1919, however, a material increase in consumption again took place. In the summer of 1920, the Secretary of the American Petroleum Institute made the following statement: "Between November first, 1918, and October first, 1919, stocks in refineries increased 6,333,786 barrels.[1] Since October first, 1919, the tide has been ebbing at an alarming rate — It is believed that the stocks of fuel oil in the mid-continent field are now negligible." (Memorandum presented to the Federal Trade Commission by the Secretary of the American Petroleum Institute). When the final accounting for 1919 came, it was found that the country had, in that year, imported 52,746,567 barrels of oil. Close to six million barrels, it is true, were exported.[2] Yet, an actual deficiency in domestic production of close to 47 million barrels appeared, a deficiency that had been made up almost entirely by imports from Mexico.

In the face of this situation, the oil industries, as well as the engine builders, were thoroughly aroused. Information about the threatened oil shortage was scattered broadcast. Studies of available resources and substitutes were made, and the question of economic utilization of oil fuel became perhaps the greatest question before the engineers of the country. Several startling facts were revealed.

In the first place, it was found that the actual oil reserve of the country was less than seven billion barrels, and since the rate of consumption for 1919 exceeded 400 million barrels, the United States, using its domestic resources alone, would exhaust its oil supply in less than fifteen years.

In the second place, the methods of production were found to be extremely wasteful. Mr. J. O. Lewis, of the Bureau of Mines, presented estimates showing that by prevailing methods of production,[1] 25 to 90 per cent of the oil in the ground was never extracted. Messrs. Chester G. Gilbert and Joseph E. Pogue, felt sure that less than 25 per cent of the petroleum under ground actually reached the pipe line and that by subsequent losses and wastes less than 20 per cent, perhaps as little as 10 per cent, of the oil in the ground was finally utilized.

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[1] All barrels in this bulletin are barrels of 42 gallons.

[2] While only six million barrels were exported in the form of crude oil, the total export of oil products from this country in 1919 was about ten times this amount. Of this quantity, according to the reports of the Bureau of Domestic and Foreign Commerce, about 23,400,000 barrels were illuminating oil, 14,000,000 barrels, gas and fuel oil; 6,200,000 barrels, gasoline; and 2,680,000 barrels, naphtha and other light distillate.

In the third place, the main reason for the increasing consumption was the enormous development of the automobile, and the fact that the automobile engine was found to operate in a very uneconomical manner, turning into power not over 5 to 10 per cent of the energy actually available in the fuel.

Finally, it was realized that much of the oil was used for purposes which could not be justified except by the fact that an enormous demand for gasoline created a surplus of less valuable oil products which had to be disposed of somehow. Oil was burned under boilers and in furnaces where only 7 to 8 per cent of the energy in the fuel was actually utilized, while with the most economical motors a utilization of over 30 per cent is possible. Very considerable quantities were consumed by locomotives at a rate of fuel utilization amounting to not over 5 to 6 per cent.

It was immediately felt that some of these conditions were remediable without injury to anybody. Though forbidding the burning of oil in furnaces and boilers might conceivably disturb the balance of the oil industry; or prohibiting some western railroads from burning oil fuel where oil fuel actually costs less than coal, might also cause disturbance; yet, the automobile engineer and the prime mover engineer in general could only gain by making their appliances more efficient and economical. Indeed, even the oil producer could have no real objection to the rational development of substitutes for petroleum as long as he could not possibly supply the amount of petroleum demanded from him. In this connection, the fact that vast areas of the country were found to be underlaid with an oil-carrying shale, capable of yielding by distillation, an oil fuel very nearly akin to petroleum and perfectly able to take its place, appeared of especial importance. The cost of production of this shale oil, at least from richer shale, moreover, did not seem prohibitive.

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[1] J. O. Lewis, Bureau of Mines Bulletin 102, page 25; Gilbert and Pogue, United States National Museum, Bulletin 102, Vol. 1, "The Energy Resources of the United States," page 69.

For some time the stage seemed set for a rational and systematic development of the fuel resources of the country and their economical utilization. Then suddenly, however, a new view of the situation presented itself. This view now seems to be gaining more and more preponderance in the public mind, to the detriment of these rational developments, and in spite of the fact that it may yet lead to complications of a political nature and on an international scale.

It was found that the United States, while being by far the greatest oil-producing country in the world, actually possessed only a small fraction of the world's petroleum resources. The most carefully scrutinized facts in this connection were brought out in an article by Mr. David White, Chief Geologist of the United States Geological Survey, in the Annals of the American Academy of Political and Social Science, for May, 1920 (page 111). The actual figures are given in Table 1. Of these figures, only those for the United States and for equally well-worked and explored areas in Europe possess fairly high degree of reliability. The rest of the figures are necessarily only extremely uncertain estimates. Nevertheless, they seem to have been arrived at by Mr. Eugene Stebinger, Chief of the Foreign Mineral Section of the United States Geological Survey, after a systematic and careful study of the geological formations and the actual conditions of production of the countries concerned, and after a comparison of these with similar conditions in the United States. Mr. White believes that the total figure given in Table 1, 43 billion barrels, is somewhat low, 60 billion barrels

TABLE 1.

OIL RESOURCES OF THE WORLD AS ESTIMATED BY EUGENE STEBINGER OF THE UNITED STATES GEOLOGICAL SURVEY.

Country or region	Relative Value	Millions of Barrels
United States and Alaska	1.00	7,000
Canada	.14	995
Mexico	.65	4,525
Northern South America, including Peru	.82	5,730
Southern South America, including Bolivia	.51	3,550
Algeria and Egypt	.13	925
Persia and Mesopotamia	.83	5,820
S. E. Russia, S. W. Siberia and the region of the Caucasus	.83	5,830

Country or region	Relative Value	Millions of Barrels
Roumania, Galicia and Western Europe	.16	1,135
Northern Russia and Saghalien	.13	925
Japan and Formosa	.18	1,235
China	.20	1,375
India	.14	995
East Indies	.43	3,015
Total	6.15	43,055
Total eastern hemisphere	3.03	21,255
Total western hemisphere	3.12	21,800
Total north of equator	5.20	36,400
Total south of equator	.95	6,655

being perhaps a more complete estimate for the total petroleum resources of the world.

Just what degree of accuracy is to be attached to this estimate it is, of course, impossible to say. As far as the United States is concerned, Mr. White thinks that a maximum estimate of its oil resources would be 10 billion barrels, or now somewhat less, so that the uncertainty would be 25 to 30 per cent. Actually the most active exploration campaign, carried out between 1908 and 1916, enlarged the visible oil reserve of the United States by only 1,200 million barrels (Gilbert and Pogue, pamphlet as above, page 68). This investigation seems to indicate that with regard to this country no very serious under-estimates should be expected.

The world's oil supply then is not inexhaustible and especially it is to be noted that there is no single country having a greater oil supply than the United States. If the oil consumption of the world should continue at its present rate, there would be oil enough for perhaps as much as a century; if the rest of the world were allowed to consume its oil without encroachment by the huge demands of the United States, it might have oil for two or three hundred years. On the other hand, the oil consumption in the United States is growing at a startling rate. With increased use of oil for automotive appliances and for ocean-going ships the rate of consumption for the world as a whole will, no doubt, also increase. It is possible, therefore, that the total oil resources of the world should be looked upon as guaranteeing a supply for only one to three generations. Nevertheless, this is a very long time.

And another point to be emphasized is that Mr. White's figures refer to oil available by present day methods of production and hence at a cost not vastly different from the present-day one. No business man would feel very much inclined to invest heavily in new and uncertain technical developments for the purpose of producing something, cheaply available also without these developments. But even from the point of view of national economic policy and statesmanship it would seem a very questionable course to try to prevent this country from securing its oil where it is to be had at least cost.

There is, however, another side to the question. The other countries of the world, noticeably Great Britain, have felt the need of safe, sufficient, and not too expensive oil-fuel just as keenly as has the United States. But while the United States had its own resources to draw on, Great Britain had no such resources. Moreover, in the older countries of Europe, long range statesmanship has through centuries of history been devoloped to a far higher degree of efficacy than in this land of yesterday. This statesmanship has, at least in Great Britain, had a steady and fostering influence on business policy and business enterprise. The consequence is that now, when the United States for the first time in its history feels the need of looking to the outside world for its oil supply, it finds Great Britain, or British capital, firmly entrenched in a majority of the oil regions. An editorial writer in the Financial News (London), of February 24, 1920, quoted by Mr. White, concludes with an estimate which he believes conservative, that Great Britain now controls no less than 75 per cent of the world's oil resources. Another British writer quoted by Mr. White directly taunts the Americans for their lack of foresight and their insane prodigality in wasting natural resources.

In some quarters, this British oil control is purely commercial; in others, it is also political. For instance, in Persia, Great Britain seems to have direct agreements with the government establishing an oil monopoly in the greater part of the country. In Mesopotamia, understandings with France establish the same oil monopoly for Great Britain and France. Holland established a monopoly in the Dutch East Indies, but the Dutch oil companies seem to be operating largely with British capital. Japan, however, restricts oil development to her own citizens; in fact, the list of countries imposing restrictions by some method or other on the free exploitation of oil by citizens of the United States (not specifically, of course), is such as to make one wonder whether it would not be best for the United States to forget the existence of the large resources of the outside world entirely. In any case, it would appear that the one field on which this country might rationally concentrate its attention is that of the North and South American continent. From Table I it will be observed that this field, exclusive of the United States and Canada, contains a total probable supply of about 13 to 15 billion barrels, or roughly, twice as much as the United States has, at the present time, available within its own boundaries. At the present rate of consumption this would mean a supply of about 50 years. Even in this field, however, Great Britain has already made itself felt. The British oil concerns operating in Mexico are well known, and British capital has long been a potent factor in South American devolopments. From our experience in Mexico we know that oil exploitation under the political conditions in Latin American countries is apt to meet with many obstacles. Lastly, if we depend on countries like Peru and Bolivia for our oil supply, an enemy in the Pacific might readily cut us off from that source of supply when it was most urgently needed.

Nevertheless, in spite of all these unfavorable things, the fact remains that somehow this country, in the midst of all revolutionary turmoils south of the Rio Grande, and in the midst of all prohibitive legislation, has succeeded in importing ever-increasing quantities of fuel oil from Mexico. Indeed, there is little doubt that such importation, not only from Mexico, but also from other countries south of us, is going to continue and grow.

On the other hand, if the countries south of us are going to be our main source of supply, there can hardly be any questioning of the advisability of the strictest economy in the use of petroleum fuel, and of diligent search for substitutes for such fuel, substitutes which may be produced from resources, or raw material, within our own borders.

While the possibilities of economy in the use of present fuels form the main subject of this bulletin, the question of substitutes should not be entirely overlooked.

To discuss either economy, or substitutes intelligently, it is necessary first to ascertain what oil fuel is used for and in what forms it is used.

ARTICLE 2.

FORMS OF OIL FUEL AND THEIR USES.

SUMMARY: *The forms in which oil fuel is used are, (1) gasoline, (2) kerosene, (3) gas oil and (4) fuel oil. The total yearly oil fuel consumption in this country now is about 400 million barrels and the gasoline consumption is one-fourth of this, about 100 million barrels. Gasoline, originally a by-product from kerosene production, has now become the most important single product and is demanded in quantities exceeding the natural supply in crude oil. In consequence, heavier and heavier material is mixed into the gasoline without yet meeting the ever-increasing automotive demand.*

There is available little more than half as much kerosene as gasoline, and kerosene can not be looked upon as a source of permanent relief. Gas oil is used, needlessly, for the purpose of enriching the candle power of illuminating gas. It should be turned to other uses. Fuel oil, constituting nearly half the amount of the original crude oil, might be looked upon as a great source of fuel for automotive purposes. Considerable parts thereof are now used, unjustifiably, for stationary steam plants and furnaces. The railroad demand and especially the marine demand are, however, to be reckoned with. Both involve very wasteful methods of utilization. Highly economical injection engines should be installed on board ship. The question in what form the automobile could use fuel oil, if it were made available for automotive uses in great quantities, is as yet an unsettled one. Only the steam automobile can burn it without radical change of either fuel or engine plant. The development of an automotive injection engine suitable for fuel oil is undoubtedly possible and proceeding, but is not hailed with enthusiasm by manufacturers having heavy investments in apparatus for producing the present-day type of automotive engine. Conversion of fuel oil into gasoline by cracking or chemical treatment is pos-

sible, and may be resorted to in increasing measure. It, however, means spending material and labor to make fuel oil suitable for consumption by an engine at present much less efficient and economical than the injection engine.

The various forms into which crude oil is converted and the quantities produced of each are given in Table 2.[3]

In addition to the products enumerated in this table, there is a great number of secondary products such as paraffine wax, vaseline, grease, etc., which, while of commercial importance, must nevertheless be regarded as mere incidentals from the fuel point of view. In fact, a great number of these products could be eliminated entirely, if the fuel situation demanded it. The same cannot be said about lubricating oil. While it is possible to exist without liquid fuel, all industry would come to a standstill without lubricants. Doctor Pogue and others have emphasized that if there ever should be a choice between lubricating oil and liquid fuels, the lubricating oil would have to come first.

The fuels themselves fall into four classes: Gasoline, (including naphtha and very light distillates); Kerosene, or illuminating oil; Gas oil, and Fuel oil. In addition to the oils already mentioned, an especially safe and high grade illuminating oil, (not usually classed as kerosene), such as light-house oil or signal oil, may be produced. The total production of such oil for certain specific purposes is, however, small and hence does not need to concern us. The four main classes will be considered in order.

TABLE 2.

THE VOLUME RELATIONSHIPS OF THE PETROLEUM INDUSTRY IN 1919.

	Millions of Barrels	Per Cent of U. S. Production of Crude	Per Cent of Total Oils Run to Stills
U. S. Production of Crude Petroleum...	378	100	.....
Imports of Crude Petroleum............	53	14	.....
Exports of Crude Petroleum............	6	1.6	.....
Increase in Stocks of Crude Petroleum...	6	1.6	.....
Net Domestic Consumption of Crude Petroleum	419	111	.....

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[3] From Joseph E. Pogue, "Material Factors in the Gasoline Outlook," Automotive Industries, August, 1920, page 375, compared with data from the Bureau of Mines.

Crude Run to Stills (Refinery Consumption)	361	.....	.....
Other Oils Run to Stills	46	.....	.....
Total Oils Run to Stills	407	.....	100
Gasoline Produced	94	.....	23.1
Kerosene Produced	55	.....	13.5
Gas and Fuel Oil Produced	182	.....	44.7
Lubricating Oils Produced	20	.....	4.9
Miscellaneous Products	40	.....	9.8
Losses	16	.....	3.9

GASOLINE

In 1919, out of a total domestic consumption of 419 million barrels of crude petroleum, there were produced 94 million barrels of gasoline. Of this quantity of gasoline 6,200,000 barrels were exported. Approximately 88 million barrels were left in this country. The greatest part of this gasoline, by far, was consumed by automotive vehicles.

The total amount of gasoline produced, as shown by Table 2, constitutes 23.1 per cent of the amount of oil run through the stills. This is far more than the quantity of gasoline naturally contained in American crude oil. In fact, even the typical gasoline oils, such as Pennsylvania oil, contain 10 to 15 per cent of gasoline in the old sense of the word rather than 15 to 20 per cent. Texas and California oils contain even less gasoline than this. In earlier years, especially before the advent of the automobile, gasoline was a product too volatile and too ignitable to be permitted in ordinary lamp oil. As the problem before the oil industry was to produce an approved grade of kerosene, gasoline had first to be distilled off from the oil. This gasoline was largely a drug on the market and could be had at a low price. In 1910, the production of gasoline in the United States was only 15 million barrels, that is, only 6.7 per cent of the total amount of crude oil consumed. While this low percentage may not be an exact measure of the natural supply of gasoline in American petroleum taken as a whole, nevertheless, it gives a fairly good indication of what constitutes a natural unstrained production of gasoline from oil distilled mainly for other purposes.

Gasoline in those days was a product distilling over below 150 deg. cent., or 302 deg. fahr. Such volatile gasoline is today used only for exceptional purposes, notably for aviation. With an ever-increasing demand for automobile fuel, the idea of what constitutes gasoline has been stretched more and more into the field of kerosene, till today the standard motor gasoline of the United States government contains fractions distilling up to 437 deg. fahr. The most recent specifications from the Report of the Committee on Standardization of Petroleum Specifications, Bulletin 5, which became effective for government purchases on December 29, 1920, are as follows:

1. — Aviation gasoline, fighting grade:
    5 per cent distilled over between 122° F. and 149° F.
    50 per cent distilled over below 203° F.
    90 per cent below 257° F.
    96 per cent below 302° F.
    End point not higher than 329° F. Loss in distillation not to exceed 2 per cent.

2. — Aviation gasoline, domestic grade:
    5 per cent distilled over between 122° F. and 167° F.
    50 per cent below 221° F.
    90 per cent below 311° F.
    96 per cent below 347° F.
    End point 374° F., distillation loss 2 per cent.

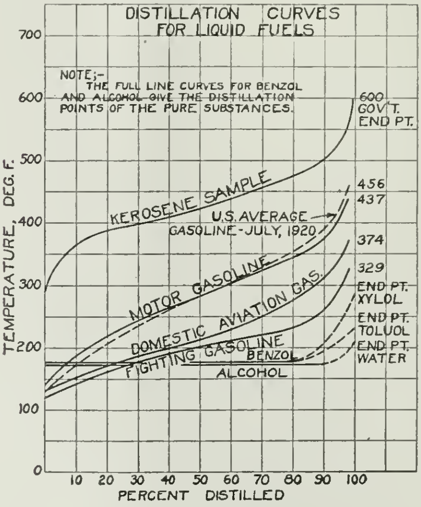

Fig. 1.

3. — Motor gasoline:
    First drop not over 140° F.
    20 per cent distilled over below 221° F.
    50 per cent below 284° F.
    90 per cent below 374° F.
    End point 437° F.
    95 per cent to be recovered as distillates in the receiver.

4. — Kerosene:
    The end point shall not be higher than 600° F.

In Fig. 1 these prescribed distillations are shown in the form of curves. As a result of the difficulties experienced by the industry in meeting earlier requirements, the present specifications for motor gasoline admit a lower volatility than before. Yet, a survey by the Bureau of Mines, covering a number of cities in the United States, shows that already in July, 1920, not only did commercially sold gasoline in many places fail to meet these more lenient specifications, but the average for the country did actually not meet the government standards. In Fig. 1 this average for the country is shown as a dotted curve. It will be noted that the discrepancy between this curve and the curve for government motor gasoline indicates a larger percentage of heavier fuel than permissible, for the end point is as high as 456 deg. fahr. To make up for this excess of heavy material a greater amount of very volatile material has also been added. The result is that the whole is a less homogeneous product.[4]

The main underlying cause of this practice is the fact that gasoline is bought and sold by specific gravity rather than by volatility. Actually, volatility rather than specific gravity is the factor influencing engine operation. In Fig. 1 are also shown the distillation curves for (pure) benzol and alcohol. Both of these fuels distill much below the average boiling point for the gasolines, yet they are both of higher gravity than gasoline. What the carbureting engine requires is a fuel which will be readily taken up and volatilized in the air. The property of the fuel in this respect is determined by two things: First, by its boiling curve, or vapor tension; second, by its so-called latent heat, that is, by the amount of heat it takes to evaporate the fuel. In no case, however, does the actual weight of a gallon of fuel enter, and this weight is what is meant by gravity. To produce a certain average demanded gravity it is possible to mix some light stuff and some heavy stuff. Only the light stuff, however, will readily evaporate; the heavy stuff remains liquid and when carried into the inlet passages and the cylinders of the engine causes all the troubles of unequal distribution, fouling with carbon, fuel leakage past the piston rings, etc., with which we are today only too familiar.

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[4] In Fig. 1 the distillation curves for alcohol and benzol are shown as straight lines. These lines correspond to the boiling points of the pure substances. Commercial mixtures containing water in the case of alcohol and toluol, xylol, etc., in the case of benzol, show a distillation curve which rises more gradually towards the heavier end.

Automobiles and Gasoline Consumption

There were in 1920 registered in the United States a total number of 8,887,572 automotive vehicles, or to be specific, passenger cars and trucks.[5] A number of states do not register trucks and passenger cars separately. On the basis of estimates by "Automotive Industries" for the year 1919, it would, however, appear reasonable to assume about 1,100,000 trucks and 7,800,000 passenger cars in the country at the present time. (See Automotive Industries, August 26, 1920, and January 13, 1921). Dr. Joseph E. Pogue (War Industries Board, Price Bulletin No. 36, "Prices of Petroleum and Its Products", in diagram, page 20), assumes a consumption of 7.2 barrels of gasoline per passenger car per year, and 47.5 barrels per truck per year. On this basis the gasoline consumption for the year 1920 would be 56 million barrels for passenger cars and 52 million barrels for trucks, a total of 108 million barrels.

Where is this amount of gasoline to come from? Mr. David White, in his oft-mentioned paper, suggests that it will never be possible to produce in this country over 450 million barrels of crude oil in one year, and that this peak of production probably will be reached in 1925. After 1925 a drop will set in. A hundred and eight million barrels is 20 per cent of 540 million barrels, and 20 per cent would be about all the gasoline which we, even with our present stretched ideas about what gasoline is, could extract out of the crude oil with old standard methods of distillation. Evidently, if the number of automotive vehicles is going to continue to increase at the present rate, new methods of converting crude oil into gasoline will have to be resorted to, even if importation does increase at an enormous rate.

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[5] These figures are taken from *The Automotive Industries* of Jan. 13, 1921, page 61. Higher figures have been seen in the public press.

There are at present in this country one passenger car or truck for every twelve people, that is to say, more than one automobile for every three families. There are certain states where there appear to be almost one car per family. South Dakota, for instance, possesses 5.24 members of population per car and Iowa, Nebraska, California and Kansas, all show figures of less than seven people per car. As the average number of persons to a family in the United States in 1910 was 4½, and is probably somewhat less now, it would appear that the "saturation limit", at least for passenger cars, has been nearly reached in such automobilized states as these. It is hardly to be expected, however, that such averages will be obtained for all states of the Union, certainly not for sections with a large industrial population, often foreign born. It would, therefore, seem questionable as to whether the total number of passenger cars existing in this country at any one time will double before the population has considerably increased.

It is more difficult, however, to estimate the maximum probable increase of trucks in the next few years. Nevertheless, to the author it seems as if an increase of 50 per cent in the total number of vehicles would suffice as an estimated maximum to be reached inside of the next five to ten years. Very much higher estimates than this have, however, been made. In fact, Mr. H. R. Cobleigh, Statistician of the National Automobile Chamber of Commerce, puts the probable registration as early as December 31, 1921, at 12 million passenger cars and motor trucks. (Memorandum submitted to the Federal Trade Commission by the Secretary of the American Petroleum Institute, "The Recent Increases in the Price of Petroleum and its Products, page 22).

Be this increase as it may, it is evident that the spread of the automobile puts a tremendous strain on the fuel situation. In fact, it is the main cause of the difficulties we are encountering.

Kerosene.

The government specifications for kerosene call only for a dry point below 600 deg. fahr., and leave the rest of the distillation curve wholly undetermined. Yet the fact that the flash point of the oil must not be lower than 115 deg. fahr., determines the lower limit of volatility very definitely. In fact, kerosene is a far more narrowly prescribed and circumscribed product than is gasoline. A sample distillation of kerosene meeting the requirements, given as a curve in Fig. 1, will probably not be very much deviated from by kerosene in general. In older specifications kerosene comprises fractions passing over between 150 deg. cent. and 300 deg. cent., that is, between 302 deg. fahr. and 572 deg. fahr. Nowadays gasoline has preëmpted a good many of the fractions that formerly would have gone into kerosene; in consequence, the end point has been pushed somewhat higher, (as the government standards show).

In 1919 were produced 55,000,000 barrels of kerosene. Of this quantity 23,400,000 barrels were exported. Thirty-two million barrels were left in this country. Ten years earlier, 1909, there were produced 40,000,000 barrels of kerosene, 25,800,000 barrels of which were exported and 14,200,000 barrels left for domestic consumption. The production of kerosene then increased very little in these ten years; and the consumption likewise very little, if compared with the consumption of gasoline.[6]

The point to emphasize is that this comparatively stationary production can in no sense be looked upon as offering any considerable amount of relief in the motor fuel situation. The domestic consumption of kerosene has increased, but has increased at the expense of exports and the total kerosene production now constitutes but little over one-half of the production of gasoline. Yet, the idea about what constitutes kerosene has nevertheless had to be stretched. Kerosene is a substance, produced especially for safe domestic use in illuminating apparatus, and to a somewhat smaller extent, in cooking apparatus. For these purposes it is excellent, but there is no reason whatever to single it out as a very suitable motor fuel. For old line carbureting engines it is not volatile enough. It requires more mixture-heating than does the present day gasoline, or it requires special "fuelizing" apparatus. The fuel converter of Ensign, to be described later, handles kerosene perfectly, and no doubt the fuel converter of the Packard Motor Car Company could readily be adapted to do the same thing. With such devices available, however, there is no particular need for limiting ourselves to the semi-volatile fractions now constituting commercial kerosene. Very likely the engines with fuel converters could handle even the heaviest fuel distillates at present produced. This situation is, however, rendered unnecessarily complicated by trying to keep all these fuel distillates apart. A much better policy might be to mix them all together under a common name of distilled motor fuel.[7] Carburetion, ignition, and starting in cold weather would be better on this mixed fuel than on kerosene alone.

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[6] Petroleum statistics are collected by many agencies and it is sometimes difficult to get figures from various sources to agree. The figure for the kerosene production in 1909 was taken from the thirteenth census, which, in general, applies to calendar years. The figure for exports was taken from the reports of the Bureau of Statistics, now the Bureau of Foreign and Domestic Commerce. These figures apply to fiscal years, ending June 30. Statistics from the Bureau of Mines, or the United States Geological Survey, also are carried out independently and usually apply to calendar years. The author does not feel competent to criticize the figures given and in general the statistics here offered will therefore have the accuracy of round numbers only.

Fortunately, there have been no very determined efforts to adapt passenger cars and trucks for operation on kerosene. There has, however, been a great deal of effort towards the adaptation of tractor engines and stationary farm engines to this fuel. In fact, the majority of these motors very probably are today intended for kerosene. The reason has naturally been the fact that the price of kerosene now is lower than that of gasoline. The more persistent the efforts are to take advantage of this fact for farm motors, the sooner the time will come when kerosene will sell at exactly the same price as gasoline. Indeed, the price of kerosene is now advancing much more rapidly than is that of gasoline. Taking the conditions in Kansas City as an illustration, we find that in the beginning of 1917 the tank wagon price of kerosene was 6.8 cents per gallon; that of gasoline, 18.2 cents per gallon. By June 30, 1920, the price of gasoline had risen to 26.2 cents, and that of kerosene to 17.6 cents. The percentage increase for gasoline was 44 per cent; for kerosene, 160 per cent. In the beginning of 1917, the price of kerosene was only 37 per cent of that of gasoline; in 1920 it had risen to 67 per cent. In that price movement the tractor demand has no doubt played a very important part; yet the tractor demand expressed in barrels is as yet a comparatively small item in our total oil fuel requirements.[8]

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[7] There are rumors that the oil industry has actually agreed on such a mixture as the standard motor fuel a few years from now.

Gas Oil

By gas oil is meant an oil used for carbureting, "water gas". In this process the oil is decomposed so that about 15 per cent remains as a liquid residue in the form of oil gas tar, and the other 85 per cent is changed into an integral part of the gas.

Such carburetion gives to the gas a somewhat greater heating value per cubic foot, which is, however, a secondary consideration. The main object of gas carburetion is to increase the luminosity of the old simple gas jet.

Such gas jets are not now used for illuminating purposes, as very much more intense light is secured by means of incandescent mantles. In the use of these the gas itself burns with a blue and almost invisible flame and no luminosity that might lead to the deposition of soot on the mantles is desired.

Unfortunately, the specifications for gas which have been incorporated in the contracts between city governments and gas companies have given no heed to this development. These contracts continue to demand a heating value which can be obtained only by carburetion. Yet, there is no need for this high heating value. Cooking can be carried on with equal convenience with a gas of lower heating value. For political reasons, or perhaps simply on account of inertia, the task of converting city authorities to a reasonable attitude in this matter has proved very hard. Consequently, oil fuel even in these days of shortage and difficulties, continues to be diverted into a channel where it is wholly unrequired. It may be admitted, however, that the total amount is not, comparatively speaking, very large, only 18,323,900 barrels[9] in 1920.

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[8] According to statements by Mr. H. H. Hill, Chemical Engineer of the Bureau of Mines, in a paper before the Independent Oil Men's Convention at Denver on September 30, 1920, there was a shortage of kerosene in 1919, as a result of unusual replenishment demands from abroad. In 1920, however, stocks of kerosene are said to have accumulated. The price figures used in the text, however, apply to conditions only a month or two before the reading of Mr. Hill's paper and should allow for the situation he speaks of.

Fuel Oil

Oil statistics usually give gas and fuel oil under one heading. The total amount of such oil produced in 1919 was 182 million barrels. This is an amount exceeding that of gasoline and kerosene combined. Here then we have a resource which would afford a considerable amount of relief in the automotive fuel situation, were it in large measure available. Our first step will then be to examine the uses to which gas and fuel oil are at present put, and to inquire into the legitimacy of such use. Such an examination has already been made as far as gas oil is concerned. Gas oil, however, is only a comparatively small item in the total amount.

The export of gas and fuel oils in 1919 amounted to 14 million barrels, again not a very significant amount.[10] Subtracting, however, this export and the gas oil, there still remain about 150 million barrels unaccounted for. In 1918 the railroads consumed 36,700,000 barrels and in 1917 roughly 42,000,000 barrels of fuel oil. The United States Navy, in the fiscal year ending June 30, 1920, consumed 5,531,932 barrels. The United States Shipping Board in 1920 called for 50 million barrels of fuel oil. Its program provides for 1,734 oil burning ships by 1922, and 30 million barrels of oil have been called for in the first half of 1921. When the American Merchant Marine, as we hope, will be what it ought to be, it will consume, according to a conservative estimate, between 70 and 80 million barrels of oil per year. The oil burning railroads and this merchant marine, plus the navy, will then require a total of 120 to 130 million barrels of oil. The comparatively small amount of 30 to 60 million barrels will be left available for other purposes. At the present time a far greater amount goes to other purposes, such as oil burning furnaces, oil burning steam plants, etc. The questions which now come up are these: (1) Which of these fields of consumption can be regarded as legitimate and what economies can be effected; (2) what prospects are there of enabling automotive appliances to operate on gas, or fuel oil?

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[9] According to estimates of the American Gas Association communicated by Mr. L. M. Fanning, Director Publicity and Statistics, American Petroleum Institute.

[10] From Reports of U. S. Bureau of Domestic and Foreign Commerce.

Oil Burning in Stationary Plants.

Oil burning in stationary steam plants and in furnaces can be defended in hardly a single case. Such consumption has occurred as a regular thing in regions where coal fuel is scarce, or as an emergency measure in other regions during artificial coal shortages, or exceptionally high prices of coal. The convenience of oil burning has, however, tended to make it permanent even after the emergency is past. One argument for oil burning in a furnace is that it gives a hot flame, easily regulated. Just as hot and almost as easily regulated a flame can, however, be produced by gas firing and nowadays especially by powdered coal. The satisfactory results obtained by this latter form of fuel for furnace work are now generally admitted. Also for stationary boiler purposes, the workability of powdered coal can be regarded as demonstrated. (See, for instance, "Power", Sept. 7, 1920, p. 358, and Jan. 21, 1919, p. 90.) However, as the science of burning coal economically and efficiently with or without automatic appliances is very well-known, there seems to be little need for discussion of the possibilities of generally substituting coal for oil for service other than marine. Oil burning should not be permitted in stationary steam plants, in the first place, because oil constitutes a particularly high grade and convenient form of fuel, without which most automotive operations would be impossible; in the second place, because there is only a limited amount of this high grade form of fuel available; and in the third place, because the steam plants utilize fuel extremely inefficiently.

All conversion of fuel energy into power is today not what it ought to be. Using the most efficient heat motors extant, we still waste two-thirds of the potential energy stored up in the fuel and turn into power only one-third. This maximum utilization is attained by so-called injection engines and can be approached by the most efficient high compression carbureting engines. A steam plant, however, is a wonder if it can utilize 18 per cent of the fuel energy. Though very good steam plants may perhaps attain a utilization of 15 per cent, the general run of good plants probably utilize only about 10 per cent. A steam locomotive turns into actual tractive power only 4 to 5 per cent.

Wasting the most valuable fuel we have at this rate is indefensible and must be stopped.[11]

Oil Burning on Railroads.

Oil burning on railroads has been adopted in regions like California and the southwestern states where oil is plentiful and coal has to be imported. As a locomotive is not the most comfortable place in the world to work in, the less strain there is put on the fireman and engineer, the safer are the people traveling on the railroads. There are far too many accidents on American railroads as it is. Oil burning does relieve the strain on the fireman very greatly. Instead of shoveling coal for all he is worth, he has simply to adjust the handwheel of a valve. One would almost feel disposed to say that this benefit derived from oil fuel, where commercial conditions have made it possible, would go far towards justifying its use. Nevertheless, it must not be forgotten that the steam locomotive wastes fuel worse than any other mechanical appliance. Moreover, from the cold point of view of dollars and cents and from the somewhat higher point of view of saving unnecessary labor, oil fuel in reality does not accomplish wonders. Whether with coal, or with oil, there must always be two men on a locomotive. Mechanical stokers, applied to locomotives so large that one man cannot fire them by hand, have been found perfectly successful. The savings effected by oil burning occur at fuel stations, in round houses, etc., and sometimes in the somewhat lower wages paid the firemen. Mr. John Purcell, of the Operating Division of the Santa Fé Railroad, has given the author some very interesting information which shows that the cost of fueling is approximately 70 per cent less with oil than with coal. The elimination of cinder removal, fire cleaning, boiler washing, etc., made possible by oil fuel, means a saving of about 10 per cent in the handling cost at terminals. All this is not negligible. Nevertheless, the argument of economy is not strong enough to offset the all-important fact that an oil burning steam locomotive simply destroys 95 per cent of the irreplaceable fuel it uses. It seems almost as a mere incident to this systematic destruction that it also turns 5 per cent into power.

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[11] Considering that coal is made available by hard human labor, as much might, in fact, be said also about coal. Nevertheless, the coal reserve is immense as compared with the reserve of oil fuel, and in any case, the question is here one of economy of liquid fuel rather than of solid.

It is, therefore, very much to be hoped that some way will be found to eliminate, or reduce, the use of oil fuel on the railroads of the West. One possible way may be by means of partial electrification of the roads with the aid of water power. Another way, not very much thought of yet, would be to introduce locomotives with oil burning apparatus of a very much more economical character. So-called Diesel electric locomotives have to some small extent, according to the author's impression, been experimented with by leading electrical manufacturing concerns. In these, the prime mover is a Diesel engine, the most economical heat engine known. This engine drives an electric generator, the current from which drives the motors on the axles of the locomotive. In spite of the electric transmission losses, such apparatus, designed to run on fuel oil, would appear to be economically sound and deserving of every attention. The same can be said of any attempt to substitute high efficiency oil motors for oil burning steam plants on locomotives. A very different judgment must be passed on the reported attempts of a prominent automobile manufacturer to introduce gasoline engine locomotives. To attempt to create a new demand for gasoline where no such demand at present exists, would appear to be a very unwise undertaking from the point of view of the fuel economic situation of the day.

Oil Burning on Ships.

Oil burning on board ships is at the present time gaining ground very rapidly, in connection with steam apparatus as well as with oil engines. As all steam motors on board ship are run condensing, which is not the case with the steam locomotive, ship plants present a heat utilization two or three times better than that of a locomotive. Even so, however, there is probably not a marine steam plant in existence which does not waste 83 per cent of the energy available in the fuel. A good many waste more. The main reason for oil burning is the labor saving effected. On naval vessels coal firing requires armies of firemen. The work, especially under battle conditions, is such as to tax the men almost beyond the limit of endurance. With oil, not only is the

TABLE 3

COMPARISON OF MARINE POWER PLANTS

I

1,000 b. hp.

	Single Screw Diesel	Single Screw Double Geared Steam Turbine		Single Screw Reciprocating Steam Engine	
		Coal	Oil	Coal	Oil
Total Engine Room Staff...	9	13	10	13	10
Relative total cost for fuel and engine room labor..	1	1.44	1.97	2.12	2.93

II

2,400 b. hp.

	Twin Screw Diesel	Single Screw Double Geared Steam Turbine		Single Screw Reciprocating Steam Engine	
		Coal	Oil	Coal	Oil
Total Engine Room Staff...	12	21	14	21	14
Relative total cost for fuel and engine room labor..	1	1.47	2.04	1.95	2.68

number of firemen enormously reduced, but the work then, instead of shoveling, barring, and poking, consists in adjusting a valve from time to time. In Table 3 is shown a comparison of the amount of labor required and the ratio of total expense for fuel and labor for three types of vessels using either coal or oil. (From "The Gas Engine", Jan. 1921). The cost is applicable to conditions met with on English ships and is based on prices both for fuel and labor that may not apply to American shipping. The number of men needed in the various cases will be very much the same the world over. It will be observed then, that on a 1000 hp. ship with reciprocating engines as well as with steam turbines, the total engine room staff is reduced from 13 to 10 by the change from coal to oil fuel. On a 2400 hp. ship the saving is even more marked, being from 21 men to 14 men. A still greater labor saving is, however, effected by introducing Diesel engines instead of steam motors. Only 9 men are needed on the smaller ship, and 12 on the larger. This is due to the fact that the boiler room staff is eliminated entirely.

As for expense, it will be seen that for steam apparatus, coal burning is cheaper than oil burning, except that on the smaller ship it appears to be somewhat more economical to use double-geared steam turbines and oil than reciprocating engines and coal. In all cases, however, the total expense is reduced very materially by the substitution of Diesel engines for steam apparatus.[12]

The saving in fuel and in labor costs is not the only item in favor of the Diesel engines. A very important consideration is the gain in cargo space for vessels of equal size which results from the absence of boilers and reduced space occupied by fuel storage. Diesel ships cost somewhat more than steam ships of the same size; yet it is found that the cost per ton of cargo space is actually less for the Diesel ship than for the steam ship.

From this review it would appear questionable whether oil burning on a *steam* ship can be economically defended, except by the argument especially applicable to conditions facing the new American merchant marine, that marine stokers or firemen in sufficient numbers cannot be secured. If by means of oil burning it is possible to keep ships manned and running which otherwise would not be manned, then of course, oil burning becomes a necessity. Moreover, the American merchant marine was conceived under the stress of war, and had to be produced with the manufacturing facilities most immediately available in the country. To supply the whole merchant marine with Diesel engines seemed to require facilities and experience which it would have taken years to develop. On the other hand, there were a number of firms able to design and deliver steam turbines at a rapid rate. From the point of view of permanent conditions these production facilities are to be very much regretted. Mr. M. L. Requa, former Oil Fuel Administrator of the United States, at a recent petroleum congress in Washington, stated decisively that steam equipment on board ships should really from now on be regarded as obsolete. From the point of view of the fuel situation of the world, this statement cannot be too strongly endorsed. Nevertheless, there are at the present time only 325 Diesel engines in actual operation on board ocean-going vessels and driving a total tonnage of 1,263,000 tons dead weight. This is a small part of the tonnage of the world, and a comparatively small fraction even of the American merchant marine. It will be a very long time before a large part of all shipping is equipped with the most economical motive apparatus now available. Under such conditions it would seem that we have to count on a marine demand as above stated of about 70 to 80 million barrels of fuel oil per year, for the American merchant marine alone. There seems to be no escape from this demand, if this marine is to be maintained under the American flag.

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[12] The fuel costs assumed in these tables are £5 per ton of coal, £10 per ton of oil for steam boilers, and £11 per ton of oil for Diesel engines. The saving effected by Diesel engines is in spite of this higher unit cost of fuel and is caused by actual fuel saving rather than by the reduction in labor expense.

Automotive Use of Fuel Oil.

All told then, at least half of the available supply of gas and fuel oil is preëmpted for unavoidable purposes other than automotive. Granted, however, a systematic campaign of education and a concerted rational fuel policy on the part of the oil and the automotive industries, not to say the public at large, there would nevertheless appear to be a great amount of fuel oil which could be diverted to automotive use. The question is, in what form should it be used?

Two solutions present themselves. One consists in developing automotive apparatus able to burn fuel oil. In England there are trucks operating satisfactorily even on coal. In this country, however, there has been little enthusiasm for the idea of adapting passenger cars and trucks to an essentially non-volatile fuel. Difficulties of distribution to all the thousands of fuel stations throughout the country present themselves to the oil manufacturer, and the difficult and expensive task of developing fuel-oil burning engines weighs heavily against the proposition in the minds of automobile builders. The author does not believe that these difficulties are insurmountable. Only scattered and insufficient effort has been devoted so far to the solution of this problem. If one-tenth of the energy devoted to the solution of ignition and carburetion problems was devoted to the problem of the oil burning injection engine, we should very likely have a satisfactory automotive engine of the latter type in a short time.

There is one form of automobile which could be made to utilize non-volatile fuel almost at once. That one is the *steam automobile*. In this bulletin a rather decided stand has been taken against steam motors in general. In automotive application the case of the steam motor is, however, very much more favorable than in any other. The heat utilization of an average automobile engine is not very much better than that of a steam locomotive. The reason for this is that an automobile engine in ordinary operation very rarely runs at full power, and very often runs at only a small fraction of its maximum power at the speed held. At these reduced loads the present day carbureting combustion engine is extremely inefficient. On the other hand, the steam engine can be run so as to maintain its efficiency very much better at reduced load. Steam automobiles in actual operation have shown a fuel consumption certainly not higher than that of combustion motor automobiles of the same weight. If now, in addition, the steam automobile can be made to run on the least volatile grades of fuel, it would seem to have a strong argument in its favor, at least until an automotive injection engine or other type of highly economical motor, arrives and presents a fuel consumption entirely out of the range either of the steam, or of the present day gasoline automobile.*

As for distribution of fuel oil and heavy distillates it would appear that a beginning at least could be made by distributing such fuel to farmers for tractor driving. The farmer, in a day's plowing, uses much more gasoline than the tank on the tractor will hold, yet, can not be running to a filling station. He has to have a certain amount of fuel present in bulk, usually in barrels. According to Dr. Joseph E. Pogue,[13] there are in the way of the general distribution of fuel oil to farmers in barrels some difficulties in the form of barrelling charges, difficulties that would require sales on a considerable scale to be overcome. Dr. Pogue, however, thinks that the distribution of fuel oil for tractor purposes, nevertheless, is a proposition sound enough ultimately to induce the efforts necessary to overcome these difficulties. Mr. R. L. Welch, Secretary of the American Petroleum Institute,[13] notes no difficulties in detail, and thinks the distribution of fuel oil to be in time entirely feasible.

Cracking

For those who are fundamentally opposed to the idea of using fuel oil directly in automotive engines, there remains the possibility of converting such oil into more volatile fractions. This conversion can be effected either by so-called cracking processes, or possibly by the process of chemical conversion. Cracking means breaking up the fuel molecules simply by the application of heat, which is often combinated with pressure. The best known cracking processes for large scale conversion of heavier hydrocarbons into gasoline are those of Burton and of Rittman. That of Burton has been applied for some time by the Standard Oil Company and the details thereof are kept more or less secret. That of Rittman was developed by the originator in the Bureau of Mines, and has been given freely to the public. (W. F. Rittman, C. B. Dutton and E. W. Dean, "Manufacture of Gasoline and Benzene-Toluene from Petroleum and other Hydrocarbons", Bureau of Mines Bulletin 114). Rittman states, (Bulletin just mentioned, page 204) that from a 42-gallon barrel of oil, by his method 33 to 37 gallons of gasoline can be recovered by a triple or quadruple conversion. This would be a yield of from 79 to 88 per cent. Just what percentage of the crude oil it would finally prove commercially profitable to convert into gasoline by all the methods of distillation and cracking it is, of course, impossible accurately to prognosticate. Messrs. Arthur D. Little, Inc. Chemical Engineers of Cambridge, Mass., in a pamphlet entitled, "The Petroleum Outlook", give a chart which indicates as their opinion a cracking limit of about 70 per cent.

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* For a description of a modern steam automobile which operates on kerosene, and which in the author's opinion, is just as easily adapted to a good grade of heavier distillate than kerosene or even to light, strained fuel oil, see S. A. E. Journal, November, 1919.

[13] In a letter to the author.

During the war a certain amount of publicity was given to a method ascribed to Captain E. C. Weisgerber, for converting heavy oil into gasoline by treatment with chemicals (or by distillation?). (See Automotive Industries, November 28, 1918, page 933). It appears that further development has been given to this method since the war and that results of commercial importance may have been attained. While no details are available at the present time, there have been some statements to the effect that by this chemical method yields of gasoline equal to that obtained by cracking methods may be realized.

The author is of the opinion that the one straightforward way to handle fuel oil is to burn it in engines adapted to run on such oil. He bases this conclusion on the fact that such engines at the present time are the most efficient, particularly at the reduced loads prevalent in automotive operation. To spend human labor in order to convert fuel oil into a fuel suitable for engines fundamentally less efficient, and in addition, to do this at a not inappreciable loss of material (4, 5 per cent at any rate) would not appear to be fundamentally sound.

ARTICLE 3.

PETROLEUM SUBSTITUTES.

SUMMARY: *As substitutes for gasoline, benzol and alcohol deserve most serious consideration. Benzol is obtained from dry distillation of coal — as in coking and coal gas manufacture. Alcohol can be obtained from various agricultural products, from waste products from lumber and paper mills and also from* *coal by way of calcium carbide. While it seems unlikely that gasoline can wholly be replaced by these products, yet, the possibility of partial replacement should not be overlooked. Engines run well and efficiently on both these fuels.*

As a substitute for petroleum in general, shale oil deserves first consideration. Of this product over one hundred billion barrels seem available — a supply sufficient for generations. Sample distillations from Colorado have shown considerable quantities of gasoline to be derivable from shale oil.

Richer shales would seem capable of yielding oil which could be sold in competition with petroleum at the present time. Poorer shales provide a reserve which seems for generations to exclude the necessity of oil prices as high as those in Europe today.

For the introduction of either benzol, alcohol or shale oil on a large scale the creation of great new industries is, however, necessary.

While we may arrive at the conclusion that a substitute for petroleum having substantially the characteristics of crude oil, will be developed, yet efforts to find substitutes have usually been concentrated where the shortage is most acutely felt, that is, on gasoline. The two main gasoline substitutes suggested are *benzol*, (benzene), and *ethyl alcohol*. During the war a very interesting substance, *cyclohexane*, was tried, but it will probably never reach commercial importance. *Ether*, produced by the action of sulphuric acid on alcohol, has been used to a considerable extent in mixtures, particularly to produce a more ready ignitibility and to make alcohol more ready miscible with other substances, notably petroleum products. It can, however, be replaced for either purpose by benzol, and at least for better ignitibility by light gasoline. As it is more expensive than alcohol, it will probably not maintain itself as a fuel.

The question of gasoline substitutes from the economical point of view resolves itself, therefore, into a question of the available supply of benzol and alcohol; from the fuel user's point of view the properties of these two fuels with regard to engine performance are also of importance.

(1) *Benzol.* Benzol (chemical formula $C_6 H_6$) is a fuel, when pure, made up of 92½ per cent of carbon, and 7½ per cent of hydrogen. It is a homogeneous product boiling at about

176 deg. fahr. (80 to 81 deg. cent.). Commercial benzol usually contains other substances such as toluol, xylol, etc., which bring the boiling point up to 200 deg. fahr. or higher. The fuel value per pound is little less than that of gasoline, (17,300, B. t. u. as against 18,000 to 19,000 B. t. u.) Pure benzol is readily combustible in engines without any trouble from knocking or carbon formation; (the latter if the compression is sufficient.)[14]. (See tests by the Automobile Club of America, Automotive Industries, Nov. 14, 1918, p. 834). Benzol, however, seems to attack both iron and copper slightly, although in this respect it is very much less injurious than alcohol. (Report by the Engineers of the London General Omnibus Company on alcohol — benzol mixtures). As it dissolves shellac, metallic rather than shellacked cork floats must be used in carburetors. The main difficulty with benzol is that it freezes at about 42 deg. fahr. Commercial benzol, however, contains a considerable amount of such substances as toluol and xylol, which lower the freezing point to about 23 deg. fahr. Finally, by the addition of gasoline it is found possible to lower the freezing point still more. In Germany during the war, a mixture of one-third gasoline and two-thirds benzol having a freezing point below 4 deg. fahr. was largely used.

Benzol is obtained from coal and is a by-product in the manufacture of ordinary coal gas. In this way in 1918, 4,400,000 gallons of benzol were produced. Exactly ten times this amount was obtained as a by-product from coke manufacture, (J. E. Pogue, S. A. E. Journal, January, 1920, page 49). In all, however, but little over 1,000,000 barrels of benzol is now produced in this country per year.

The amount of benzol obtainable from coal may vary considerably. In estimating the maximum supply of benzol available, a yield of two gallons per ton is usually assumed. The total bituminous coal production in 1918 amounted to somewhat over 500,000,000 tons. (It was only 430,000,000 in 1919). If all this coal were coked, there would then be available about one billion gallons or 24 million barrels of benzol. This is less than one-fourth of the present gasoline demand. The amount of capital and enterprise necessary to introduce coking of the greater part of all of the coal mined in the United States need hardly be dwelt upon.

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[14] Frequent statements have been seen that benzol gives a fluffy carbon deposit if compression is not quite high. The motor used in the tests mentioned was a standard four-cylinder 3″ x 5″ Continental engine, however.

Nevertheless, benzol, as a partial substitute for gasoline, should always be seriously kept in mind. Being a by-product, it may be sold at a price that depends perhaps more on market conditions than on actual cost of production. In some of the larger cities of the country, for instance in Chicago, benzol mixtures are at the present time supplied to motorists in not inconsiderable quantities.

(2) *Ethyl Alcohol.* Alcohol (chemical formula $C_2H_5.OH$) is a substance consisting of 52.12 per cent carbon, 13.14 per cent hydrogen and 34.74 per cent oxygen. Alcohol furnishes very much less heat energy per pound than does either gasoline or benzol. (10,800 B. t. u. as against 18,000 B. t. u. for gasoline.) Alcohol can, however, without knocking be run with a higher degree of compression in the engine than can gasoline, or kerosene (unless special means for eliminating the knocking in the latter case are adopted). High compression leads to more efficient fuel utilization; consequently, in carbureting engines as made and operated at the present time, a pound of alcohol will furnish almost as much power as a pound of gasoline.

The tests previously quoted in connection with benzol, carried on by the engineers of the London General Omnibus Company, showed alcohol to have so serious a corrosive effect on iron and on copper that lead lining of fuel tanks and fuel ducts had at last to be resorted to. In cold weather considerable trouble was found in starting the engine on account of the fact that a great deal of heat is required for the evaporation of alcohol. Moreover, all commercial alcohol contains from 5 to 10 per cent of water by volume. This water separated out in the inlet passages and had to be drained off.

Even with compressions as high as 123 pounds as compared with 90 pounds for gasoline, the London tests showed a higher mileage per gallon for gasoline than for a mixture of 50 per cent alcohol and 50 per cent benzol. Especially during cold weather, the balance in favor of gasoline was marked. Six buses running on a mixture of 50 per cent benzol and 50 per cent alcohol obtained an average of 6.05 miles per gallon, while the fleet average for gasoline was 7.19 miles per gallon.

Nevertheless, the engineers of the Omnibus Company stressed the point that the engines consumed less heat energy when running on the mixture than when running on gasoline, the margin being 2 per cent in favor of the mixture in cold weather and 12 per cent in warm weather. In their final conclusion, they appear to consider alcohol-benzol mixtures as deserving of serious attention as a motor fuel. This conclusion, however, would appear to apply preëminently only to conditions in Europe; it would be less applicable under price conditions prevailing for liquid fuels in America.

Quite similarly an alcohol-ether mixture, Natalite, was found satisfactory and economical motor fuel in South Africa. The price for this mixture was about 62 to 75 cents per Imperial gallon (at pre-war rates of exchange), as against $1.00 to $1.12 for gasoline. Again, it must be noted that the alcohol mixture would be entirely out of the question under price conditions in the United States.

Alcohol produces no carbon in the engine cylinders and may even be regarded as a carbon remover. It should be run with considerable air excess to avoid the formation of corrosive intermediary combustion products, such as aldehyde, or acetic acid.

Alcohol is a substance which, as a rule, has been obtained from vegetable products and for this reason the supply is not limited by an "unmined reserve", as is the case with petroleum. Alcohol, especially alcohol manufactured from potatoes, has already in Europe, notably in Germany, found a considerable application as an automotive fuel. One of the strongest pleas for alcohol has, however, been that it can be manufactured from a number of by-products available in large quantities and at present of comparatively small value. Such by-products are "blackstrap" molasses, waste liquor from sulphite paper pulp manufacture, sawdust, straw, corn stalks, etc. Alcohol can also be manufactured from coal by the way of calcium carbide. That is, where water power in considerable quantities is available at a low price, coal can, by means of this power, be converted with the aid of water and of limestone alone into a liquid fuel.

All these are not only suggested possibilities, but are actual processes in the majority of instances practiced at least some-where even on a commercial basis. The question is simply whether in the United States any of these possibilities should be expected to yield alcohol in quantities sufficient to make it a material factor in relieving the threatened gasoline shortage, at a price which would not look unreasonable during the transition period from gasoline to alcohol.

(A) *Alcohol from Corn, Potatoes, and other Farm Products*

With an average production of only 28.8 bushels of corn per acre, figures of Dr. Wiley (U. S. Department of Agriculture, Farmer's Bulletin No. 268, page 20), would indicate a possible yield of almost two barrels of alcohol to the acre. (2.7 gallons to the bushel.) Fifty millions of acres would then, without any particularly high grade type of farming, supply all the alcohol necessary to replace gasoline entirely. The total improved farm acreage in the United States in 1910 was 478 million acres and the total land area in farms was 879 million acres. Of this latter acreage, 50 million acres constitute only 5.7 per cent and of the improved acreage a little more than 10 per cent. In other words, it cannot be said that agriculture would be taxed unduly if it were called upon to supply raw material for alcohol in the form of corn.

Very similar is the situation with regard to the production of alcohol from potatoes. Figures by Wente and Tolman (U. S. Farmer's Bulletin No. 410, page 18), indicate a yield of 1.3 gallon of 90 per cent alcohol per one hundred pounds of potatoes: that is, about three-quarters of a gallon of 95 per cent alcohol per sixty pound bushel. To produce two barrels of 42 gallons each of alcohol per acre the quite reasonable yield of 112 bushels of potatoes per acre is all that is necessary.

A somewhat different aspect of the situation is obtained, however, if price is considered. Dr. Wiley estimates that with corn at 40 cents per bushel the cost of raw material would be 15 cents per gallon of alcohol. Allowing 15 cents for labor and 10 cents for profits, he arrives at the selling cost of 40 cents per gallon. A price of 40 cents per bushel for corn is certainly not attractive to the farmer. As prices go today alcohol from corn would probably cost the consumer at least 80 cents at the filling station, and alcohol from potatoes would hardly be cheaper. (With potatoes at 50 cents a bushel, a gallon of alcohol would cost 67 cents for raw material alone.) In figuring on cheap alcohol from potatoes in this country, potato culls at 15 cents a bushel have sometimes been spoken of. This would mean alcohol at about 40 cents a gallon.

It is possible that the price of vegetable alcohol could be lowered materially by the use of certain special low priced materials, such as cassava. This is a semi-tropical plant which, however, grows well in some southern states, yielding anywhere from 4 tons per acre to five or six times this amount. Cassava is very rich in starch and other fermentable matter. A conservative estimate is that by means of cassava 3.5 barrels of alcohol could be produced per acre. Nevertheless, the root does not seem to stand transportation excessively well, a serious obstacle in the way of collection and large scale production.

With gasoline at European prices of between one and two dollars per gallon, it can readily be seen that alcohol from primary farm production is, indeed, a very sensible substitute to discuss. In the United States, however, the situation, especially in view of the shale oil reserve, to be dwelt on later, appears to offer few possibilities for this form of substitute.

(B). *Alcohol from Waste Products.*

(1) *Sawdust.* There seem to be plants at the present time in actual operation in the United States producing alcohol from wood or wood waste with a yield of 15.4 to 22.8 gallons of alcohol per ton of dry material. (Automotive Industries, January 8, 1920, page 81). The production from sawdust is not as easy as from more coarsely shredded wood, but nevertheless appears to be a success in Europe. The total amount of lumber sawed in the mills of the United States in 1918 was 29 billion board feet. The census figures for 1909 give a figure much higher than this, namely, 44 billion 510 million board feet, corresponding to roughly 3 billion 700 million cubic feet. Counting as much as 10 per cent of this as saw dust available and allowing 30 pounds per cubic foot as the weight of dry lumber, there would in 1909 have been available 6 million tons of sawdust. From this about 120 million gallons, or approximately 3 million

barrels of alcohol, could have been produced, a mere drop in the bucket as compared with our automotive demand for gasoline.

(2) *Sulphite Liquor.*

Increasing amounts of wood pulp are being converted into pure cellulose by means of the so-called sulphite process for the production of higher grades of paper. Waste liquor from the sulphite boilers contains somewhat less than one per cent of alcohol, and would give a yield of about 10.7 gallons of alcohol per ton of cellulose. (Automotive Industries, January 8, 1920, page 81.) The total quantity of pulp produced in the United States was in 1909 626,000 tons,[15] corresponding to a potential yield of about 6 million gallons of alcohol (143,000 barrels). It is quite possible that this alcohol could be sold at a fairly low price. Evidently, however, even with liberal allowances for increases in production of cellulose, the amount is not of very great importance in the present fuel situation.

(3) *Molasses.*

Three gallons of black-strap molasses yield about one gallon of alcohol. Mr. E. Humboldt (Power, Sept. 9, 1919, page 420) asserts that a large plant can manufacture alcohol from this source at a price of 25 cents per gallon. As the main source of supply for the world, Cuba is usually mentioned. The available supply of such molasses from this island is put at 200 million gallons per year, an equivalent of 70 million gallons of alcohol. The supply of such molasses from the United States would be lower, especially in view of the fact that it has been found possible to extract sugar from beet sugar molasses by means of a new process. At best, the total amount of alcohol derivable from Cuban and North American molasses would probably not be much in excess of two million barrels, again a fairly insignificant amount compared with the demand for gasoline. The price for black-strap molasses was before the war something like 3 cents per gallon, meaning less than 10 cents per gallon of alcohol. Now the price would be more than twice this amount.

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[15] From U. S. census statistics.

(4) *Straw, Corn Stalks, Etc.*

A great many statements regarding the immense amount of fuel alcohol obtainable from corn stalks and from straw have appeared in the popular and in the technical press. One figure accredited to the chief chemist of the United States Department of Agriculture is as high as 940 million gallons or 22 million barrels per year from straw alone.[16] The possibility of converting waste straw into fuel has, by the Department of Agriculture, been deemed serious enough to lead to the establishment of an experimental laboratory designed for the investigation of the question. Yet, whatever the possibilities are, very great difficulties are in the way of the wholesale utilization of straw. These difficulties may, possibly, to be sure, be more in the nature of difficulties of collection and transportation of the raw material, than of difficulties of conversion.

Ten or fifteen years ago, rather optimistic opinions were voiced regarding alcohol production from corn stalks. Difficulties are here encountered not only in point of collection and transportation, but also in view of the high moisture content of the stalks, and the amount of chemicals needed for the conversion process.

(C). *Alcohol from Coal.*

It appears that in Switzerland already at the present writing alcohol is being manufactured from coal by way of calcium carbide, at a cost of only 1.1 mark per gallon — even at the pre-war rate of exchange less than $0.30 — (Automotive Industries, Jan. 8, 1920, p. 82). As the price of power seems to be put at something less than two pfennigs (one-half cent at pre-war rate) per kilowatt hour, and the coal at 36 marks ($8) per ton, a cost of $0.30 seems incredibly low.

The amount of power necessary for operation on a scale sufficient to meet the United States demand for gasoline would, however, be enormous. The figure given in the article above referred to is 24,000 kilowatt-hours per thousand gallons of alcohol. On this basis about 16 million horsepower would be necessary to produce 100 million barrels per year. The total available water-power of the United States on an all-year basis has usually been put at 25 million horsepower. Estimates on a half-year basis run up to about 50 million horsepower yet undeveloped. In any case, it is evident that the major part of the country's water-power would have to be applied to the production of automotive fuel, if carbide alcohol were the fuel in view.

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[16] Automotive Industries, 1920, p. 495.

Conclusions Regarding Benzol and Alcohol. Evidently there are potential sources of both benzol and alcohol, rendering these fuels well worth keeping in mind as important possibilities in future fuel problems. A rational utilization of our coal resources might mean general coking with full recovery of all products of distillation. Lignite, a product similar to coal, but not yet as perfectly mineralized, of which there are large deposits in the West, decidedly calls for such methods of exploitation. Such methods of utilization mean benzol in large quantities and at a price which may be kept quite low on account of the simultaneous recovery of other valuable by-products.

Alcohol from carbide, from tropical vegetation, and to some extent from various waste products, may also be obtainable in considerable quantities. Here, however, a further increase in price of gasoline would probably be needed to stimulate production.

In view of the foreign petroleum reserve and our enormous shale oil resources now to be considered, the author doubts whether such an increase will for a long time to come be justified. Abroad, however, alcohol may play a much more prominent part, and make it quite desirable for American manufacturers bent on foreign trade to adapt their engines to operation also on this fuel.[17]

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[17] The trend of engineering developments depends so much upon the personality of those who control the necessary capital expenditures, that sometimes one hesitates to draw any conclusions whatever on the basis of plain technical and economic facts. The author has heard that the leading engineers of some very great industrial corporations are personally interested in alcohol from agricultural waste as against shale oil. What development such a combination of inventive ability and money power may lead to can, of course, not be predicted.

Fuel Mixtures.

Even in the United States a number of fuel mixtures were tried during the war, and some publicity was given to certain more successful ones. One such mixture is, for instance, the so-called Alcogas. This substitute, as prepared for aviation purposes, was tested by the Bureau of Standards (Report No. 89, National Advisory Committee for Aeronautics — Automotive Industries, Apr. 29, 1920). It consists of 40 per cent alcohol, 35 per cent gasoline, 17 per cent benzol, and 8 per cent other ingredients. This is, however, stated not to be the composition of the fuel prepared for commercial or passenger-car use. The Bureau of Standards found alcogas in general to give a somewhat smoother operation of the engine and ability to stand much higher compression without knock. The engine showed greater power on alcogas than on gasoline, and better thermal efficiency. In view of the fact that less energy is contained in a pound of the composite fuel than in a pound of gasoline, more pounds of alcogas were, however, consumed to attain a given power at the same compression. By introducing higher compression for the alcogas than gasoline could carry without knock, the fuel consumption was brought, in both cases, to an approximately equal number of pounds per brake horsepower hour.

Some very thorough investigations of alcohol-benzol mixtures were carried out by the London General Omnibus Co. and have already been referred to (p. 31).

The most comprehensive investigations of mixtures of alcohol and benzol with petroleum products have been carried out in Germany. (Automotive Industries, Sept. 9, 1920, p. 501.) The most satisfactory mixture so far discovered seems to be that of one part kerosene, one part benzol, and one part alcohol. This mixture gave more miles per gallon than either benzol or gasoline alone, and gave, under German price conditions, 12 per cent more miles per dollar's worth of fuel than the second best mixture. It is interesting to note that this second best mixture was that of one part benzol, and one part alcohol, and that this mixture not only ranked high in dollar economy, but actually gave more miles per gallon than any other mixture.

In Germany, under war conditions, some very radical propositions for the replacement of gasoline by other products were tried out. Acetylene was given a careful test. So was also naphthalene. In England a mixture of alcohol with ether, sold under the name of Natalite, was experimented with.[18] Of all these possibilities, those of the benzol-alcohol mixtures are probably of most permanent interest, and the figures from the article in the Automotive Industries above referred to are, therefore, here reproduced in full.

TABLE 4.

TESTS BY PROFESSOR VON LOEW IN GERMANY ON FUEL MIXTURES.

Full Power of the Motor in Hill Climbing.

	Maximum Speed, Kilometers Per Hour	Distance in Meters Covered on ½ Liter of Fuel
Ethyl alcohol	30	1070
Methyl alcohol	38	1030
Gasoline	40	1350
Benzol	40	1460
1 Benzol + 1 alcohol	40	1470
1 Benzol + 2 alcohol	38	1350
1 Benzol + 3 alcohol	35	1190
1 Benzol + 4 alcohol	30	1060
1 Benzol + 5 alcohol	30	1030
1 Benzol + 1 kerosene + 1 alcohol	40	1020

Economy Tests on Level Road at a Speed of 40 Kilometers (25 miles) Per Hour.

Used Fuel	Distance Covered with 1 Liter Fuel, Kilometers	Miles Per Gallon	Price Per Liter, Mark	Distance Covered on 1 Mark's Worth, Kilometers
Gasoline	5.8	13.7	0.38	15.7
Benzol	7.1	16.7	0.375	18.3
1 Benzol + 1 Alcohol	7.5	17.7	0.358	20.9
1 Benzol + 2 alcohol	7.2	17.0	0.352	20.4
1 Benzol + 3 alcohol	7.0	16.5	0.349	20.0
1 Benzol + 4 alcohol	6.6	15.6	0.377	19.0
1 Benzol + 5 alcohol	6.0	14.2	0.345	17.3
Alcohol	5.4	12.7	0.31	15.8
1 Benzol + 1 Kerosene + 1 alcohol	7.3	17.2	0.31	23.5

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[18] Automotive Industries, 1920, p. 59. Compare p. 32.

COLLOIDAL FUEL.

A very interesting new departure in the line of fuel mixtures may here be mentioned, although it is not destined to take the place of gasoline. This is the possibility of mixing finely ground coal into fuel oil, or into a mixture of fuel oil and tar, and thereby producing a combination which flows like a liquid, and can be atomized in ordinary oil burners, yet may contain as much as 30 per cent of solid coal. In tests described in "Power," of April 29, 1919, p. 662, coal ground to pass through a 300 mesh screen, was mixed with fuel oil in such a manner that the final product contained either 30 per cent coal with 69 per cent fuel oil, and 1 per cent so-called fixateur, or 45 per cent oil and 20 per cent tar, together with fixateur.

This colloidal fuel was burned in so-called mechanical burners, producing atomization by pump pressure only. With a fuel temperature of 150 deg. fahr., and an oil pressure of 150 pounds per square inch, the atomization of the colloidal fuel was stated to be as good as that of the fuel oil alone.

SHALE OIL.

Public opinion regarding the possibilities of shale oil as a substitute for petroleum generally has gone through a number of violently changing phases in the last few years. While the existence of oil shale deposits in the United States has long been known and while the commercial exploitation of such oil shale for the manufacture of products closely similar to those derived from petroleum, has long been an established fact in Scotland and in other places, yet the fact that shale oil in this country actually represents a reserve of liquid fuel many times larger than our original petroleum resources came almost as a fairy tale to the general public when in 1917 and 1918 popular periodicals began to publish it. Yet, this is a fact based on thorough scientific exploration. The shale oil reserve in Colorado alone is estimated at about 20 billion barrels, and that of southwestern Indiana at 100 billion barrels, five times as large.[19] Such a reserve cannot be exhausted for generations. A feeling of intense relief is apt to be experienced when a person worried by the gasoline stringency first hears this. Gradually, however, it begins to be realized that, while the shale oil is there, it would be necessary to develop a mining industry of proportions equal to those of coal mining in order to supply by shale oil the United States' demand for liquid fuel. It was also found that a good many pioneer attempts to exploit the shale oil reserve had proved commercial failures. Then the fact was revealed that the world outside of the United States still contains large stores of ordinary petroleum, producible by drilling and pumping merely, and at prices not very different from those obtaining in this country now. From that time on the watchword of men in or close to the oil industry seems to have become: "Access to the foreign field; nobody has as yet produced shale oil commercially at a profit!"

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[19] See United States Geological Survey, Bulletin 641, (1916). pp. 140 and 322.

In this swinging back and forth of opinions and views, it would be of importance to get some facts to go by. The Colorado School of Mines, under the leadership of its president, Mr. Victor C. Alderson, seems to have made a special task of supplying the world with such facts. Some of these facts are here given, necessarily without any criticism on the part of the author.

To begin with, it appears that the cost of production in Scotland, as indicated by the following figures, supplied by Mr. Victor C. Alderson in the Quarterly of the Colorado School of Mines, October, 1920, page 7, is:

Cost of Mining	$1.25 to $1.50
Cost of Retorting	0.37 to 0.43
Cost of Refining	1.95 to 2.43

From these figures we may conclude that in Scotland the cost of retorting is only about one-third of the cost of mining.

Applying these data to conditions in America, and assuming the cost of mining a ton of shale to be as high as $2.00, the cost of retorting ought not to exceed $1.00 per ton and the total cost for mining and retorting not $3.00 per ton. The cost per gallon of shale oil produced at this rate would be as follows:

Yield per ton, gallons	10	20	30	50
Cost per gallon	$ 0.30	$ 0.15	$ 0.10	$ 0.06
Cost per barrel	$12.60	$ 6.30	$ 4.20	$ 2.52

From this table it would appear that with yields of 50 gallons per ton or more, the cost of the crude shale oil becomes negligible compared with the present selling price of gasoline as well as of kerosene; and even with a yield as low as 20 gallons per ton there would yet be an ample margin of profit in case the crude is worked mainly for motor fuel selling at prices now prevailing. When the yield sinks as low as to 10 gallons per ton, of course, the price of the crude becomes equal to or greater than the present selling price of gasoline. Even then, however, a price of motor fuel of more than 50 cents per gallon would seem unnecessary. Now, a yield of 20 gallons per ton is about the average to be expected from Colorado shales, (see Bulletin 641 F of the United States Geological Survey, by D. E. Winchester). There are, however, not an inconsiderable number of locations in Colorado, as well as in California, where yields of 50 gallons and more per ton have been found. In Canada, a recent circular by Mr. Alderson speaks of samples yielding as much as 123 gallons per ton. If such richer shales are at first attacked, it would seem that shale oil products could readily be developed at the present time in competition with petroleum. As the petroleum resources give out, the poorer grades of shale might be considered. Finally, when all other resources have been exhausted, if that time comes, such low grade shales as those of Indiana, yielding only about 10 gallons per ton, will still give motor fuel at a price considerably less than the price at which it is selling in Europe today.

These figures are based on estimated cost of production, and are, if anything, too high. Actually, according to the circular of Mr. Alderson, there is now at Elko, Nevada, a plant financed by Mr. R. M. Catlin of Franklin Furnace, New Jersey, at which shale is being mined at a cost of about $1.25 per ton. This plant is regarded as on a commercially productive basis, with a daily output of 100 barrels of crude. A more ambitious undertaking is the one of the United States Producers Refining Company in California, which owns 1,000 acres of shale oil land. According to estimates of Mr. E. W. Hartman, President of this concern, a plant of 5,000 tons daily capacity, which he contemplates, can mine and retort the shale for 65 cents a barrel, the marketable products being 20 gallons of gasoline, 2 of distillate, and 2 of coal oil.

Mr. Alderson sums up his circular with a statement that some very strong financial organizations, such as the Union Oil Company of California, the Pure Oil Company, the Ohio Cities

Gas Company, the Carter Oil Company, The Standard Oil Company, the Midwest Oil Company and the W. P. Hammon organization, have now entered the field of shale exploitation. The interest of financial organizations of such strength in shale oil developments, insures, he thinks, the early establishment of the industry on a commercially productive basis, so that the United States need have no apprehension whatever about its domestic supply of crude oil, "virtually for all time".

The question will then be, What is the character of shale oil? Will it yield today naturally more or less gasoline or motor fuel than ordinary crude oil?

While a great deal of information is available on this point at the present time in the form of distillation tests carried out for private interests, we can here only give data from nine samples of Colorado shale distilled by Mr. D. E. Winchester. These data are found in Table 5. (From Bulletin No. 641 F, of the United States Geological Survey, page 156.)

In as far as the information in the table can be regarded as typical for Colorado shale oil, it would seem that this oil would yield from 6 to 12 per cent of the highest grade of aviation gasoline, with an average of perhaps 10 per cent; more than twice this quantity of motor gasoline as at present specified by the government; and a total motor distillate, including all gasoline and kerosene fractions, of from 38 to 61 per cent of the crude. Very likely in the near future ordinary motor fuel may contain fractions even heavier than those of kerosene. When this comes to pass, the shale oil represented in the table would yield by simple distillation probably between 50 and 60 per cent of fuel for carbureting engines. Additional quantities can, as Mr. Winchester's bulletin shows, be obtained by cracking. In other words, the shale oil investigated by him is, if anything, superior to present day average petroleum for production of ordinary automotive fuel.

Should this now mean that all worry ought to be thrown to the winds and that we should look to the future with a firm assurance that no further thought needs be given fuel difficulties?

In the author's opinion such an attitude would be entirely indefensible. As has already been pointed out, the exploitation of the shale oil resources means the gradual development of a mining industry on a scale wholly commensurate with that of coal

TABLE 5.

RESULTS OF FRACTIONATION OF SHALE OIL.

Sample No.	4	6	10	18	19	27	32	51	57
Begins to boil at ° C.	80	52	50	70	72	65	80	70	54
Distillation (cubic centimeters):									
to 100° C.	6	10	2	2.5	2	4	2.5	7	4
100 to 125° C.	2	.5	3.5	1	1	1.5	3.5	1	2
125 to 150° C. (302° F.)	2	1.5	6.5	2.5	8	1.5	1.5	1	3
Total gasoline	10	12	12	6	11	7	10.5	9	9
150 to 175° C.	2	2	8	6	6.5	3	6	2	5
175 to 200° C.	2	4	6	5	5	3	5	4	4.5
200 to 225° C. (437° F.)	5	4	7.5	5	5	4	6	4	5
225 to 250° C.	7.5	6	7	5	5	5	8	6	5
250 to 275° C.	6	6	7.5	7	7	7	8.5	6.5	7
275 to 300° C. (572° F.)	6	10	13	7	7	17	9	13	12
Total kerosene	28.5	32	49	35	35.5	39	12.5	35.5	38.5
Total distillate	38.5	44	61	41	16.5	46	53	44.5	47.5
Total residuum	61.5	56	39	59	53.5	54	47	51.5	52.5
Amount of oil used	100	100	100	100	100	100	100	96	100

Specific gravity at 60° F.—									
Crude	0.8937	0.8850	0.9138	0.9290	0.9327	0.8946	0.8838	0.9126	0.9126
Gasoline	.7947	.7769	.8090	.7974	.8202	.7849	.7568	.7838	.7605
Kerosene	.8602	.8466	.8260	.8742	.8876	.8722	.8524	.8682	.8538
Residuum	.9695	.9643	.9881	.9894	1.0160	.9684	.9368	.9695	.9628
Asphalt, per cent. by weight	1.35	.82	2.82	1.10	3.62	2.49	.17	1.40	1.03
Paraffin, per cent. by weight	7.70	6.93	2.22	3.72	1.63	4.56	4.70	9.21	4.00
Sulphur, per cent. by weight	.54	1.06	....	....	....	.73	1.42	.41	.69
Nitrogen, per cent. by weight	1.848	.887	2.198	1.549	1.643	1.267	1.849	1.820	2.135
Unsaturated hydrocarbons—									
Crude, per cent	....	....	82	86	81.6	....	72	....	....
Kerosene, per cent	55	55	64	71	71	61	57	62	58

mining. If this development is carried out gradually and systematically as the petroleum resources of the world at large become exhausted, the transition from petroleum to shale oil may take place in the course of the next 25 or 40 years, and no stringency, or violent rise of prices, may be necessary. Without such a systematic development the reverse will be true.

This is, however, not the whole story. In these days the social conscience has become quickened. We begin to ask ourselves what influence our business and industrial operations have on the life of the nation and of the race. It is not unimportant to the sound social development of this country that some hundreds of thousands of miners are to be added to its laboring forces and are to be recruited perhaps very largely from poorly assimilated foreigners. The conditions in our mining regions have never been ideal. Strikes on a national scale, tying up whole industries and threatening widespread calamity have occurred repeatedly. Actual labor wars with incidents too sad for description have gone on for years in Colorado and other parts of the West. It is not unimportant that a large population of human beings will be forced to spend their working hours underground, often exposed to danger, quite regularly to a certain amount of discomfort, in order that the nation at large may enjoy the pleasure and convenience of the automobile. Against such an arrangement the social conscience should react. At this minute then, when we see before us within our own borders, immense resources of mineral oil fuel for generations to come, it is incumbent upon us to resolve more firmly than ever that the exploitation and utilization of these resources shall be carried out with the maximum amount of economy possible.

To the engineer belongs the task of reducing to a minimum the onerous effort necessary to produce the nation's motor fuel. Some suggestions as to what can be accomplished in this connection with apparatus already existing and awaiting only reasonable modification and improvement, are the object of this bulletin.

CONCLUSION TO PART I

The conclusions arrived at have been largely indicated in the summaries. Briefly stated they are:

(1) The domestic production of petroleum in the United

States fails to meet the domestic demand. The shortage is increasing and is mainly made up by imports. This may lead to political entanglements and dangerous situations.

(2) The main reason for the shortage is the ever-increasing consumption of gasoline by automobiles. Automobile engines operate mostly at reduced load and are very inefficient at such load.

(3) Alcohol and benzol deserve attention as substitutes for gasoline, but there is little prospect of producing these substitutes, at least in the near future, in quantities rendering them of other than secondary importance in the relieving the automotive fuel shortage.

(4) Kerosene is not available in quantities sufficient to make it a great source of relief, could the engines be made to handle it.

(5) The marine and railroad demands make serious inroads on the amount of fuel oil available for automotive purposes. Yet, by most efficient use of fuel oil for every purpose, and by eliminating entirely the consumption of fuel oil where coal will answer, notable quantities of fuel oil may be made available. To make this fuel oil adaptable to automotive engines, either the fuel oil must be converted to lighter oils by cracking or chemical treatment; or oil burning engines must be introduced. Developments along both lines have begun. The so-called injection type of engine is of especial interest as it is now the most economical engine in existence and is especially distinguished by economy at reduced load.

(6) Shale oil promises relief in the shortage of oil fuel generally. It is present in quantities sufficient for generations. In some places at least it contains gasoline in considerable quantities. Developments for shale oil production on a larger scale seem to have started.

PART II.

FUEL UTILIZATION IN COMBUSTION ENGINES

SUMMARY

In this part the nature of fuel, of combustion, and of the heat value of fuel is explained; and the heat values and air requirements for a number of fuels are given. The function of expansion in converting combustion heat into power is then considered and the necessity for early completion of the combustion and of maximum possible expansion for good fuel utilization is stressed. It is emphasized that compression is of importance to fuel economy only by enabling expansion, and it is pointed out that the fuel economy of Diesel engines is better at reduced load than at full load because the expansion at reduced load is greater than at full load. The variation in mixture requirements of present day carbureting engines for best economy is discussed.

The evil effects of the burnt residue on the fuel economy of throttling carbureting engines at reduced load is pointed out and various means proposed for overcoming these effects are referred to. The progress of combustion is considered; likewise the nature of knock and the methods of overcoming knock. The difficulties resulting from poor evaporation and distribution of the fuel are taken up and various methods of fuel conversion and charge heating are scrutinized. The influence of friction losses on economy at reduced loads forms the subject of one article; the influence of vehicle speed and gear ratio, that of another.

Steam engines and injection engines can run on fuels heavier than those which a carbureting engine can utilize. Both these classes of engines show better maintenance of economy at reduced load than does the present day carbureting engine. The possibilities of these two types of prime movers for automotive purposes are discussed and the strong points of each are pointed out.

ARTICLE I

FUELS AND AVAILABLE FUEL ENERGY

SUMMARY: *The chemical constitution of alcohol, benzol and petroleum fuels is given. The nature of combustion is explained, as is likewise the meaning of energy, and of heating value, or fuel value. The fuel values, as well as the air requirements for a number of liquid fuels, are given in a table. The gain in economy resulting from complete combustion is pointed out.*

The explanations furnished in this article are of an extremely elementary character and may seem superfluous to a technically trained engineer. In point of fact, however, so much light is shed not only on the qualitative happenings, but also on the quantitative energy relations in combustion processes, by a clear understanding of the molecular occurrences, and so little attention has until recently been given to these occurrences even by technically trained engineers, that the subject matter has been deemed worthy of inclusion.

1. *The Chemical Constitution of Fuels.*

For the proper understanding of the factors making for fuel economy it would appear necessary, in the first place, to understand what fuels are and just how energy dormant in the fuel is set free for motive purposes. In Table 6 is given a list of liquid fuels with some data of interest. Let us, even at the risk of some repetition, consider the three main classes of fuels: petroleum, or similar substances, alcohol and benzol, which have been reviewed in the preceding chapter.

Of these ethyl alcohol, (chemical formula C_2H_5OH), aside from the fact that it usually contains some 5 to 10 per cent of unavoidable water, is a single chemical compound made up of 52.12 per cent of carbon, 13.14 per cent of hydrogen and 34.74 per cent of oxygen.[1]

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[1] In a chemical compound even the most minute particles, or so-called *molecules,* still contain the constituent elements such as carbon, hydrogen and oxygen, in the same proportions. The molecules in a liquid, or still more in a gas, are hardly held together at all. In fact, in the gaseous state they dash about quite freely along rectilinear paths, impinging on each other and recoiling from each other as perfectly elastic

TABLE 6

Heat Values and Air Requirements

Fuel, with illustrative analysis	Average lower heat value, B. t. u. per pound	Theoretical air requirement, lb. per pound of fuel	Boiling Pt. deg. fahr.	Heat of Evaporation, B. t. u. per pound
Gasoline (14.9% H, 85.1%C.)	18000	15	140–437	150?
Kerosene (14.12% H, 85.20% C, 0.6% O. N.	19000	14.8	292–600	250?*
Gas Oil (12.65% H, 86.3% C.)	18000	14.4	300–750?	300?*
Fuel Oil (12.5% H, 86.3% C, 1.2% O, etc.)	18000, or more	14.0?		
Crude Oil (84% C, 12 H, 1% O, S, N.)	17000–21000	13.9		
Ethyl Alcohol (95% by weight) } Pure alcohol	10800	8.6 } Ex-	Pure alcohol	
52.12% C,		cess		
Ethly Alcohol (90% by weight) { 13.14% H,	10200	8.2 { advis-		
34.74% O		able	172	372
Methyl Alcohol (37.5% C, 12.5% H, 50% O.)	8400	6.5	150	480
Benzol (Commercial, 91.5% C, 7.8% H.)	17300	13.3	176	167
Creosote Oil (80.11% C, 9.7% H, 8.9% O, N, 1.3% S)	15600	12.5		
Gas Tar, Coke oven tar (from dry distillation especially in vertical retorts, 89.45% C, 6.59% H, 3.96% O, N, S.)	(14700)–15700	12.3		
Water Gas Tar (91% C, 7.4% H, 1.6% O. N.)	16200	13.0		
Naphthalene (93.75% C, 6.25% H.)	17300	13.0	422	
Ether, Sulphuric, (65% C, 13.5% H, 21.5% O)	15900	11.3	96	162
Toluol (91.5% C, 8.5% H.)	18300	13.5	232	150
Xylol (90.6% C, 9.4% H.) (average approx.)	18500	13.8	288	146

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* Total, including heating of liquid above average atmospheric temperature

Substances made up of single chemical compounds are characterized by definite physical properties, for instance, specific gravity, boiling point, freezing point, etc. Pure ethyl alcohol has a specific gravity of 0.806, a boiling point of 172 deg. fahr. and a freezing point of 202 deg. fahr. Since the water mixed with alcohol raises both specific gravity,[2] boiling point and freezing point of the mixture, it is possible with the aid of available tables to tell from the specific gravity the exact amount of water present.

Benzol (chemical formula C_6H_6), as such, is also a single chemical compound consisting of 92.5 per cent carbon and 7.5 per cent hydrogen. Its boiling point is 176 deg. fahr. Its specific gravity is 0.874, and its freezing point 42 deg. fahr. All commercial benzol, however, contains other substances, whch in general have a tendency to raise the boiling point and lower the freezing point. In Germany, where benzol and benzol mixtures are more standardized than in the United States, commercial benzol, as a rule, contains from 10 to 25 per cent toluol. (See Automotive Industries, Sept. 9, 1920, page 502.) These benzol mixtures show parts distilling over at temperatures from 185 to 280 deg. fahr. and freezing points, appearing to vary between 23 and 33 deg. fahr. For this reason there are in use in Germany special so-called winter benzols, which are mixtures of two-thirds benzol or benzol and toluol, and one-third either gasoline or alcohol. These winter benzols have freezing points of below 4 deg. fahr.

All petroleum products whether gasoline or kerosene or heavier oils, consist of a great variety of chemical compounds. Pennsylvania oils are mainly made up of so-called paraffins which, if liquid at ordinary temperatures, contain from 15.06 to 16.67 per cent hydrogen and from 84.94 to 83.33 per cent carbon. The more hydrogen they contain, the lower is their specific gravity, freezing point and boiling point. The lightest one, Pentane, boils at 99 deg. fahr. and has a specific gravity of about 0.627. The heaviest one, Hexadecane, has a boiling point of 549 deg. fahr.

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diminutive balls. The *atoms*, on the other hand, which build up the molecules, are held together in certain more or less fixed relative positions, by electrical or chemical forces, often of enormous intensity.

[2] Smithsonian Tables, page 224. The scources of chemical and physical data in this bulletin, when not specially given, have usually been either the Smithsonian Tables, or Landolt & Börnstein's Tables.

and a specific gravity of 0.775. This heaviest fraction freezes at 64 deg. fahr.

Other American oils contain hydrocarbons less rich in hydrogen. For instance, the so-called *olefines*, or *ethylenes*, very common in Californian oils, contain 14.29 per cent hydrogen and 85.71 per cent carbon. Exactly the same proportions of carbon and hydrogen are found in the so-called *naphthenes* which form the major part of Russian oils.

In view of the fact that the present day petroleum products are mixtures of chemical compounds of such widely varying constituents and proportions, it is not possible to single out any one property of these mixtures as determining all other properties. The olefines and naphtenes are heavier than paraffins of the same boiling point and specific gravity. For instance, the naphtene cyclohexane, with a specific gravity as high as 0.77 has an average boiling point of only 178 deg. fahr. A paraffin of the same specific gravity would boil at approximately 453 deg. fahr.

As stated before, the property that interests us most in connection with carbureting engines is the volatility of the fuel. Fuel specifications for such engines might very well bear on volatility alone.

Nevertheless, it has become customary in the United States to characterize fuel largely by specific gravity, and to do this with the aid of the entirely arbitrary so-called Baumé scale. It may be well to give an idea about what this Baumé scale means in specific gravity and weight per gallon, even though the use of the scale as a criterion for quality of fuel is not countenanced. Very complete tables covering this matter are obtainable from the Bureau of Standards (Circular No. 57, United States Standard Tables for Petroleum Oils). Only a very much abridged table is given here for rough and ready reference. (See Table 7.)

Specific gravity in the United States usually in the case of oils means gravity measured at 60 deg. fahr. as compared with that of water at 60 deg. fahr. All oils grow lighter with increasing temperature and the above-mentioned tables of the Bureau of Standards give figures supplying the standard so-called 60°/60° gravity when the gravity at any other temperature has been taken.

TABLE 7

Degrees Baumé	Specific Gravity (60°/60°)	Pounds per Gallon
10	1.0	8.328
15	.9655	8.041
20	.9333	7.772
25	.9032	7.522
30	.8750	7.286
35	.8485	7.065
40	.8235	6.857
45	.8000	6.661
50	.7778	6.476
55	.7568	6.300
60	.7368	6.134
65	.7179	5.976
70	.7000	5.827
75	.6829	5.685
80	.6667	5.549
85	.6512	5.420
90	.6364	5.296
95	.6222	5.178
100	.6087	5.066

Actual distillation curves for commercial gasoline and kerosene have been given on page 12, Fig. 1.

2. *Combustion and Fuel Values.* To break up the molecules of chemical compounds it is necessary that the atoms constituting these molecules be subjected to attractive forces from the outside greater than those which hold them together. Such forces may be supplied by the attractions of atoms in the molecules of other substances. The only necessary condition is that these other molecules be brought into close proximity with those they are destined to break up.

In all ordinary combustion the oxygen of the air is the agent tearing apart the molecules of the fuel, and forming with the atoms of the fuel new molecules which make up the so-called combustion gases. This attractive power of the oxygen asserts itself, however, only to a negligible extent at ordinary temperatures. When a substance is heated up, it means that the molecules are agitated. In setting fire to a fuel a certain part thereof is mostly evaporated and gasified. This heated and gasified part mixes intimately with the oxygen of the surrounding atmosphere. Collisions between the molecules of the evaporated fuel and the molecules of the oxygen result. In these collisions, or close approaches, the oxygen atoms break up the fuel atoms. New compounds are formed by atoms rushing violently onto each other. Intense agitation results. This agitation becomes apparent as high temperature and incandescence, the thing we call fire.

By energy is understood the amount of work a body is able to do. By work again, scientifically speaking, is meant the transport of a substance a certain distance by the exertion of a certain force; more exactly the work is defined as the product of the force and the distance moved. Thus, if a pound of water descends a certain distance in a water-fall, the work this pound of water is able to do is equal to one pound times the height of the fall. If the fall is 10 feet high, then every pound of water going over that fall is able to do 10 foot-pounds of work. One ton of water would be able to do two thousand times 10, that is, 20,000 foot-pounds of work, etc.

The energy may be either energy directly available in the form of motion, or it may be so-called *potential* energy, not yet set free. Thus, if the water is allowed to fall freely, it will acquire velocity. This velocity energy may be taken up by a water wheel of suitable construction and converted into useful work. The water, however, even before it goes over the fall, simply by virtue of the fact that it is stored at a level above the fall, has potentially the power of doing the same amount of work. The one condition is that it reach the verge of the fall and go over.

The same conditions apply in the case of a fuel. We know that if the fuel is brought into contact with oxygen and set on fire, a violent molecular agitation will result. This molecular agitation represents a certain amount of motional energy which can be converted into useful work. Potentially this energy is present in the fuel even before it is set on fire. It is what we call the *heat value* of the fuel. Manifestly, however, the amount of agitation, or work-doing power, developed per pound of fuel is not dependent on the fuel alone. It is dependent also on the amount of oxygen available. In stating the energy derivable from a pound of fuel, it is, therefore, necessary to define the conditions under which this energy is going to be developed.

The general practice is to assume that there is available at the combustion sufficient oxygen, not only to break up all the molecules of the fuel completely, but also sufficient to form with the fuel atoms the combustion gas containing the maximum amount of oxygen.

Hydrogen can with oxygen form only one compound stable enough to maintain itself under combustion conditions. This compound is water vapor. In water vapor two atoms of hydrogen are combined with one of oxygen. Chemically an atom of hydrogen is denoted by H, an atom of oxygen by O and water vapor is written H_2O.

Carbon can burn either to carbon-monoxide, which is a union of one carbon atom with one oxygen atom; or to carbon-dioxide, which is a combination of one atom of carbon with two atoms of oxygen. The chemical sign for carbon is C, and carbon-monoxide is written CO, carbon-dioxide CO_2.

In determining the so-called heating values of a fuel, the fuel is enclosed in a small reaction chamber with a quantity of oxygen sufficient for complete combustion to the compound most rich in oxygen. This whole reaction chamber is submerged in a known quantity of water. Combustion is started, for instance, by means of an electric glow wire and the temperature rise of the water is measured. The amount of heat necessary to raise the temperature of one pound of water one degree Fahrenheit is called one British thermal unit, abbreviated B. t. u. Since heat is fundamentally simply molecular agitation, and since the thing we are ultimately interested in is the work-doing capacity of the fuel, it is necessary to ascertain just how many foot-pounds one B. t. u. is equivalent to. This has been done in various ways and the so-called mechanical equivalent of heat is now known with an accuracy of about 1 in 1000. The value assumed by Goodenough (Steam Tables, page 20) is 777.64 foot-pounds per B. t. u., by Marks and Davis, (Steam Tables, page 93) — 777.52 foot-pounds per B. t. u. For practical purposes the value 778 is close enough.

Lower and Higher Heating Value.

A later article on the combustion processes in heat engines and the heating values of the fuel under the conditions in combustion engine cylinders shows that the heating values are not the same for combustion under constant pressure and for combustion at constant volume; also that they vary to some extent with temperatures of combustion and heat development. Here will be explained only what is meant by "lower" and "higher" heating value, since these are terms commonly met with in specifications for fuels and discussions about fuels.

The cooling water in the combustion calorimeter is kept at a temperature so low that the water formed by combustion of the hydrogen in the fuel is condensed. In actual combustion practice the cooling of the combustion products is never carried to a point where the water vapor formed will condense. In the calorimeter the condensation of the water vapor sets free a certain amount of "latent" heat which goes to raise the temperature of the cooling water, just as does the heat of reaction proper. The total heat so measured is called the *higher* heating value of the fuel. In practice, the latent heat of the water vapor will not be available for heating purposes and may, therefore, just as well be subtracted from the heat value of the fuel once for all. The heat value obtained by such subtraction is called the *lower* heating value of the fuel. It is this lower heating value in which we are mostly interested.

The latent heat of the water vapor at the temperatures of condensation in the calorimeter may be assumed roughly equal to 1,000 B. t. u. per pound, or somewhat more. The amount of water vapor formed per pound of fuel burnt is, for gasoline and kerosene with 14 per cent hydrogen, 1.26 pounds; for pure ethyl alcohol, 1.18 pounds; and for benzol, about 0.7 pounds. The difference between the lower and the higher heat value is, therefore, for petroleum products roughly 1,000 to 1,300 B. t. u. per pound; for pure alcohol 1,200 B. t. u. per pound; and for benzol 700 B. t. u. per pound.[3]

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[3] To condense a gas it is necessary first to reduce the molecular velocities to a point where these velocities no longer suffice to carry the molecules past their mutual spheres of attraction. This lowered velocity means drop in temperature. The instant, however, the attractive powers get the better of the velocities, the molecules "fall" onto each other and in this way acquire new velocity, just as does a projectile falling back to the earth. This velocity energy must be carried off, or the liquid would be hotter than the gas. No further condensation could take place. The up-shot of it all is this: As long as condensation occurs, all the cooling medium can do is to hold the temperature steady at a point where molecular attraction just barely overcomes the velocity energy of the molecules. With pressure constant, no drop in temperature takes place. It is only when the condensation is complete that the temperature of the liquid commences to drop again. The "fall" energy of the molecules,

3. *Air Requirements in Combustion.* In Table 6 are given not only the heat values of the fuels, but also the air requirements for complete combustion. The fuel values presuppose combustion to carbon dioxide. With this assumption and also with the assumption that every particle of oxygen in the air actually enters into combination, the total amount of oxygen required for combustion of any fuel of known chemical composition is easily determined on the basis of the following requirements.

1 pound of carbon requires 8/3 pounds of oxygen.
1 pound of hydrogen requires 8 pounds of oxygen.
1 pound of sulphur requires 1 pound of oxygen.

For every pound of oxygen there are needed 4.35 pounds of air. In this way are obtained the theoretical air requirements given in the table.

In general, it is difficult to bring all oxygen into combination. Some of it will pass through unconsumed. In order, therefore, completely to utilize the fuel, it is necessary to provide an excess of oxygen. In the case of gas, or well mixed and evaporated liquid fuels, this required air excess may, however, be very small. For the proper understanding of combustion phenomena it is always of great importance to know whether air was present in excess, or whether air was insufficient. In combustion engine parlance a mixture of fuel and air containing air in excess is called a lean mixture; on the other hand, a mixture containing fuel in excess, that is to say, air insufficient for complete combustion, is called a rich mixture. The dividing line between lean and rich mixtures might be placed at the theoretical amount of air required. Thus in the case of gasoline, mixtures containing more than 15 pounds of air would be regarded as lean mixtures, while mixtures containing less than 15 pounds of air would be rich mixtures.

In practice the distinction between a lean and a rich mixture is apt to be drawn simply on the basis of power development of the engine, without actual knowledge of the air-fuel ratio. On

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which must be continually carried away at condensation, is called the *latent heat of condensation.* At evaporation just the reverse takes place. A quantity of heat exactly equal to the "fall" energy must be imparted to the molecules in order to carry them out of their mutual spheres of attraction. This heat does not serve to increase temperature. It is called the *latent heat of evaporation* this basis a mixture containing 15 lbs. of air would often be considered quite "lean", i. e., somewhat lacking in power.

The question comes up as to what amount of heat is made available in the case of incomplete combustion. In Article 17 of Part III this question is given more detailed discussion. Here it is sufficient to remark that, for rough and ready information on the subject, it may be assumed that all carbon first burns to carbon-monoxide and all hydrogen to water vapor. If then there is any oxygen left, some of the carbon-monoxide burns further to carbon-dioxide. Under this assumption we can estimate the percentage loss in available energy caused by incomplete combustion.

The conditions for combustion of petroleum fuels will be as shown in the following tabulation:

Pounds of air per pound of fuel ...	10	11	12	13	14	15
Percentage of available energy	53	63	73	82	91	100

From this table it will be seen that a mixture containing only ten pounds of air per pound of fuel will, per pound of fuel, develop less than 60 per cent of the energy obtained from a mixture with 15 pounds of air per pound of fuel. In other words, to obtain the same amount of energy, we would have to burn

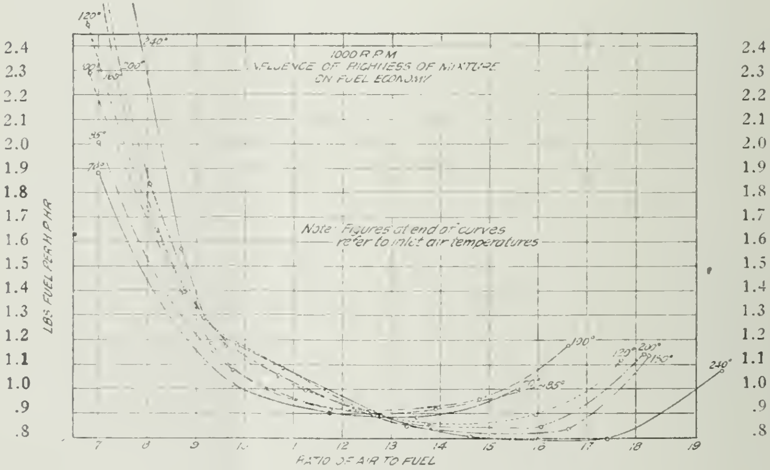

Fig 2.

about 70 per cent more fuel in the case of the over-rich mixture than in the case of the normal mixture with just sufficient air for complete combustion.

In carbureting engines there are secondary influences in favor of the richer mixtures, especially at reduced load; yet in a general way, the above conclusion is borne out by tests on the subject. The results from some such tests carried out at the suggestion of the author in the laboratory of the International Harvester Company are reproduced in Fig. 2. The fuel consumption per horse power hour, for instance with air at 200 deg. fahr., as will be seen, is about 0.85 pounds, with 15 pounds of air per pound of fuel, and as high as 1.1 pounds with 10 pounds of air.

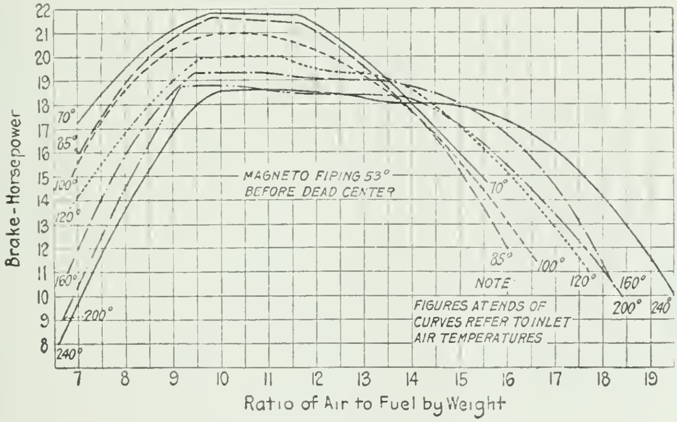

Fig. 3 — Speed 1000 r. p. m.

In spite of this fact, however, the inclination of the motor-using public is always to use the uneconomical over-rich mixtures. The reason for this use is shown in the curves in Fig. 3, from the same set of tests. These curves indicate that the engine gives more power with the richer mixtures than with the leaner mixtures. This becomes more pronounced the cooler the air is and the more throttled the engine is. Under ordinary conditions of driving the engine is very rarely run with wide-open throttle. Hence, especially during starting up periods when the engine is cold, the temptation is ever present to enrich the mixture unduly. These conditions will be studied somewhat more in detail in the article on carburetion and mixture heating. It is necessary only to say here that the main reason for the greater power output with rich mixtures lies in the greater ignitibility of these mixtures and in the more rapid pressure development therewith.

In the so-called Diesel and solid injection type of engines, ignition is secured simply by heating the air by compression to a point where the fuel catches fire. Such engines ignite very readily even with the leanest mixtures. In this fact lies one of the reasons for their low fuel consumption.

ARTICLE 5

CONVERSION OF HEAT ENERGY INTO POWER

SUMMARY. *The function of expansion in converting heat energy into power is explained, and the range of heat utilization by various classes of prime movers is given. A graph showing the relation between expansion and pressure and temperature drop in an average combustion engine reveals the fact that in our present-day combustion engine processes the heat can be only imperfectly utilized with manageable expansion ratios.*

1. In the preceding paragraphs the fact has been set forth that by combustion a violent agitation of the molecules in the combustion gases is produced. This agitation is, however, wholly chaotic. It is not immediately apparent how such a disordered motion can be converted into the well-regulated work of the piston in a reciprocating engine. The method consists simply in utilizing the battering of the molecules against a wall.[4]

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[4] Conditions obtaining in gas and steam turbines have been of great interest in the study of the actual physical processes involved in the conversion of heat energy into power. The steam turbine today is the most important prime mover for stationary purposes, and is gaining ground for marine purposes. The gas turbine has been an engineering problem so difficult of solution that by a good many responsible men it has been considered closely akin to perpetual motion. It now appears that the gas turbine has actually reached almost the stage of commercial exploitation for stationary purposes in Germany. Possibly an automotive gas turbine is not entirely out of the range of possibilities. The author has devoted a good deal of attention to this problem, and would welcome any chance to investigate — experimentally — these possibilities. His experience so far, however, leads him to the impression that, for the present at least, fuel economy in the automotive field and with liquid fuel will not be increased by the turbine *as a prime mover*. It will, therefore, here be left out of consideration. (For more about gas turbines see p. 178.)

The sum total of these impacts on any given area in unit time is the pressure on the wall. As long as the wall stands still, the molecules dashing against the wall are hurled back with undiminished velocity. The minute the wall commences to move under the pressure, the velocity of the recoiling molecules is reduced. This reduction means that the temperature of the gas drops. In other words, chaotic molecular energy has been absorbed into the orderly movement of the piston. In case the combustion is not complete when the piston begins to move, the continued combustion may impart new speed to the slowed up molecules. In case the combustion is complete, however, no such reinvigoration takes place, and the reduced velocity of the molecules will make itself felt both as reduced temperature and as reduced pressure. As the piston moves out, however, the space behind the piston increases and the molecules have to spread themselves over a greater volume. The gas is rarefied, there is a smaller number of molecules per cubic inch, and from this effect the number of impacts on any square inch of piston area is reduced. There is then a double cause for pressure reducton: the slowing up of the molecules, and the increased distance between the molecules. The drop in temperature is determined by the former cause alone. In consequence of this situation, the pressure drops faster than does the temperature.

For any given quantity of gas the relation between the temperature drop, pressure drop and the increase in volume are perfectly definite. The relations often assumed to obtain in the working cylinder of a combustion engine are illustrated by the curves in Fig. 4. It will be seen that when the volume increases ten times, the pressure drops to about five per cent of the initial value, and the temperature only to fifty per cent of the initial value. These curves give an extremely clear picture of the occurrence when heat energy is converted into power. The gas expands and drives the piston outward; the pressure drops. Very soon it reaches values too low to make its further utilization, at least in reciprocating combustion engines, possible. The engines must be designed to stand the highest pressures occurring in the cylinder. These pressures call for parts of considerable strength and weight. If in addition, the dimensions must be such as to accommodate the very great volumes resulting from considerable expansions, then the low pressure finally resulting will hardly be enough to overcome even the frictional resistances of these big and heavy engine parts. In actual combustion engine practice (with one exception) volumetric expansion ratios as high as ten are rarely met with. Seven to eight forms about the limit. It will be seen that under such circumstances it is not possible to convert even half of the heat energy in the gas into useful power.

With respect to utilization of expansion, turbine machinery is superior. Steam turbines can take steam at 300 pounds pressure and more, and expand it down to a pressure of only one-half pound per square inch; a pressure expansion ratio of 600. If such an expansion were feasible in gas turbines, it would be theoretically possible to convert over three-fourths of the available heat energy into work. So far no practical design enabling such expansions has been evolved. Moreover, turbine machinery presents much greater turbulence, cooling and friction losses than reciprocating machinery, and consequently gas turbines actually designed and tested have not as a rule shown a fuel economy approaching that of reciprocating gas engines. (See, however, page 178).

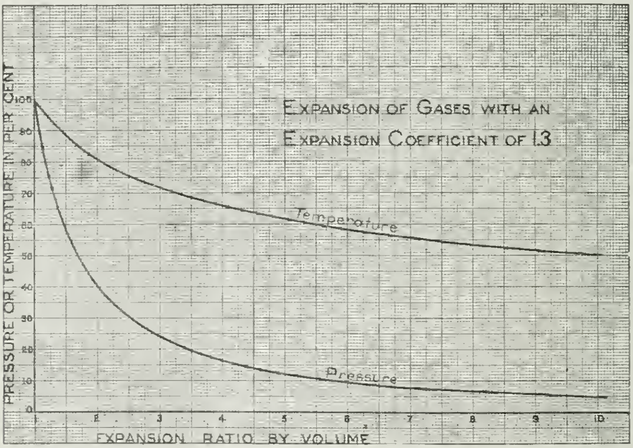

Fig. 4.

The expansion ratios and the heat utilizations up to the present moment attained by reciprocating engines are about as follows:

Type of Engine	Volume Expansion Ratio	Heat Utilization	Corresponding Consumption of Petroleum Fuel Lbs. per hr.
Ordinary Kerosene Engine	3–4	0.16–0.22	0.6 –0.8
Ordinary Gasoline Engine, Full Power	4–5	0.22–0.26	0.5 –0.6
Aeronautical Gas Engine	5–7	0.24–0.29	0.45–0.55
Solid Injection Engine	10–13	0.29–0.33	0.4 –0.55
Diesel Engine	7–10	0.29–0.36	0.36–0.45

The reason that the Diesel engine, in spite of lower expansion ratio, as a rule shows a somewhat better fuel economy than the solid injection engine, lies in the fact that in the Diesel engine the combustion has a tendency to be somewhat more rapid so that the heat development can be more fully utilized by subsequent expansion.

In combustion engines the pressure from which the gases expand is produced partly or wholly by actual compression. In Diesel engines, combustion commences only very close to dead center and continues for a considerable fraction of the stroke, while the piston moves out. All the combustion can do is to counter-balance by increased molecular agitation the pressure drop that would occur from expansion. A combustion at constant pressure takes place. The maximum pressures occurring do not materially exceed the compression pressure.

In ordinary "explosion" engines, combustion takes place more or less suddenly in the clearance space and to a great extent at practically constant volume. A pressure rise in direct proportion to the temperature rise takes place. Maximum pressures are reached which may be from two to five times the compression pressure.

The compression is a charge on the engine output and the Diesel "cycle" is as such less efficient than the "explosion" cycle. On account of the greater expansions possible with the high compressions used for ignition purposes, the fuel economy of the Diesel engine is actually better than that of the explosion engine.

APPENDIX ON EXPANSION.

The idea that it is the expansion which turns heat energy into power and that the combustion for best conversion should be completed as early as possible in the stroke is not immediately apparent even to many engineers familiar with thermo-dynamic reasoning. They know that, if a gas expands isothermally, that is, with no temperature drop at all, then the whole amount of heat used to maintain this temperature is converted into work. This would appear to be 100 per cent utilization. What more can be asked for?

The conclusion is correct — but only as far as it goes. There are actually electro-chemical processes by means of which 100 per cent heat utilization can be secured by something equivalent to isothermal expansion. In gas engine work, however, the situation is somewhat different. For the heat applied to be turned entirely into useful work, the isothermal expansion must be carried out at atmospheric temperature. Otherwise there will be some heat left and going to waste in the exhaust. In combustion engines this expansion at atmospheric temperature can not be carried out. Yet, even if it could be carried out, an adiabatic expansion, that is to say, an expansion with no heat absorption during the expansion, would be superior from the point of view of economy. It should be observed that if a gas can expand at all, it must be at a super-pressure to begin with. This pressure once given, the gas is able to do work without any heat addition at all. In other words, from a gas under pressure not only can 100 per cent heat utilization be obtained, but actually a heat utilization infinitely greater than 100 per cent. Indeed, it is possible even to extract heat from the surrounding atmosphere and convert it into work. As a gas is expanded from atmospheric temperature, it cools below atmospheric temperature. At the end of the expansion stroke it can, therefore, absorb heat from the surrounding atmosphere. This heating of the gas will give further expansion, which again, theoretically at least, is convertible into work.

Without at this point going into the theory of this matter any further, the one safe fundamental idea for combustion engine men to hold on to is that the way to turn heat into work most economically is first to introduce the heat at increased pressure and then convert the heat into work by the expansion of the combustion gases. In the case of carbureting engines there are cases where a more slow burning fuel admits of more compression without knocking than a faster burning one By the use of such higher compression a better heat utilization may be obtained with the aid of the slow burning than with the aid of the fast burning fuel. This fact, however, in no wise invalidates the statement that the maximum possible expansion after the completion of combustion means the maximum possible heat utilization.

ARTICLE 6

CARBURETION AND COMBUSTION IN CARBURETING ENGINES

SUMMARY: *Automobile engines are mostly operated in a throttled condition. Such throttling is shown to mean a very serious increase in fuel consumption per horsepower of output. The reasons hereof are touched on and some proposed remedies are mentioned. Experimentally, extremely important increases in the mileage obtainable from a gallon of gasoline have by some devices been achieved.*

With engines as they are, it is necessary for the best economy that the carburetor should furnish a leaner mixture when the engine is less throttled and a richer mixture when the engine is more throttled.

The carbureting engine such as generally used in automotive vehicles is characterized by the fact that the output is governed entirely by throttling. In ordinary operation of trucks and automobiles only a comparatively small fraction of the possible engine output is used; hence the engine is mostly operated in a throttled condition. This fact leads to inefficient fuel utilization for a variety of reasons. From experiment, the conditions obtaining have been set forth very lucidly by Mr. P. S. Tice, in an article entitled "Carburetion Requirements of a Typical Gas Engine", (Automotive Industries, June 24, 1920, page 1452).[5]

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[5] Though very complete and valuable investigations of carburetor performance have been carried out by Mr. O. C. Berry, while professor at Purdue University (Standards of Carburetor Performance, Trans. A. S. M. E., 1919, p. 333), Mr. Tice's curves have here been selected because they seem to give a more condensed statement.

In Figs. 5 and 6 is given a redraft from Mr. Tice's original curves showing mixture requirements with varying degrees of throttling. The pressures obtaining in the inlet manifold are marked on the curves in millimeters of mercury. Atmospheric pressure at sea level is 760 millimeters. The maximum pressure obtaining in the inlet manifold, as shown by the curves, is 685 millimeters, and throttling is carried to less than half of this pressure, or to 300 millimeters.

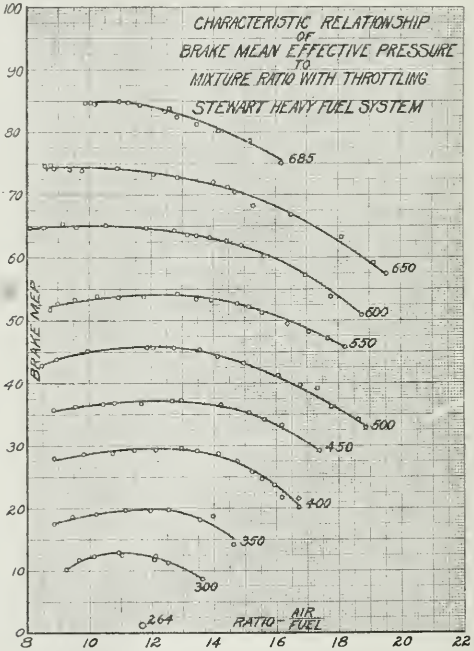

Fig. 5.

In Fig. 5 the effect of such throttling on the power of the engine is shown in terms of brake mean effective pressure. It will be seen that the brake mean effective pressure drops from 85 pounds per square inch at 685 millimeters manifold pressure to about 13 pounds at 300 millimeters of pressure. In other words, throttling to 300 millimeters reduces the output to 15 per cent of the maximum possible. This would probably be the output required for comparatively slow driving on an excellent level road. Fig. 6 now shows that the fuel consumption per brake horsepower hour rises from 0.56 pounds at the maximum output to as much as 1.42 pounds at the minimum output, an increase of over 150 per cent. It may be observed also that it is very much more difficult to hit the mixture ratio giving best economy in the throttled condition than it is in the unthrottled condition. Varying the air fuel ratio from 14 pounds of air to 19 pounds of air at full load changes the fuel consumption very little. Varying the ratio, for instance, from 12 pounds to 14 pounds in the most throttled condition increases the fuel consumption 28 per cent. One reason for this situation is the increased importance of the friction losses at reduced load (see Article 9). The main reason, nevertheless lies probably in the fact that the engine does not expel the burnt gases completely. A certain amount is left in the clearance space at practically atmospheric pressure, no matter how rarefied by throttling the fresh charge is in the inlet manifold. As a result,

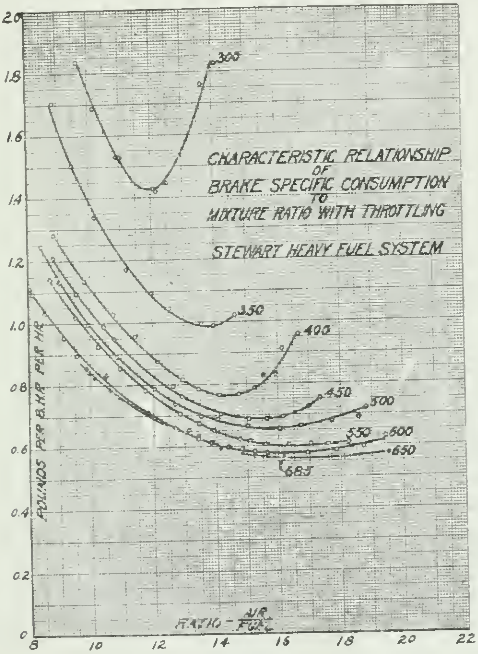

Fig. 6.

the mixture in the engine cylinder will contain a higher and higher percentage of burnt gas the more the engine is throttled down. This pollution of the mixture makes ignition less positive and flame propagation more slow. There is less opportunity for the heat developed to be converted into work by subsequent expansion.[6]

Now, it is a generally observed fact that rich mixtures give more rapid combustion than lean mixtures; hence with rich mixtures a greater percentage of the heat developed is converted into work than with lean mixtures. With the charge fouled by throttling, this better heat conversion compensates for the waste of fuel due to incomplete combustion. With the less fouled charge the combustion is fast enough, even with leaner mixtures, to give satisfactory heat conversion; the requirement of sufficient air for complete combustion then becomes the dominating factor for fuel economy.

The most obvious way of lessening the evil effects of the burnt residue is to reduce the amount of such residue, i. e., reduce the clearance space of the engine. Such reduction, however, increases the compression, and may lead to knocking. (Various means for the prevention, or mitigation, of knock are now known and will be discussed later.) Another way to reduce the amount of fouling is by the avoidance of throttling partly, or entirely. One manufacturer referred to in an editorial in the Automotive Industries of January 27, 1921, accomplishes this by dividing his charge in two parts — one, a throttled part carrying the fuel; another, an unthrottled part introduced through a separate valve, carrying only pure air. Ignition is secured even with the leanest mixtures by introducing a sufficient amount of the fuel carrying part into a pocket containing the spark plug. Various modifications of this scheme, more or less efficacious, have been proposed.[6a] The above-mentioned editorial states that the extra air valve just referred to doubled the mileage per gallon with a car of conventional design.

A third interesting way for reducing the effect of the burnt residue at low load has been tried by Mr. A. L. Nelson, (Journal S. A. E., February, 1921, page 103). This consists in delaying the closing of the inlet valve and simultaneously introducing a higher compression ratio than usual. At low speed, when knocking would be most pronounced, the blow-back through the valve will keep the compression down and prevent the detonation. At high speed the inlet inertia of the charge will reduce the blow-back and allow a higher compression to build up with beneficient effects on power and economy.

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[6] For a complete discussion of the effect of charge dilution and throttling in general, see Mr. Tice's original paper, with references to Fourth Annual Report of the National Advisory Committee for Aeronautics, Report 49, Part V, Plots 22 and 23; and to Bureau of Mines Technical Papers 43 and 121.

[6a] One such is the auxiliary air valve of W. L. Dempsey.

A now well-demonstrated way to accelerate combustion and improve operation even with throttling, lies in the introduction of multiple spark plugs. With multiple spark plugs, therefore, the variation in mixtures would in all likelihood be less pronounced with varying load.

It is very conceivable that a good way to provide correct mixtures for carbureting engines would be to tinker less with the carburetor and more with the ignition system. As automotive engines are mostly constructed today the carburetor manufacturers would, however, have to provide two regulations which may, or may not, be performed by the same mechanical device. In the first place, the carburetor should be adjusted to give the leanest possible mixture that the engine will carry in an unthrottled condition. This adjustment means best economy at full load. In the second place, there should be an adjustment enriching this mixture somewhat as the pressure in the manifold goes down.

The first requirement, which means a constant mixture ratio for various flows, seems to be met fairly well within certain limits by some of the carburetors now on the market. (See investigations by Charles E. Lucke, Second Annual Report of the National Advisory Committee for Aeronautics, pages 471, etc., also Report No. 49, Fourth Annual Report of the same committee, investigations by Messrs. P. S. Tice and H. C. Dickinson). At least it would seem that these carburetors without much change could be made to meet this requirement, if they are not doing so already. The second requirement seems also to be met to some extent by some carburetors; however, in a less premeditated and well-organized way. Thus both the Stewart-Warner and the Stromberg carburetor tested by Mr. Tice showed a decided inclination to mixture enrichment with falling load under throttle:

and possibly the Zenith carburetor could be made to show the same characteristic (even at atmospheric pressure), if the relative proportions of the two nozzles were changed. Likewise some carburetors of entirely different type tested by Lucke, such as the Kingston and especially the Newcombe, show tendencies to enrichment of mixture with increase in pressure drop across the carburetor. These carburetors are not here singled out for special recommendation to the public. There are other carburetors which probably could be made to give, or are giving, equal or better results if properly adjusted with the highest possible fuel economy in view. They are here mentioned to indicate that the problem does not seem insoluble, and that it can be solved in a variety of ways not involving weird and complicated carburetor designs with a multiplicity of moving parts.

The matter, however, is one calling for scientific design and scientific research on the part of carburetor manufacturers every bit as much as for invention.

ARTICLE 7

DISTRIBUTING AND EVAPORATING DEVICES.

SUMMARY: *The steadily decreasing volatility of the fuel for carbureting engines has made the evaporation of the fuel and its distribution to the various cylinders a matter of considerable difficulty. Unsatisfactory working of the engine, dilution of the lubricating oil and formation of carbon are the results. Heating the air or the mixture by means of the exhaust is the usual remedy tried, but this does not prevent difficulties during the starting up period. Fuel converters, using a partial "pre-combustion" to evaporate and gasify the fuel, are less open to this objection. Heating the air or the mixture reduces power on the test stand, but experiments by the Bureau of Standards seem to indicate that, with present-day fuels, heat applied either to mixture or to air improves acceleration. Fuel economy is in all cases improved by heating, as the heating enables satisfactory running on leaner mixtures.*

In the early days of automobiling, with highly volatile gasoline as a drug on the market, the carburetor needed to be nothing else than a metering device. There was hardly any doubt but that, even in cold weather, the gasoline would be almost completely evaporated and thoroughly mixed with the air, most likely already in the inlet passages and certainly at the end of the compression stroke.

With present day gasoline this is no longer true. Even in summer weather probably not over half of the gasoline is evaporated in the inlet manifold (See Article 15, Part III), and in winter weather but a small fraction is volatilized even at the end of the compression stroke. With kerosene as fuel these conditions are, of course, even more accentuated. As a result, the distribution of the fuel is not uniform to all the cylinders of a multiple cylinder engine. In order to make all the cylinders fire, it is often necessary to enrich unduly the mixture in some of the cylinders. The fuel settles in pools in the inlet passages and is swept along into the different cylinders in an irregular manner; sometimes the extreme ones are starved or flooded, sometimes the intermediary ones. A slight difference in height, inclination, or curvature of the inlet passages has an entirely disproportionate effect on the fuel consumption.

The fuel collected in the cylinders during the starting up periods, before enough has evaporated to ignite, flows past the piston rings, dilutes the lubricating oil in the crank case, and produces engine wear, vibration, etc. Other parts of the fuel are disassociated, and form carbon deposits on the pistons and spark plugs. In short, a host of troubles have arisen in consequence of the decreased volatility of the fuel, which have so far by no means been wholly overcome by the engine designers.

The most immediate way to try to cure the troubles would appear to be the jacketing of the inlet passages by the exhaust, or the heating of the air by the exhaust. Both means have been tried. The air heating, for reasons that are shown in Article 16 of Part III, has an unfavorable effect on the power of the engine. Heating the mixture to a point where heavier gasoline, or especially kerosene, is completely evaporated, may mean a power reduction of as much as 10 to 15 per cent—compared, of course, with the power the engine is able to develop with a fuel capable of evaporating in cold air.

In any case, air or mixture heating by means of the exhaust is a means available only after the engine has once started. Churning the engine over on a cold winter day for several minutes before it finally starts and then running it for a number of more minutes in the choked condition with an over-rich mixture, until finally the exhaust has had a chance to accomplish the necessary evaporation, is enough to cause all those troubles of carbon formation and oil dilution in the crank case that have been previously referred to.

For this reason it would be highly desirable to develop means for enabling proper fuel evaporation and distribution from the very first minute the engine turns over. One way to accomplish this is, of course, to carry a small amount of especially volatile gasoline, or ether, for starting purposes. As this method naturally involves a complication in the form of double tanks, double pipes, and necessary cocks and valves for doing the switching from one fuel to the other, it means added chances for leakage and is not immediately incorporable in present-day vacuum feeds, etc.

The most radical way to avoid the trouble would be, as has been done in the case of the Bellem and Bregeras engines in France, to develop the injection type of engine for automotive purposes, so as to do away with carburetion and evaporation in a manifold entirely. If this method is introduced, then, as pointed out before, not only shall we be able to handle present day gasoline in our engines, but we may use almost any kind of liquid fuel, reasonably fluid and free from dirt.

Nevertheless, while this is not only the author's opinion, but the opinion of a number of responsible engineers, it is an opinion that possibly will not determine the line of action to be adopted by the great majority of engineers manufacturing automotive machinery along lines established today. For these engineers it would then seem that the salvation would lie in the introduction of fuel conversion. This fuel conversion may take two forms: Either refineries will be asked by cracking or by chemical processes to produce a more volatile fuel than they are now producing, if they can do so; or, the fuel must be volatilized by apparatus forming a part of the automotive power plant itself. In view of the extreme importance of this issue, two such devices at present in use will herein be described—that of Mr. O. H. Ensign, and that applied to the recent automobiles of the Packard Motor Car Company.

Fuel Converters.

The Ensign fuel converter is shown in section in Fig. 7, (as described in a paper before the Society of Automotive Engineers, reproduced in the S. A. E. Journal for July, 1920, on page 19). This fuel converter uses as a metering device a sort of whirlpool chamber, into which the air enters tangentially and acquires a considerable whirling velocity. The velocity as such causes a pressure drop in the air and the whirling motion in addition sets up a centrifugal force tending to create a vacuum around the fuel discharge openings D. The volatilized parts of the fuel are carried up thru the main passage and into the engine. The non-volatilized parts flow along the wall L, reach the grating U, and drop down into a combustion chamber below this grating. A certain amount of air, or volatilized mixture, also enters this chamber thru the passage V, shown dotted in the main sketch. A spark plug A sets fire to the fuel in the chamber and a combustion takes place which, owing to insufficient air supply, is very incomplete. The combustion products will contain a considerable amount of carbon monoxide, some permanent gases and possibly some fuel volatilized by the combustion. This hot mixture issues thru the stack Z, and is mixed with the main body of the charge passing up and around Z. In consequence of the application of heat from the walls of the tube Z, as well as from the admixture of the products of partial combustion just referred to, a complete volatilization of the

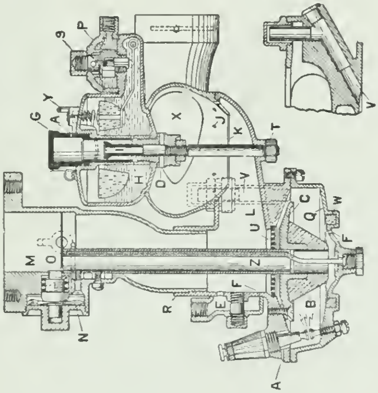

Fig. 7.

whole charge takes place and the same is carried in a slightly pre-heated state into the cylinder.

This device has found considerable application on the Pacific Coast for the burning of so-called distillate, and has also proved

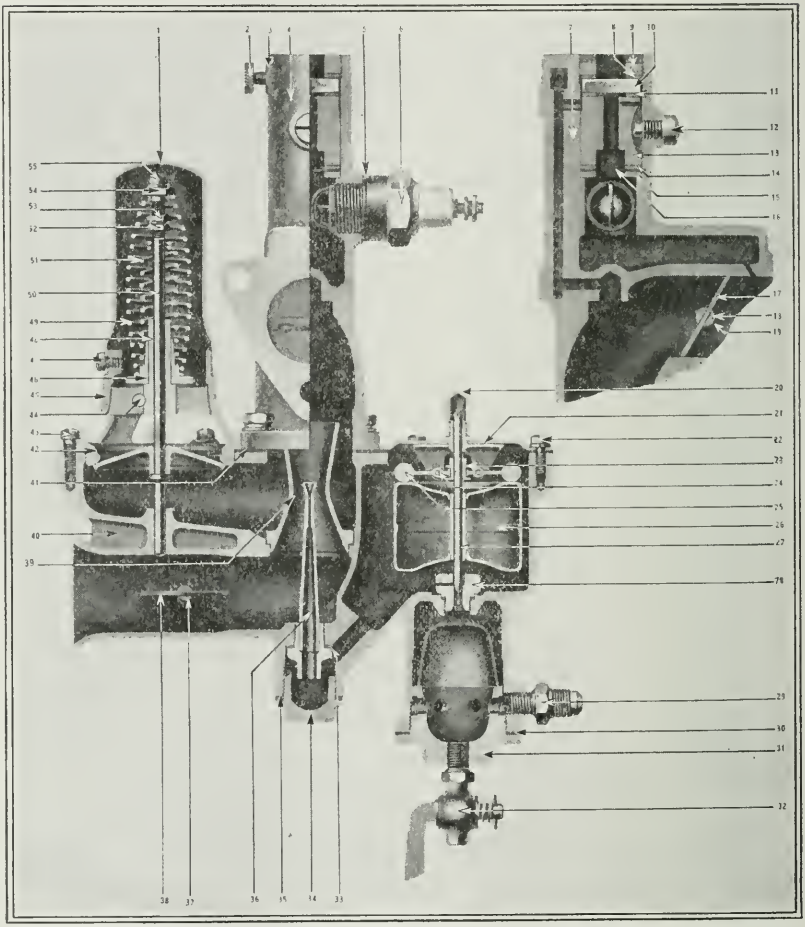

FIG. 8 — Packard Carburetor and Fuelizer.

itself applicable to kerosene. It does not, however, appear to be uniformly successful in all sizes, and does as yet seem to be in the experimental state as far as gasoline is concerned. Nevertheless, it is a device worthy of every consideration.

Very different in design, although not radically different in principle is the "fuelizer" used on the twelve-cylinder Packard Motor Car. This device was described in detail by Mr. J. G. Vincent in the S. A. E. Journal, November, 1919, page 345. It contains an auxiliary carburetor located in a by-pass to the main carburetor. The mixture from this carburetor is fired by a spark plug and is discharged in a burnt, or semi-burnt, state into the main charge. The more the engine is throttled the greater is the proportion of the air going through the auxiliary carburetor, and the greater is the fuel converting and heating effect on the charge. The more open the throttle is, the less is the proportion of air going through the auxiliary carburetor, and the less is the "fuelizing" and heating effect on the charge.

This condition is consistent with the general observation that an engine running at full load is better able to handle non-volatile fuel than a throttled-down engine.

A simplified form of this "fuelizer" is used on the Packard single-six car. It is shown in Fig. 8. Here there is no auxiliary carburetor, but a certain portion of the main mixture itself is by-passed around the throttle, 17-18-19, is ignited by the spark plug 6 and reënters the main stream through the small hole shown.

The writer believes that fuel-converting devices of this kind deserve most serious attention in case our automotive fuel is going to remain what it is, or become increasingly non-volatile. Unfortunately, there appear to be certain difficulties in the way of the patent situation which may cause a delay in the general adoption of such apparatus. In the meantime, air heating and manifold heating in various ways have been subject to thorough investigation by the Bureau of Standards and by a number of manufacturers.

Air or Mixture Heating.

Air heating by means of exhaust has, of course, for a very long time been applied to kerosene burning engines. Owing, however, to the reduction in full load power incident to such air heating, efforts have been made to apply the heat to the fuel rather than to the air. In the Stewart-Warner heavy fuel system, the fuel is sprayed by the carburetor into an exhaust heated bulb and issues from there into the air stream of the inlet manifold.

The Stewart-Warner carburetor appears to be a metering device capable of maintaining a very uniform mixture ratio in consequence whereof the heating effect of the fuel on the air is very uniform. Curves furnished by the company show that with kerosene a temperature of about 180 deg. fahr. is maintained in the inlet manifold and with gasoline about 140 deg. fahr. These temperatures come fairly close to those necessary for complete evaporation of the fuel, especially in the case of gasoline. If, instead of first evaporating the fuel and then heating the air by the fuel, the air going to the carburetor is pre-heated so as to give the same final temperature in the manifold, the temperature of pre-heat for the air would be about 240 deg. fahr. for kerosene and 190 deg. fahr. for gasoline. The evaporation of the fuel in the carburetor would then bring the mixture temperature down to the point noted.

An air temperature of 190 deg. fahr. for gasoline must be regarded as a very high one, and an air temperature of 230 deg. fahr. for kerosene is also not low. The curves in Fig. 3 on page 59 indicate that such temperatures might reduce the full load power of the engine as much as 15 or 16 per cent below the power on cold air.

Mr. A. L. Nelson, in a most noteworthy paper already referred to, (S. A. E. Journal, February, 1921, also reproduced in Automotive Industries, January 27, 1921, page 162), describes a corrugated or ribbed inlet elbow, by means of which fuel evaporation without considerable air heating is said to be obtained. Only a very slight reducton in power is said to occur with this device.

Messrs. O. H. Ensign and D. L. Arnold have tried to reduce the temperatures at the intake port still more by introducing a whirling action in the charge passing through the inlet manifold. The heavier fuel thus centrifugated out is thrown against heated walls and is subsequently "wiped" back into the main stream by the atomizing action at a sharp edge. Good results are said to have been obtained with this system (S. A. E. Journal, November, 1920, p. 456, 457).

In any case the substitution of mechanical atomization in the inlet passages for volatilization means extensive experimentation with every new size and type of engine developed. Even minute changes in dimensions or shape of inlet passages may mean very considerable differences in economy and in general operation.

In the author's opinion the carbonization of the cylinders and the dilution of crank case oil are troubles serious enough to warrant "heroic" measures, if only positive and dependable results are obtained. And the fact seems to be that there is not a very excessive amount of heat available in the exhaust, for mixture heating, especially not at reduced load. Moreover, it appears that test stand power at wide open throttle is by no means identical with accelerative power on the road.

In this connection, tests by Messrs. James, Dickinson and Sparrow of the Bureau of Standards are of extreme importance. (S. A. E. Journal, August, 1920, page 133).

These tests show that, especially for the leaner and more economical mixtures, higher temperatures at the intake ports lead to improved acceleration. Particularly, if the whole manifold is heated, the charge should be very hot for best results. These tests were carried out with a six-cylinder engine at speeds and loads corresponding very nearly to average operating conditions in an automobile run at 15 to 25 miles per hour. They form a safer guide for the designer of engines for automobiles and trucks than do test stand figures for steady load, especially at full power. For the tractor engineer, of course, the full load power is more nearly representative of actual working conditions and for him the efforts to keep down inlet manifold temperature at any cost may consequently be more justifiable. Even for him, however, it should be of more importance to know in what condition the engine is after a certain term of operation in the field, with the system of charge distribution he uses, than it is to know what display figures he can obtain on the stand.

ARTICLE 8

COMPRESSION AND IGNITION

SUMMARY: *By compression the temperature of the charge is raised and may reach the ignition point of the mixture. This is the standard mode of ignition in Diesel and other injection engines, but it requires compression pressures of from 200 to 600 pounds per sq. in. By carbon deposits, local hot spots, etc., premature ignition may result in carbureting engines, this*

trouble being particularly serious with present-day fuel of low volatility. Especially inimical to the improvement of economy by higher compression is, however, in this latter class of engines the phenomenon of fuel knock. This is probably in essence an intense, local partial combustion accompanied by high local pressure. Knock may be abated by good mixing, turbulence, suitable shaping of the combustion chamber, and by the use of multiple spark plugs, etc. An interesting discovery is knock suppression by the addition of small quantities of chemicals to the fuels. For these additions the General Motors Research Corporation has introduced the trade name "compressionizers". The question whether these popularly so-called "dopes" may be produced cheaply enough, in sufficient quantities, and of suitable quality to find general adoption for all automotive fuels is, as yet, an unanswered one. The injection engine should receive careful consideration.

The value of compression in internal combustion engines was by no means clear to the early engineers. Compression as such, of course, is work done by the engine and is a charge against the output. By introducing compression, however, the average pressures during the expansion, or working stroke, are increased nearly in proportion, and as the pressures during the compression stroke are only one-third or one-fourth of those during the expansion stroke the net pressures available are from 66 to 75 per cent of the expansion pressures and will go up also in almost direct proportion to the compression used. As the conversion of the combustion energy into power is due mainly to the expansion of the combustion gas with attendant drop in temperature, the fuel economy, as has been emphasized before, increases in direct proportion to the expansion; and as again the expansion ratio in the great number of combustion engines is practically the same as the compression ratio, this means that the economy increases directly with the compression.

When a gas is compressed, it is heated up, and actually, if friction and cooling losses are neglected, the whole compression work is at the end of the compression present in the gas in the form of heat. It is, therefore, possible to carry the compression to a point where the temperature produced is sufficient to ignite the fuel in a combustible mixture. This is regularly done in so-called Diesel and solid injection engines. In these, the fuel is injected into the heated air only at the end of the compression stroke. In this connection it becomes of interest to know what are the temperatures of "inflammation" of the various fuels mixed with air. Such information is given in Table 8.

TABLE 8

INFLAMMATION POINTS OF FUELS MIXED WITH AIR.

After H. Holm, Zeitschrift für Angewandte Chemie, 1913, p. 273.

For gases.
Dixon & Coward, Journal, Chemical Soc. London, 1909, 95, 1.

Substance	Inflammation Point, Degrees C.	Inflamamtion Point, Degrees F.
Ammonia	(780)	(1435)
Coal gas (Holm).................	(600)	(1110)
Hydrogen (Holm).................	(470)	(880)
Gasoline	415	780
Kerosene	380	715
Gas oil............................	350	662
Roumanian Residue	380	715
Lubricating Oil—		
Engine oil.....................	380	715
Compressor oil................	410	770
Lignite Tar oil....................	370	698
Solid White Paraffine.............	310	590
Coal Tar	500	932
Tar Oil	580	1075
Ether (sulphuric)	400	752
Aldehyde	380	715
Ethyl alcohol	510	950
Acetone	570	1055
Benzol	520	970
Xylol	500	932
Aniline	530	985
Cellulose	360	680
Peat, air dry.....................	280	536
Lignite	250	482
Coal	390	734
Anthracite	440	824
Hydrogen	580–590	1070–1090
Methane	650–750	1200–1380
Ethane (Dixon & Coward).....	520–630	970–1170
Ethylene	542–547	1010–1020
Acetylene	406–440	760–825

On the basis of the figures in Table 8 the author has attempted to compute what compression pressure, or what initial temperature, would be necessary to produce inflammation of such fuels as kerosene and gas oil. To be more exact, the necessary temperature of inflammation has been assumed to equal 400 deg. cent. or 750 deg. fahr. and the compression has been considered to proceed with a heating ratio such as observed in actual combustion engines. The figures so derived are given in Table 9.

TABLE 9

Volumetric compression ratio	Pressure compression ratio	Compression pressure, lbs. per sq. in. abs. (Initial pressure = 14 lbs. abs.)	Final temp. °F (initial temp. = 70° F.)	Necessary initial temp. to produce ignition, degrees F.
10	15.8–20.0	222–280	380–600	305–144
15	25.9–33.8	362–475	445–740	245–75
20	36.6–49.2	512–689	505–840	205–30
25	48.0–66.0	670–925	550–935	175–0
30	59.0–83.2	825–1165	585–1010	150– –25

The higher pressures and temperatures found in this table correspond to a compression exponent of 1.3 and may be obtained in a well-warmed-up cylinder compressing a dry mixture and with piston rings that do not leak. The lower figures correspond to an exponent of 1.2. An exponent even somewhat less than this is shown by diagrams of the Bureau of Standards obtained from a well-known truck and tractor engine of careful design and workmanship.[7] Very possibly the evaporation of the fuel during the compression stroke is here mainly responsible for the slow pressure rise. In so-called injection engines a cold engine or leaky rings should be held responsible.

The table shows that under unfavorable conditions compression pressures even as high as 800 pounds might not as such suffice to ignite kerosene, if the air temperature before compression is 70 deg. fahr. In Diesel engines the actual compression pressure would not run much over 600 pounds; yet ignition is usually positive, even at starting, and in spite of the cooling effect of the entering charge. This ignition may be partly due to the influence of the walls. It appears that almost any solid material can promote ignition "catalytically".

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[7] From blue prints furnished to writer.

Nevertheless, many modern so-called solid injection engines, or semi-Diesel engines, while they run perfectly without special ignition devices when once heated up, must depend upon such extra devices for starting. One arrangement is to heat a part of the cylinder, or a special bulb, by a torch; another, to circulate hot water through the cylinder jacket. Finally, in a recent American engine the initial ignition is accomplished by an electrical heating element in the cylinder. In all cases, these ignition devices need not be kept functioning after the engine is once in regular operation.

As above noted, in all these engines the fuel is not carried into the cylinder with the entering air, as in an ordinary carbureting engine. The question which now comes up, is, what are the conditions for ignition in this latter kind of engines.

Ignition in Carbureting Engines.

From the number of complaints and discussions heard today about such things as pre-ignition, knocking, etc., one would almost think that the greatest difficulty in present day carbureting engines is to keep the charge from igniting too early. Actually, as has been seen, the more volatile fuels require higher temperatures for inflammation in air mixtures than do the less volatile ones. If the attempt were made to ignite a volatile gasoline by heat of compression only, compression as high as five to six hundred pounds might be needed. Yet, designers of kerosene engines have long found it necessary either to prevent "pre-ignition" by water injection or to keep the compression down to 50 or 60 pounds. As the composition of gasoline is developing today, there is a tendency to reduce the compression even for gasoline engines toward values of this order of magnitude in order that dissatisfied customers may be avoided.

All these troubles are not due, however, to ignition by heat of compression as such, but are due to ignition from some outside source. To begin with, it may be that in the course of operation certain parts, such as the exhaust valves, or the head of the piston, or even imperfectly cooled parts of the heads or cylinder walls, may become hot enough to ignite the charge prematurely. Imperfectly designed or placed spark plugs have no doubt often been responsible in this respect.[8] If the engine gets badly coated with carbon, then the trouble is aggravated. Heat is conducted away from such carbon much less readily than from the metal parts and ignition from semi-incandescent carbon accumulations results. This trouble is accentuated by the low volatility of present day fuels and by the flooding of the cylinders with liquid fuel during starting up periods. There is not sufficient air to burn this fuel. The heat of the first explosion, however, causes it to "crack" or dissociate, and converts it into carbon deposit.[9]

The pre-ignition trouble from hot spots other than carbon may in a large measure be overcome simply by good design, in which respect at present undoubtedly much progress has been made. The main source of knocking now lies in the regular ignition system, and the trouble may not consist in too early ignition, but rather in partial ignition.

The reason that kerosene and gas oil inflame at a lower temperature than does light gasoline, or a permanent gas like methane, lies in the fact that the molecules of the heavier petroleum hydrocarbons are, so to speak, more loose-jointed than those of the lighter ones. The atoms are strung together in long, snaky chains. By the agitation incident to ignition these chains are easily shaken to shreds and in the general break-up of the structure, the oxygen finds ready points of attack. This easier ignitibility of the heavier hydrocarbons would at first sight appear to be only an advantage, if the fuel mixture at the instant of ignition were homogeneous and homogeneously heated throughout. With the heavier fuels, however, the mixture is apt to lack uniformity. There may be certain pockets here and there where an especially ignitible mixture occurs. It is possible also that the fuel mixture adjacent to certain hot parts of the combustion space has become more heated than others. As ignition takes place, the flame may at first spread quite slowly from the spark plug through the combustion space. The instant it hits such a pocket of readily ignitible, or overheated charge, this part of the charge may all at once be brought to the point of inflammation and be touched off almost explosively. A considerable local pressure rise takes place before the adjacent cooler mixture parts have time to recede, and this local super-pressure dashes through the combustion space like a wave, hitting walls, rebounding, and setting up sounds of detonation. The engine "*knocks*".

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[8] For some interesting experiments on pre-ignition caused by spark plugs, see Journal S. A. E., Feb. 1920, p. 129. Sparrow: Preignition and Spark Plugs.

[9] It is a well-known fact that a good deal of the so-called carbon found in cylinders may be made up of road dust, or metal particles from the engine elements. A considerable part of the actual carbon accumulated during the starting-up periods may subsequently burn up. The fuel, however, to a very large extent furnishes the cement holding these accretions together, although the lubricating oil also plays its part.

The remedy for this disturbance would, of course, seem to be, in the first place, homogeneous mixing; then turbulence, tending to prevent the formation of overheated — or overrich — pockets; lastly, shaping the combustion space in such a manner as to facilitate the orderly propagation of the flame through the charge.

The influence of turbulence in facilitating combustion has long since been known.[10]

Researches by Prof. Harold D. Dixon in England, first published in 1893, and subsequently in 1903, in the Philosophical Transactions of the Royal Society of London, show, however, that even a flame propagation, which begins in a perfectly steady and orderly manner, may speed up and at last reach a state of detonation. When this state is reached, the chemical reaction seems to take place in a very thin so-called wave front, or zone of combustion. In this zone a tremendous local pressure is set up; this high pressure region travels through space with a velocity in excess of the velocity of sound, and finally on hitting a wall gives the resounding hammer blow that is called "knock".

Without any knowledge of Prof. Dixon's investigations the engineers of the Dayton Metal Products Company, (now the General Motors Research Corporation), had in conjunction with the Bureau of Mines, and at the instigation of Mr. C. F. Kettering, commenced a research on the true nature of knocking in engines, with a view to finding some way of suppressing it. These investigations subsequently by Mr. H. C. Dickinson of the Bureau of Standards and Mr. H. L. Horning were connected up with the work of Prof. Dixon in England. Out of this coöperation of able men a great deal of light was shed on the subject of knocking. For a comprehensive account of this whole matter a paper by Mr. Thomas Midgley, Jr., in the Journal of the Society of Automotive Engineers for December, 1920, should be consulted. (See also a paper by Prof. Dixon on "Alcohol as an Engine Fuel", in the same number of the same Journal). Here it must suffice to say:

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[10] See experiments of Prof. Hopkinson in England, cited by Dugald Clerk, London Engineering, July 4 and 11, 1913, pp. 28 and 58. For some more recent observations in this respect see Honoman and McKenzie, Journal of the Society of Automotive Engineers, February, 1920, page 122. A very interesting paper on turbulence is to be read at the summer meeting of the Soc. of Aut. Engrs., by Mr. H. L. Horning, just as this bulletin goes to press (May, 1921).

1. That the detonation seems to depend on the preponderant combustion of hydrogen and that it, therefore, is apt to be found especially in fuels very rich in hydrogen; preëminently, however, in the more loose-jointed hydrogen fuels, those that can be most readily disrupted. The combustion in the rapidly progressing wave is too fast for the carbon to be consumed, and the carbon in consequence separates out. The exhaust becomes black and the power of the engine drops, due for one thing to reduced heat development from incomplete combustion.

2. That the fuels containing fewer hydrogen atoms are less apt to knock, and especially that those in the molecules of which the atoms are tied firmly together in rings present hardly any tendency whatever to knock. The typical fuel of this class is benzol. Another is cyclohexane, a fuel brought to the attention of the engineering world by the investigations in Dayton.

3. That it is possible by the addition of certain substances to the fuel, to suppress the knock. One such substance is iodine. For commercial purposes the Dayton engineers have among other things proposed certain so-called benzol derivatives, aniline, xylidine, etc. Mr. H. R. Ricardo, in an article just received, recommends toluol in considerable quantities.[2]

The end in view is to make it possible for the engine to carry a very high compression without disturbed operation and without loss of power due either to too early ignition, or to too disturbed or incomplete combustion. Both benzol and cyclohexane are fuels capable of carrying compressions of close to two hundred pounds, which are about as high compressions as can profitably be introduced. Higher compressions may be necessary for auto-ignition in injection engines, but lead to leakages around piston rings, heavy dimensions, considerable friction losses, etc.

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[2] According to oral statements of Mr. Midgley, the General Motors Corporation proposes to introduce as a trade name for these knock suppressers the word *"compressionzers."* For more about Ricardo's work, see pp. 85 and 88.

However, while benzol is a commercial fuel, it is not yet available in quantities sufficient for general consumption; and cyclohexane, an artificial fuel produced from benzol, will probably always remain too expensive to be considered for commercial use.[11] The proposition of the General Motors Corporation is, therefore, to sell to the motor users these anti-knock materials, or "compressionizers", which may either be added to ordinary gasoline or kerosene to the extent of two or three per cent in order to give a non-knocking fuel, or may be injected into the engine at times when it shows a particular tendency to knock. This is the case, for instance, when the engine runs slowly at full power, as in hill-climbing. Some of these substances, such as aniline, are said to be efficient also as carbon removers and may be sold to the public to begin with, at least, for use in ordinary engines on the strength of this property. Later on, the idea is, of course, for the motor trade to introduce engines with increased compression and higher economy, made possible by the use of these doped fuels. Mr. Midgley[12] speaks of about 50 cents' worth of aniline being sufficient for 500 miles of automobile operation, a charge of one-tenth of a cent per mile. Little can be said against a charge so infinitesimal, when such important results are obtained. A somewhat different situation is, however, presented if a general doping of gasoline to the extent of perhaps two or three per cent is proposed. Such an addition would necessitate the creation of a whole new industry for the production of the dopes and the public would be subjected to an extra charge probably amounting to a few cents per gallon, in order to enable engine design for better economy.

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[11] Both benzol and cyclohexane have the disadvantage of congealing above the freezing point of water. A mixture of both in the ratio of 20 or 30 per cent benzol to 80 or 70 per cent cyclohexane congeals only at a temperature down to 40 deg. below zero fahr., and was tried out during the war under the name of Hecter Fuel. Tests by the Bureau of Standards, Automotive Industries, June 10, 1920, p. 1336, show this fuel, however, to be so slow-burning that it actually gives less economy than gasoline. In fact, at least in fast airplane engines, it passed out into the exhaust with its available heat only imperfectly developed. It does, however, with the higher compression that it is able to carry, give 10 per cent more power than gasoline.

[12] The efficacy of fuel doping by means of small quantities of chemicals has been flatly denied by H. R. Ricardo (See Automotive Industries, April 14, 1921, p. 805). At the time of sending this bulletin to

Therefore, while the author is full of admiration for the work done by the Dayton engineers, and while their results may indubitably be acclaimed as a new and important victory for American experimentation, at the same time it does not appear that their method of improving fuel economy should make it superfluous for the engine designer to try to attain improved engine economy simply by the direct method of adequate design. One way in which this can be done, with ordinary carbureting engines, was arrived at by experimentation independent of that concerned with knocking, but has been further elucidated by these latter researches. This method consists in the combination of a number of spark plugs with a suitably shaped combustion space. Messrs. Honoman and McKenzie of the Bureau of Standards have succeeded in directly measuring the velocity of flame propagation in a Liberty engine cylinder under actual working conditions. (Journal of the Society of Automotive Engineers, February, 1920, page 119.) From their results it would appear that the velocity of flame propagation is of the order of magnitude of 21 feet per second through distances of about one inch from the spark plug. The velocity becomes greater as the flame proceeds and is about 41 feet per second

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press complete details of Ricardo's investigations on this subject have not reached the author.

Mr. Midgely orally makes the following statement:

"The use of 0.4% diethyl–telluride with commercial Pennsylvania gasoline (end point over 400 deg. Fahr.) makes this fuel a perfectly satisfactory fuel to operate with a compression ratio of 7:1. In tests at McCook Field mean effective pressures of 165 lbs. per sq. in. were attained at full load by means of addition of 7 per cent of aromatic amines to Standard Domestic Grade aviation gasoline, with a Liberty engine of 6.5 to 1 compression ratio."

over distances 4 inches from the spark plug. Now, for full utilization of the heat of combustion, it would be desirable that the flame should spread through the combustion space in the time occupied by, say, one-tenth of a revolution. In Table 10 are given the cylinder dimension, the engine speeds, corresponding to a car speed of twenty miles per hour, and the time corresponding to one-tenth of a revolution for a number of well-known cars. It will be observed that the time in which it would be desirable to have the combustion complete itself is little more than 5/1000 of a second. In this short time interval the flame, according to the investigations just referred to, travels only one to two inches from the spark plug. This means that unless the spark plug is located right in the middle of the cylinder head and the combustion space is very thin, the flame will not spread through the cylinder of any of the engines noted, in the time desired. Thus also from the point of view of sufficiently rapid combustion, it will be desirable to have two spark plugs. A gain in power resulting from such an arrangement has already been pointed out on page 69. Now if these spark plugs be located on opposite sides of the combustion space, the flames from the two plugs will travel against each other, and will most probably impinge on each other and complete the inflammation before they reach the detonation stage. Actually tests by Captain Hallet at McCook Field (Journal of the Society of Automotive Engineers, November, 1920, page 477), showed that even

TABLE 10

Name of Car	Cylinder dimensions	Engine speed at 20 miles, r. p. m. (1920 Models)	Time in seconds for 1/10 revolution
Buick	3⅜ x 4½	815	.0074
Cadillac	3⅛ x 5⅛	853	.0070
Dodge	3⅞ x 4½	875	.0069
Essex	3⅜ x 5	1066	.0056
Ford	3¾ x 4	811	.0074
Hudson	3½ x 5	970	.0062
Hupmobile	3¼ x 5½	1020	.0058
Overland	3⅜ x 4	1010	.0059
Packard	3 x 5	840	.0071
Pierce–Arrow	4 x 5½	725	.0083
Pierce–Arrow	4½ x 5½	680	.0088

in an L-head truck engine, knocking badly at 500 r. p. m., all knocking was suppressed instantly when a second spark plug was put into action.[13]

Water Injection.

It has long been known that water injection reduces knock and makes it possible to carry a higher compression. Water injection is, however, a complication, and the carrying of clean water for injection purposes in addition to fuel, is a considerable inconvenience in automotive operation. It should also be observed that water acts as a knock remover simply by absorbing in its evaporation some of the heat of combustion. If no increase in compression accompanies the water injection, both power and economy of the engine are, therefore, apt to suffer. The reputed efficiency of water in small quantities as a carbon remover has been rendered highly questionable by tests of the Bureau of Standards, (Automotive Industries, 1919, page 481).

Appendix. — Too late for complete consideration in the main text, there is appearing in Automotive Industries a series of articles by Mr. H. R. Ricardo of England. In the first of these articles (Means of Materially Increasing Thermal Efficiency; Auto. Ind., February 24, 1921, p. 456), Mr. Ricardo, besides dealing with practically all the suggestions for improvement in economy referred to above, recommends in addition the use of cooled exhaust to prevent knocking. He states that by this device volumetric compression ratios as high as 7 can be used with aviation gasoline, and that with such compressions fuel utilizations of close to 30 per cent are attained. As a knock suppresser, or, perhaps, rather, as a non-knocking material, Mr. Ricardo especially recommends toluol (toluene), although quantities as high as 20 per cent must be added to the fuel.

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[13] Just as this bulletin is going to press there appears in the S. A. E. Journal for May, 1921 (p. 487) an article by Dr. H. C. Dickinson, containing some interesting information on the effect of two spark plugs on detonation. According to this article it is possible, with certain spark timing, to produce detonation with two spark plugs and avoid detonation with one. The conclusion is drawn that the nature of the combustion and the pressure development depend not so much on the timing of the spark as on the timing of the actual "inflammation," as brought about by the spark. Two spark plugs give a smaller interval between spark and inflammation than does one. It is possible for one spark plug to produce inflammation so late in the stroke that the combustion occurs mainly under falling pressure, that is to say, under conditions not conducive to detonation. Two spark plugs fired with the same timing might cause the inflammation to come earlier at, or near the peak of the pressure, under conditions more conducive to detonation. If this is true, however, then low power and economy should certainly be caused if detonation is avoided by the use of one spark plug instead of two. On the other hand, if detonation is avoided by means of two spark plugs, combustion earlier in the stroke should be secured and, hence, good power development.

ARTICLE 9

INFLUENCE OF FRICTION LOSSES.

SUMMARY: *Friction losses in most internal combustion engines consume only 10 to 20 per cent of the indicated power at full load. At the reduced engine output of ordinary automobile driving, these losses, however, may consume as much power as does the propulsion of the car. Piston friction forms the main part of these losses and some attempts to reduce friction losses by suitable piston design are referred to.*

The friction and the mechanical losses in airplane engines were investigated very carefully by Mr. J. Edward Schipper at McCook Field, (Automotive Industries, page 1338, June 10, 1920.) The mechanical losses of a great many English automotive engines are given in Clerk and Burl's "Gas, Petrol and Oil Engines", Vol. 2. As a rule, it is found that the friction losses of automotive engines vary between 10 to 20 per cent of the actual work of the gases behind the piston at full load, so that only 80 to 90 per cent of this indicated work is available as brake work at the shaft. Of these friction losses, piston friction is found to constitute practically 50 per cent. In other words, from 5 to 10 per cent of the indicated power goes to overcome the friction of the piston against the cylinder walls.

To some this might not seem like a very great amount. Unfortunately, however, these frictions do not decrease with reduction of load. In ordinary city driving of automobiles the engine output perhaps is only 15 per cent, or roughly one-seventh of the maximum output. Supposing the friction losses to remain the same as at full load, they would then amount roughly to 35 to 70 per cent of the engine output. In other words, under unfavorable conditions nearly as much fuel would be burnt to overcome piston friction alone as is consumed in driving the car.

Reduction in this piston friction has, therefore, become a matter of great importance. To begin with, the tests of Mr. Schipper show that the friction power increases at a more rapid rate than does the horsepower output. At 1000 r. p. m. the total mechanical losses of the engine investigated by Mr. Schipper were only 13 per cent; at 2000 r. p. m. they had risen to 21 per cent of the indicated horsepower. In other words, for good efficiency, engines should not be run at very high speeds. Many prominent designers have, however, attempted to evolve special piston designs tending to reduce the piston friction. One such design which has become rather famous, is that which Mr. Ricardo applied to his tank engines, of which as many as 3,000 were used by Great Britain in the last year of the war. (For description see Automotive Industries, February 20, 1919, page 407). In this design the side thrust of the connecting rod on the piston pin is taken on a special guide sleeve of smaller diameter than the piston itself. The piston is short and the lubrication thereof is accomplished simply by oil mist carried up through slots by the air going to the carburetor.

That such a scanty lubrication should be sufficient to keep down piston friction and seal up the rings seems surprising. Actually, however, as reported by Mr. Hermann Lemp of the General Electric Company before the S. A. E., (S. A. E. Journal, May, 1920, page 354) it has in Diesel engines been found that a heavy oil may serve simply to gum up the rings and cause either sticking or leakage. His remedy was to pump pure kerosene by means of a Kipp lubricator onto the piston rings. In spite of the high maximum pressure, this gave satisfactory sealing and good operation generally.

Some piston designs not quite as radical as those of Mr. Ricardo are described by Mr. H. L. Pomeroy in the January number of the Journal of the S. A. E. for 1920. He refers, for instance, to the well-known Zephyr piston, now becoming noted for its lightness and its low friction. In this piston the ring of the bearing surface around the wrist pin is cut away entirely. In designs of Mr. Ricardo other than that of his tank engine the piston is provided with long bearing surfaces only where the side thrust is actually to be taken, but not at right angles to this thrust.

Mr. A. L. Nelson, in a paper already referred to (Journal S. A. E., February, 1920, Automotive Industries, January 27, 1920), describes other pistons of ingenious and refined design, intended to reduce at once friction and piston slap.

While piston friction forms the main part of the friction losses, crank shaft and auxiliary losses in the single cylinder engine tested by Mr. Schipper amounted at 1700 r. p. m. to 21 per cent of the total mechanical loss. This loss may seem considerable, but it will, however, form a much smaller percentage in multiple cylinder engines, since the number of bearings does not increase in direct proportion to the number of cylinders.

ARTICLE 10

THE POSSIBILITIES OF STEAM

SUMMARY: *The fuel utilization obtained by steam has inherent limitations because of the heat spent in evaporating the water. By the use of high pressures and superheats, (and, also, in stationary plants, by condensation at high vacuum), the fuel economy of steam plants is, however, much improved. Actually, steam automobiles in existence use no more fuel per mile travelled than do combustion engine automobiles of equal weights. The steam automobile can, however, readily be made to consume even the most non-volatile liquid fuels; in fact, in England steam-automobiles are run on coal.*

It has already been hinted that, if the heat derived from combustion is used for the production of steam, the conversion of water into steam consumes by far the major part of the heat, and leaves but little available as motional energy in the steam itself. The part of the heat gone to motional energy can, however, be increased by super-heating the steam. That is, the steam, after being generated in a boiler, is taken, usually, to a system of pipe coils where it is heated by outside flame to a temperature more or less above the boiling point of the water.

If the steam is subjected to condensation after it is used in the engine, then the amount of energy at least theoretically available, is very much augmented In steam turbine work it has been found possible to keep the pressure in the condenser as low as one-half pound per square inch. A pressure of one pound per square inch is quite common. To maintain such low pressures it is necessary, however, to have available a copious supply of fairly cold water. Such a supply is not available in automotive practice. The steam in steam automotive vehicles is now mostly condensed. This condensation, however, is carried out in an ordinary air-cooled radiator which leaves the water at a temperature fairly close to its boiling point and at atmospheric pressure. This is done simply to avoid waste of water and the necessity of frequent replenishing of the water supply. In the cylinders themselves of automotive steam engines, either of the so-called "uniflow", or of the "compound" type, the steam is not expanded to anything approaching atmospheric pressure. It is doubtful whether these engines will ever turn into power over 60 per cent of the energy available if the steam were expanded to this pressure.

TABLE 11

BOILER PRESSURE 600 POUNDS PER SQUARE INCH ABSOLUTE

Pressure in pounds per square inch abs.	Temperature of steam in degrees Fahrenheit	Theoretical maximum utilization in %[44]	60% of theoretical maximum in %
600	486.5	23.2	14.0
600	600	24.0	14.5
600	700	24.7	14.8
600	800	25.4	15.3
600	850	25.8	15.5

TABLE 12

TEMPERATURE OF STEAM 700 DEG. FAHR.

Pressure in pounds per square inch abs.	Temperature of steam in degrees Fahrenheit	Theoretical maximum utilization in %	60% of theoretical maximum in %
300	700	20.9	12.6
400	700	22.3	13.4
600	700	24.7	14.8
800	700	25.8	15.5

In Tables 11 and 12 are given some data showing the fuel utilization obtainable with steam expanded from the boiler pressure noted to a pressure of 16 pounds per square inch absolute, also practical heat utilizations equal to 60 per cent of the ideal ones. It will be observed that a heat utilization between 15 per cent and 16 per cent is the maximum looked for. This can be obtained, however, either by using a higher super-heat with a somewhat lower pressure, or by using a high pressure with a somewhat lower super-heat. Nevertheless, it will be observed that under the assumptions made, it is more fatal to let the pressure drop from 600 to 300 pounds than to let the temperature of the steam drop from 700 deg. fahr. to 486.5 deg. fahr. This latter is the temperature of saturation, that is to say, no super-heat at all.

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[44] Feed water temperature 125 deg. fahr.

The difference between the theoretical maximum utilization and the practically obtainable utilization involves boiler losses, losses due to incomplete expansion, finally frictional losses and cooling losses of the engine.

The utilization here arrived at is low compared with those possible with internal combustion engines as noted on page 63. The steam engine has, however, one advantage over at least the carbureting combustion engine: The efficiency is much better maintained at reduced load if the reduction of load can be brought about by what is termed reduced cut-off; i. e., by reducing the amount of steam admitted into the cylinder at every stroke without reducing the pressure. This means increasing the expansion with reduced load, hence a better economy, as long as cooling losses or friction losses, compared with the available steam work, do not become too preponderant.

Steam automobiles with high grade steam plants are, therefore, found to give a fuel mileage fully as good as that of internal combustion cars of equal weight and power. Nevertheless, the maximum possible economies with steam have their hard and fast limitations. It is possible that by the introduction of internal combustion engines designed with a special view to economical operation at reduced load, heat utilizations could be obtained not within the reach of the steam motor.

The steam motor has, however, another very important advantage. This is that without very many uncertain and complicated devices it can be made to run on almost any kind of liquid fuel and can, in fact, be run on coal. Most of the steam automobiles at present developed run on kerosene. The fuel atomizing systems of at least some of these automobiles are, however, such that there should be no difficulty in using even crude oil or light fuel oil instead. In England some steam vehicles run on coal and do not appear to be very inconvenient to take care of. One steam plant, developed by Thomas Clarkson and described in the Automotive Industries of November 28, 1918, page 919, requires, in a 3-ton truck, replenishment of fuel every 12 to 15 miles and of water every 30 to 60 miles. This is probably far more trouble than an American truck driver would care to submit to. It, however, is hardly more work than could reasonably be expected from a tractor operator. The distribution of fuel oil or crude oil to automobile users generally may not be feasible. There should, however, be no insuperable difficulties in the way of distributing such fuel in barrels or tank wagons to the farms.

For the description of a very interesting modern American automotive steam plant see Journal of the S. A. E., November, 1919.[15]

ARTICLE 11

THE POSSIBILITIES OF INJECTION ENGINES

The injection engine in large size is at present a perfect success for marine and for stationary service, and is rapidly gaining ground. It is at present the most economical prime mover built, and meets the requirements of the present fuel situation in the two points where the carburcting engine so far has failed most signally: (1) *It can utilize even cheap, heavy, non-volatile fuel, such as crude oil, residue, and even coal tar;* (2) *its high fuel economy is well maintained at reduced load. The various types of injection engines are described, and at-*

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[15] In the field of prime movers depending on the evaporation of liquids there are some interesting new developments, which may perhaps some day increase the economy of these plants considerably. Just at present it would appear that the general introduction of high pressure and high superheat in steam plants generally will give very considerable improvements in fuel utilization, and this by the application simply of good high grade engineering rather than by more uncertain inventions. At any rate, in discussing the possibilities of fuel economy by steam in automotive practice, the writer has deemed it best to concentrate the attention on the more certain and immediate possibilities.

tempts to adapt these engines to automotive purposes are reviewed. It is found that objections in point of weight and complication are being overcome, and have, in fact, in some instances been overcome.

While in the field of automotive engineering, as the fuel becomes less and less volatile, the carbureting engine is giving rise to an increasing amount of troubles and dissatisfaction, the injection type of engine is making triumphant progress on land as well as on the sea for power production on any kind of liquid fuel. It may be well, therefore, to cast a glance at the characteristics of this type of engines, and eventually on the chances of such engines being adopted also for automotive purposes.

In the injection type of engine, air alone is charged into the cylinder at or before the beginning of the compression stroke, and this air alone is subject to compression. Usually near the end of the compression stroke fuel is injected through a spray nozzle of one kind or another in a state of more or less fine subdivision. The method of fuel injection and the method of ignition separate the injection engines into a number of different groups:

1. *Vaporizer Engines.* Engines actually vaporizing the injected fuel in so-called vaporizing chambers into which the air is subsequently compressed. As soon as sufficient air enters combustion takes place, the heat of the vaporizing chamber sufficing for ignition. This is the low pressure type of oil engine, the compression pressure ranging as a rule between 80 and 200 pounds.

2. *The Hot Spot or Semi-Diesel type of Engine.* In this type the fuel is sprayed into the cylinder towards the end of the compression stroke and is not actually vaporized. It, however, is sprayed against a heated spot or against a heated bulb and the heat of this bulb or spot is sufficient to cause ignition. The compression pressure of such engines may range from 200 to 300 pounds per square inch.

3. *The Diesel type of engine.* In this type of engines the compression pressure is carried to five or six hundred pounds per square inch, and the heat of compression is sufficient to ignite the injected fuel. The injection of the fuel is accomplished by means of air of still higher pressure than that of compression. A very fine subdivision of the fuel results, as

well as excellent mixing with the combustion air. As a result, this type of engine is able to operate on the very heaviest type of fuel, even on coal tar. The multistage compressor needed to provide the injection air is, however, a serious complication, and the injection air, on expanding into the cylinder, has a cooling effect which tends to render very high compression pressure necessary for ignition. The Diesel type of engines is now manufactured in all parts of the world, and is the dominating type of marine combustion engine.

4. *Solid-injection engines.* To avoid the complication of the high pressure air compressor, determined efforts have of late been made to inject the fuel in high compression engines simply by pressure on the fuel itself. This pressure may be maintained steadily on a small fuel storage, as is done in the recent Doxford cold starting engine.[16] Usually, however, the pressure is only momentarily maintained by the fuel pump in forcing through the injection valve a measured quantity of fuel. In America, a representative of this class of engine is the Price-Rathbun engine, now manufactured, for instance, by the Ingersoll-Rand Company. An injection device of a peculiar kind is used in the Hvid-Brons engine, sold by the Sears-Roebuck Company in Chicago. Abroad, the solid injection engine of the Vickers Company has gained particular attention in the marine field.[17] The compression pressure used in solid injection engines varies all the way from 200 pounds in the Price-Rathbun up to almost practically full Diesel pressure in the Hvid-Brons type of engine. The tendency is to keep the pressure as low as possible, but in so doing the designer often finds it necessary to provide special starting arrangements in order to set the engine going and keep it going until it is warm enough to fire by heat of compression alone. Thus in the Doxford engine hot water is circulated in the water jackets before a start is attempted; in the case of the Price-Rathbun engine an electrically heated ignition plug is provided, which, however, can be disconnected after the engine is once in operation.

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[16] The Engineer, July 18, 1919, p. 64.

[17] For an American description of the Vickers principle see for instance a paper by Fred C. Ziesenheim at the summer meeting of the S. A. E., 1921.

The highest economy obtainable by heat motors today is obtained by the last two classes of engines, the Diesel and the solid injection engine, with the Diesel probably yet somewhat in the lead.

The Diesel engine was originally conceived to burn even coal dust and to burn it at such a low temperature that no water jacketing was necessary. For this purpose a "constant temperature" cycle was proposed, in which the fuel was to be burned during the expansion stroke at such a rate that the temperature rise resulting from combustion would be evenly counter-balanced by the temperature drop resulting from expansion. This proved impracticable. The typical Diesel cycle is now a constant pressure one; that is, the drop in pressure resulting from expansion practically counter-balances the rise in pressure which would result from combustion in an unvarying space. In solid injection engines the combustion takes place to a considerable extent in the clearance space, a marked rise in pressure resulting. The fuel is nowadays the heaviest kinds of liquid fuel.

The first attempt to apply the Diesel engine to marine purposes was probably made by the firm of Nobel Brothers in Petersburg, Russia, as early as 1903. Not very long after the first experimental Diesel engine had appeared before the public, this firm had on the Volga two vessels running, the Vandal and the Sarmat, equipped with Diesel engines for propulsion. These boats were of 1100 tons capacity. Not much later the same far-sighted firm dared to equip tankers of much greater tonnage with Diesel engines for open sea navigation on the Caspian. A great many troubles were encountered, but finally conquered. The first ocean-going vessel equipped with Diesel engines was built by the firm of Burmeister & Wain for the East Asiatic Company in England, and made its first trip in 1912. In the same year Dugald Clerk and G. A. Burls give ten other Diesel ships as either built or building, all by firms on the European continent.

We cannot here follow further developments. Suffice it to say that today there are running on board ship 325 Diesel engines which propel a total of 1,263,000 dead-weight tons. (From "The Gas Engine", 1921, page 13). The Diesel ship is no longer an experiment. In fact, as already stated, at a recent fuel congress in Washington, Mr. M. L. Requa, former Oil Fuel Administrator of the United States, very seriously put the question whether steam machinery for marine propulsion should now be regarded as obsolete, at least in connection with oil. There is little question about the justification of this stand from a fuel-economy point of view. The Diesel engine can, in continuous commercial operation, turn into power 30 per cent of the fuel energy; on the test stand more. There is no question that from 50 to 70 per cent of oil fuel used for marine propulsion by means of steam could be saved by the introduction of injection engines. Other advantages resulting from Diesel engine drive have been noted on page 24.

While during the stress of war the United States Shipping Board did not have time to wait until Diesel engines could be supplied for all the ships contemplated, there is little question that the economic advantages of the Diesel engines are now fully appreciated, and that these engines will make steady progress in the marine field. For power production from oil also in stationary plants the injection engine may be expected progressively to replace the steam plant, provided oil burning in stationary plants should at all be tolerated. Another question, however, is: Should any relief by means of the injection engine be looked for in the automotive field?

There is no use denying the fact that the Diesel engine is a very complicated engine, and that many troubles were experienced in the early stages of its development, on account of the high pressure occurring in connection with very high temperatures. Very high grade engineers, indeed, have been found to be necessary for the successful running of large size Diesel engine plants.

Nevertheless, let it be said at once that if the Diesel engine is complex, so is perhaps in no less degree the ordinary carbureting engine. Nobody thinks of a high pressure compressor on an automotive injection engine. With this feature eliminated, the two elements that may present unusual difficulties are the fuel pump and the injection valve. True, the high pressures on the pump and the high temperature on the valve introduce problems here that may require a great amount of effort for final solution. Nevertheless, it may well be asked whether the complications here in store are any worse than those presented by the carburetion and ignition system of the ordinary automotive engine.

As for pressures, the explosion pressures in an airplane engine are of the same order of magnitude as the pressures that need occur in an injection engine. The efforts of the leading engineers in America in the automotive engine field are now directed towards the introduction of a compression approaching 200 pounds. This is what doped fuels, or multiple spark plugs, or any other knock-avoiding scheme proposed is intended to accomplish. Such a compression is, however, just as high as that used in the Price-Rathbun oil engine.

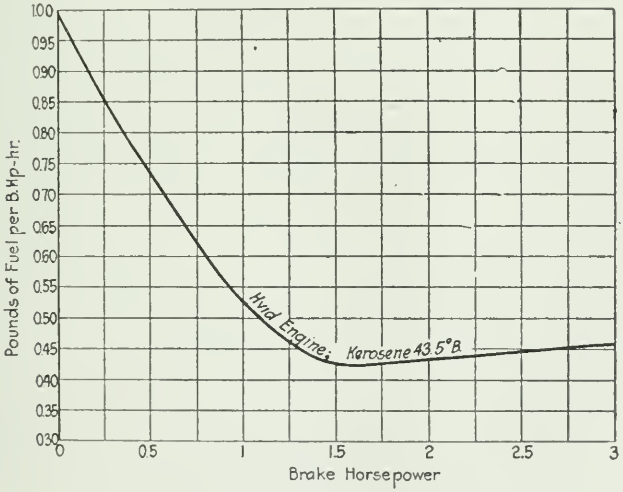

FIG. 9.

The popular impression is that injection engines are very bulky and very heavy. Actually this is true simply because the engines so far produced have been marine engines or stationary engines designed for low speed and with no particular regard to lightness. As early as in 1911 (Zeitschrift des Vereines deutscher Ingenieure, Sept. 30, 1911, page 1644) there was described a light Diesel engine constructed by Swedish engineers for the Nobel Company in Petersburg. This engine weighed only 22 pounds per horsepower of output in comparison with weights of from three to five hundred pounds for marine Diesel engines. Also small cylinder dimensions and high speeds do not appear to be beyond the range of the injection engine. On page 864 of the Transactions of the American Society of Mechanical Engineers for 1919, Mr. E. B. Blakely describes a small Hvid engine of three inch bore and four and one-half inch stroke running normally at 1100 r. p. m. and performing satisfactorily at 1500 r. p. m. Since the fuel consumption curve of this late engine is of particular interest, it is reproduced in Fig. 9. From this curve it will be seen, not only that the fuel consumption is very low, the minimum value corresponding to a heat utilization of 30 per cent; but, in addition, that the minimum value occurs at half load, and that the fuel consumption at one-third load appears to be only 15 per cent higher than at full load. Such a fuel consumption curve is in marked contrast with curves for carbureting engines, which usually show a consumption at one-third load about 150 per cent higher than at full load. Mr. Blakely admits that a four-cylinder Hvid engine did not show as rapid acceleration as a carbureting engine normally would. He maintains, however, that it maintained its torque remarkably at reduced speed. Be this as it may, Mr. Blakely's tests on this small engine, as well as others carried out on Diesel engines proper, (e. g. F. Romberg, Zeitschr. Ver. deutscher Ing., Nov. 5, 1910, p. 1897), show that it is by no means unreasonable to think of injection engines of automotive speeds and dimensions.[18]

This, however, is stating the case feebly. For, as a matter of fact, the automotive injection engine has made its appearance already, even in a place where it would perhaps least of all be expected, namely in the air. Already before the war Prof. Junkers in Germany had become known for his opposed piston injection engine. It appears that during the war, this engine was adapted to aeronautical purposes. A description thereof appears in the Automotive Industries of Oct. 7, 1920, p. 718. It is a 2-cycle engine, the scavenging air being furnished by a separate pump and the exhaust being swept out through sets of ports uncovered by the two pistons as they move out. According to an article in the Scientific American, this engine uses a compression of only 210 pounds and secures positive ignition by means of a spark plug. The article in the Automotive Industries states that other injection engines of this type are now being developed in Germany with weights of less than two pounds per horsepower.

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[18] Mr. H. R. Ricardo, in the article reproduced in the Automotive Industries on February 24, 1921, speaks of a successful experimental carbureting engine of 8–inch bore and 11–inch stroke running at speeds as high as 1350 r. p. m. Such dimensions are desirable, at least, for the large outputs common in aeronautical engines. With cylinder dimensions of this magnitude the application of the injection principle should not be difficult.

Modified injection engines have been introduced in France and in England for operation on kerosene. In these the kerosene is injected during the suction stroke, the object being not ignition by compression, which is only 80 pounds, but simply positive introduction of the fuel. Ignition is brought about at the end of the compression stroke by means of a spark plug. In France, in June, 1918, the first and second places in a competition of kerosene burning automobiles were won by two Unic cars having injection engines of this type, as designed by Bellem and Bregeras. These two cars were the only ones to complete the competition successfully. Stationary engines of the same type are now manufactured by the Caffort Company of Paris (for description see Automotive Industries, April 29, 1920, page 1009).

In England, a similar engine, built by Blackstone and Company, was seen in operation at the recent tractor demonstration (Automotive Industries, September 30, 1920, page 659). In this engine the injection is accomplished with the aid of compressed air furnished by a small plunger compressor.

Finally in the Automotive Industries of March 3, 1921 (page 501), it is reported that the Deutsche Automobil Konstruktions Gesellschaft of Germany has adopted the Steinbecker type of injection engine for automotive purposes. This engine works on a principle somewhat similar to the Brons-Hvid (For a description see also Power, Oct. 12, 1920, page 584).

All these developments may be only slight beginnings. Nevertheless, the sum total of all this successful work in connection with the palpably awakening interest in the injection engine in many quarters is sufficient proof that talk about an automotive injection engine can no longer be regarded simply as a waste of time. The thermodynamical advantages of the injection engine have long been known, and have not been disputed. The mechanical difficulties now seem actually to have been overcome. Very likely much more experimentation is necessary before a commercial automotive engine of injection type will find acceptance by the motor using public. Owing to the fundamental soundness of these developments from the point of view of general fuel economy, they, however, should not be belittled or depreciated by automotive engineers, but should wherever possible, be countenanced and encouraged, and should in all cases be received with interest and without prejudice.[19]

ARTICLE 12.

EFFECT OF CAR SPEED AND GEAR RATIO ON FUEL ECONOMY

SUMMARY: *The influence of car speed on fuel economy has not been so carefully investigated as one might think. The experiments made indicate, however, that, at least within certain limits, economy is gained by moderate speed, even by speeds falling below 20 miles per hour.*

A low gear ratio between engine and rear axle, i. e., a low engine speed, is conducive to economy. This means that for any given car speed the engine should be run with as little throttling as possible. Over-powered cars are very wasteful of fuel.

It may be questioned whether it is advisable to try to make the automobile-using public tinker with the machine so as to secure best economy. The carburetor setting most apt to secure such economy under ordinary driving conditions is not easy to hit. To the mixture heating and distributing systems the car owner can do little or nothing after the car is once bought. The same applies to the mechanical parts having a vital influence on friction losses.

There is, however, one thing which has a very considerable influence on fuel economy, and at the same time is entirely in the hands of the automobile user and of nobody else. This is the speed at which the car is driven. It is peculiar that on this vital factor so little experimental work has been done. On September 9, 1920, the editor of the Automotive Industries states in his paper (page 531), that the only figures in his possession regarding the variation in fuel economy with speed were obtained with a pair of Cadillac cars on the Indianapolis speedway. These can not be regarded as normal driving conditions for an ordinary automobile. The data there obtained were as follows:

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[19] At the summer meeting of the S. A. E. in 1920, Mr. E. A. Sperry made announcement of the development of a compound Diesel engine of very light weight. The author is not in a position to state whether this development can yet be regarded as sufficiently out of the experimental stage to be considered a complete technical success. It would appear, however, that it is sufficiently far advanced to deserve serious consideration.

Speed, Miles Per Hour ..	14.15	24.5	33.9	43.5	52.9	62.8
Fuel Economy, Miles per Gallon	20.6	18.6	16.3	13.6	10.8	8.8

From these figures it would appear that the slower one drives, the more miles per gallon are obtained — an easy rule to remember, and certainly a very safe one from every point of view. The editor, however, expresses as his personal opinion that on good roads the most economical driving speed with cars in average condition lies between 25 and 30 miles per hour on high gear. Some tests by the Northway Motor & Mfg. Co., on a Cole 8-cylinder car actually showed 18.3 miles per gallon, at 20 miles per hour and 17.65 miles per gallon at 30 miles as against 16.6 miles per gallon at a speed of 10 miles per hour. (Automotive Industries, 1920, p. 781).

In view of the scanty information on the subject it is of so much greater importance that Mr. A. L. Nelson, in his repeatedly mentioned paper in the S. A. E. Journal of February, 1921, on page 111 and 113, gives some curves showing best economy at about 15 miles per hour, with engines having a compression ratio of 4.25 to 1 and at about 17 miles per hour for engines having a compression ratio of 5 to 1. For higher speeds the economy drops rapidly. Owing to the importance of this matter, the curves for the lower compression ratio are here reproduced. (Fig. 10).

The reason for the improvement in fuel economy with reduced speed seems to lie wholly in the fact that the brake horsepower required increases with a power of the speed higher than one. According to curves furnished by Mr. Nelson and reproduced in Fig. 11, the fuel consumption per brake horsepower decreases very materially with the speed of the engine even up to 2100 r. p. m., a car speed of 60 miles per hour. In view hereof Mr. Nelson's findings may not be applicable to all kinds of road and driving conditions. Especially it should be noted that an engine not in the best of condition is apt to operate better at high speed than at low speed. At very low speed it may be apt even to lose compression and hence a good deal of the driving power developed by the fuel. In the statement of the editor of the Automotive Industries above referred to, the remark is made that the engines of the cars with which the tests on the Indianapolis speed-way were carried out, probably were in a most excellent condition. The great lesson for the public to learn from all this is, then, perhaps, to have their engines well cared for and thereafter drive at a moderate speed.

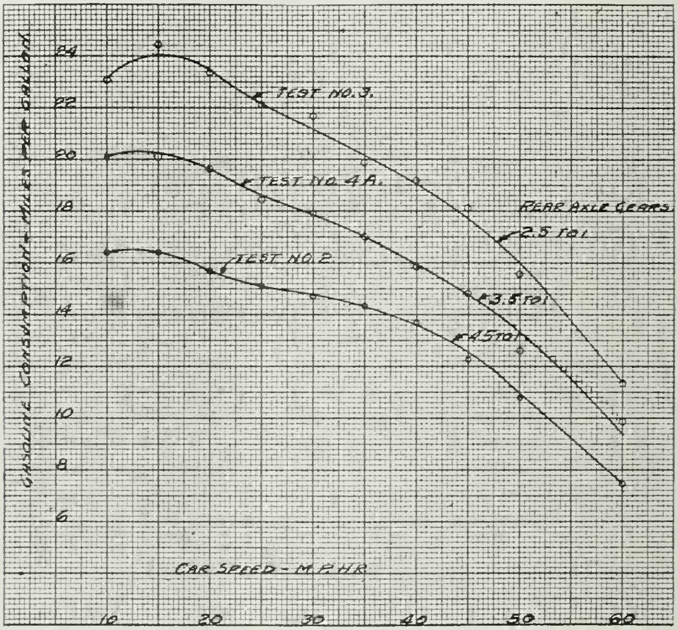

FIG. 10 — Curves Showing the Relation Between Car Speed and Fuel Consumption Using Pistons Having a Compression Ratio of 4.25 to 1.

The curves in Fig. 10 also show the effect of various gear ratios in the rear axle. It is observed that the lower the gear ratio the better the economy. The reason for this has been given earlier by Messrs. J. H. Golten and Allan Neuman in an article in the Automotive Industries of December 9, 1920, page 1167. The reason is that the fuel economy of a carbureting engine varies but little with the speed, but varies considerably with the manifold pressure, that is to say, with the degree of throttling. The more throttled the engine is, the poorer is the economy. Now it is possible to secure the same horsepower output for a certain needed tractive effort either by running the engine at a higher speed with a higher degree of throttling, or by running it at a lower speed with less throttling. The latter

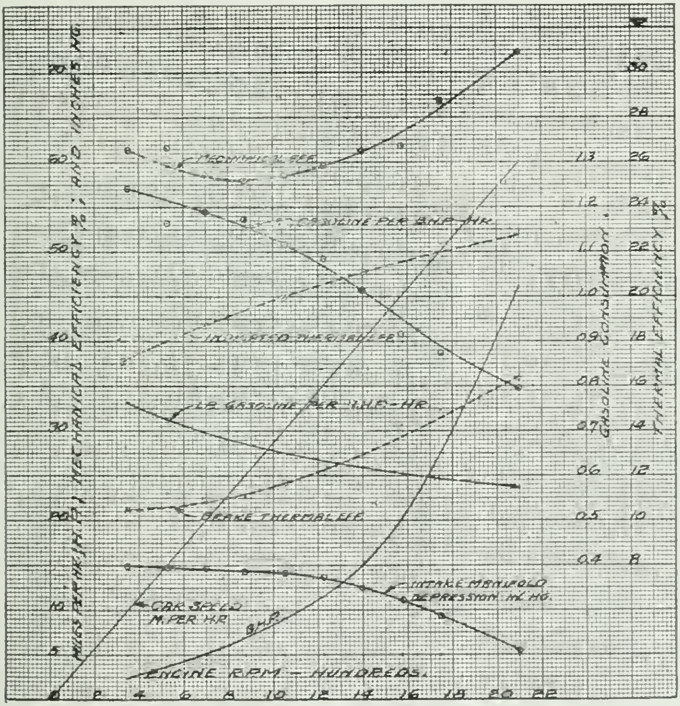

FIG. 11 — Engine Characteristic Curves Obtained with a Constant Car Speed, a 3.5 to 1 Rear-Axle Gear-Ratio and Pistons Having a Compression Ratio of 4.25 to 1.

is the condition under which the engine gives better economy. To secure this condition the gear ratio should be as low as possibly consistent with satisfactory operation of the engine from the point of view of firing and distribution.

Naturally, the less throttled the engine is for any given speed the more likely it is also to be taxed to the limit of its power. Hence normal driving on high gear with a low gear ratio in the rear axle means more frequent gear shifting for increased power needs. This is something that the operating public will have to get used to, if automobile manufacturers generally will design their cars in harmony with the requirements for best economy. It is something that has been characteristic of European design for a long time. The worst offender against good fuel economy is the overpowered car which most of the time runs in a heavily throttled condition.

To lessen the inconvenience of gear shifting, engineering invention might well direct itself towards the development of some automatic or semi-automatic device to do away at least with the difficulty of selection. Some such designs have already appeared, (For an example see the description of the Campbell All-in-mesh gear-set, Automotive Industries, March 13, 1919, p. 596). Hydraulic, as well as electric transmissions have also been mentioned (see, for instance, Automotive Industries, 1920, p. 1429, also reports of W. F. Bradley regarding German automotive developments, same journal, 1920, p. 1052).

NOTE ON MULTIPLE SPARK PLUGS.

In reading the page proof of this bulletin, the author feels that the case of multiple spark plugs has been stated too feebly. It is true that Ricardo found it possible by spark advance to equal the power increase resulting from multiple spark plugs. It is also true that at the Bureau of Standards knock in some cases was produced by two spark plugs where one gave no knock. Nevertheless it can hardly be doubted that by means of multiple spark plugs inflammation and combustion can be concentrated more closely to the dead center than can be accomplished by only one plug. This should make for a good diagram, without knock, at all loads and speeds and should render the engine more flexible, i. e., should tend to make it operate smoothly without spark shifting under a variety of throttle conditions. As for the power increases obtainable at least under some conditions see the article by Captain Hallet, quoted on page 87 (S. A. E. Journal, Novemeber, 1920.)

CONCLUSION TO PART II

The causes of inefficiency of the present-day carbureting engine arrange themselves in five groups:

(1) Difficulties of carburetion and of distribution of the mixture to the various engine cylinders with present-day fuels.

(2) Pre-ignition and knocking troubles, making difficult the introduction of compressions sufficient to give economies comparable to that of injection engines.

(3) Difficulty in securing good ignition and power with sufficient air excess to give complete combustion.

(4) Charge dilution and combustion delayed by the burnt residue —a trouble accentuated by the large clearance volume necessary to avoid knock, and by the throttling method of governing.

(5) The general operation of automobile engines at reduced load, at which the troubles in groups (3) and (4) become especially serious.

Progress has been made in the overcoming of these difficulties, part of it unexpected and brilliant. The fundamental requirements for economy — sufficient expansion, complete combustion early in the stroke, low friction losses, etc., — are becoming increasingly understood, and from such understanding further progress must certainly follow.

In this connection, however, it should not be forgotten that there are prime movers now — the steam engine and the injection engine — possessing many of the desirable characteristics which the carbureting engine lacks, notably the ability to burn fuel of low volatility and to run efficiently at reduced load, and that some, on the whole, quite promising steps have been taken to adapt these engines to automotive work.

PART III.

SCIENTIFIC DISCUSSION OF COMBUSTION ENGINE PROCESSES.

SUMMARY: Part III contains a number of not very closely connected articles, the main contents of which are as follows:

Article 13 shows that a gas is not cooled by being passed through a throttle valve, and that throttling does not lower the temperatures in a combustion engine. If knock is reduced by throttling, the reason is that knock is dependent on actual pressure as much as on temperature.

Article 14 estimates the cooling effect due to the evaporation of the charge and gives theoretical and experimental proof that this cooling may amount to from 60 to 70 deg. fahr.

Article 15 gives a method for estimating the temperature necessary for complete evaporation of the fuel in a mixture of fuel with air The temperature is shown to be of the order of magnitude of 120 to 130 deg. fahr. for present day gasoline and of 200 to 220 deg. fahr. for kerosene. Unheated air will hardly evaporate more than half of the gasoline entering a cold engine even in summer time, and only a negligible fraction on a cold winter day; hence the well-known troubles such as crankcase dilution, carbon formation, etc.

Article 16 shows the reduction in power with air heating to be due to reduced ratio of explosion pressure to compression pressure in consequence of reduced ratio of the corresponding absolute temperatures.

Article 17 deals with the heat developments in combustion engines, discusses the reactions occurring, gives tables for heats of reaction, and finally shows the variation of heat development with varying strengths of mixture.

Article 18 gives a refigured entropy chart and directions and examples covering the use of this chart for analysis of the occurrences in actual engines, and for the computation of the probable performance of untried motors, such as gas turbines.

Article 19 deals with the efficiencies attainable in combustion engines, derives the air-cycle efficiency, and cites efficiency formulas of Tizard and Pye, and of Ricardo. The ideal efficiency of an injection engine is calculated. The possibilities of improvement in maximum engine efficiency are found to be not excessively great. Experiments with combustion batteries in Germany and high temporary heat utilizations obtained are mentioned.

ARTICLE 13.

PASSAGE OF A GAS THROUGH A THROTTLE VALVE

SUMMARY: *The throttling of a gas produces no temperature drop. The end temperature of compression should be the same in a throttled as in an unthrottled engine. If knock is relieved by throttling, then knock is dependent on the pressure attained rather than on the temperature.*

In current gas engine discussions one often finds the opinion that a gas cools in passing through a throttle valve; hence that a lower compression temperature would be reached when the engine is throttled than when it is run open.

In actuality, however, an appreciable cooling takes place only if the gas retains a considerable velocity after passing through the throttle If the velocity after throttling is of the same order of magnitude as before the throttling, then no cooling takes place.

The law governing these relations is simply that the total energy contained in the gas, including the velocity energy, must be the same before and after the throttling. The velocity energy expressed in B. t. u. per pound is $\frac{v^2}{2gE}$.

Here v is the velocity in feet per second, g the acceleration due to gravity = 32.2 ft./sec.^{2}, and E the mechanical equivalent of heat, (usually taken at 778 foot pounds per B. t. u.).

If this velocity energy is acquired by temperature drop, i. e., by the conversion of heat into kinetic energy, then we have

$$(1) \qquad \frac{v^2}{2gE} = c_p\,(T_1 - T_2)$$

T_1 is the temperature before throttling, T_2 the temperature after throttling, c_p the so-called "average specific heat at constant pressure"[1] in the temperature range between T_1 and T_2.

The velocity corresponding to various temperature drops is computed with the aid of this formula in Table 13. The value of c_p has taken equal to 0.24.

TABLE 13

v = ft./sec.	50	100	200	300	400	500	1000
Temp. drop deg. F....	0.2	0.8	3.3	7.5	13.3	20.7	83.0

It will be observed that for velocities such as occur in inlet manifolds, say those up to 300 ft. per second, the temperature drop is altogether negligible.

The compression temperature reached in an engine is dependent only on the pressure ratio and not on the absolute value of the pressures. If poorer ignition results from throttling, then this is due to greater dilution of the charge by the burnt residue, as already explained (page 67); possibly, also to poorer flame propagation with reduced pressure, although this last mentioned point is less certain.

Knocking seems to depend on actual pressure attained at least as much as on temperature.

ARTICLE 14

COOLING BY THE CHARGE

SUMMARY: *The evaporation of the charge is capable of producing a cooling effect amounting to 50 or 60 deg. fahr. if the evaporation is complete. This cooling effect is naturally*

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[1] The designation of c_p in the above equation as the specific heat at constant pressure has been criticized on the ground that the pressure manifestly drops. In any one temperature range we have in thermodynamics, however, to deal with only two specific heat values. The lower is usually styled the "specific heat at constant volume." The higher is the specific heat at constant pressure. The difference between the two is the quantity R/E, which might be called the displacement work per degree temperature change.

reduced by the imperfect evaporation of present-day fuels. It may be masked by heat from the engine parts. Experiments on a truck and tractor engine show it, however, when the air heating is at its maximum, to be of quite the magnitude computed.

The cooling of the mixture in passing through the carburetor is due to the heating and evaporation of the fuel. If the temperature of evaporation is t_e and the initial temperature of the fuel is t_i, then the heat necessary for heating the fuel to the point of evaporation is per pound

$$c(t_e - t_i),$$

where c is the specific heat of the liquid fuel. The values of c for some fuels is given in Table 14. (From L. Schmitz, "Die flüssigen Brennstoffe").

TABLE 14

SPECIFIC HEATS, c, OF LIQUID FUELS

Kind of Fuel.	Temperature of Determination, deg. fahr.	Specific Heat, c, B. t. u. per lb. and deg. fahr.
Pennsylvania Crude, sp. g. 0.810.		0.50
California crude, sp. g. 0.908...		0.398
Light gasoline..................		0.487
Kerosene	64 to 210	0.455 to 0.498
Naphthalene (solid)............	104 to 122	0.326
Naphthalene (liquid)...........	176 to 185	0.396
Solar oil, gas oil...............		0.416 to 0.419
Alcohol	104	0.648
Alcohol	176	0.769
Benzol	70 to 160	0.436
Toluol	54 to 210	0.440

The heats of evaporation of a number of liquid fuels have been given in Table 6 on page 50.

The total heat consumed is the sum of that gone to heating of liquid and of that gone to evaporation.

As an example, the heat required to raise the temperature of light gasoline from 70 deg. fahr., and evaporate it would be

$$0.487\ (200 - 70) + 133 = 196 \text{ B. t. u. per pound.}$$

In the next article it will be seen that under the conditions obtaining in the inlet manifolds of carbureting engines, even kerosene might be completely evaporated at a temperature not much in excess of 200 deg. fahr. The total heat required to evaporate present-day petroleum fuels in ordinary mixtures will then be around 200 to 250 B. t. u. per pound. Suppose a somewhat over-rich mixture of thirteen pounds of air per pound of fuel; then the heat required for heating and evaporation of the fuel would be, say, 200/13 = 15.4 B. t. u. per pound of air. If the specific heat of the air at constant pressure is 0.24, then the cooling would be $\frac{15.4}{0.24} = 64$ deg. fahr.

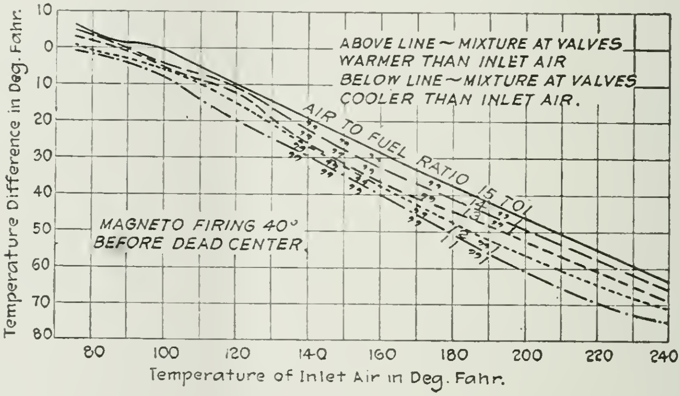

FIG 12.

In Fig. 12 (from a paper "A Tractor Engine Test" by C. A. Norman and B. Stockfleth, S. A. E. Journal, February, 1920) is given a set of curves showing the actually measured temperature drop through carburetor and inlet manifold for various mixtures of commercial gasoline and air, and various inlet air temperatures.

It will be seen that with inlet air temperatures as high as 200 to 220 deg. fahr., the temperature drop is of the order of magnitude just computed. This temperature drop shows fairly complete evaporation. At lower temperatures the drop in temperature becomes much less. This is due partly to incomplete evaporation, partly to heating of the charge by the hot engine and the exhaust manifold. At an air temperature of 90 deg. fahr. and less this last effect preponderates so that a heating of the mixture takes place.

ARTICLE 15

THE EVAPORATION TEMPERATURES OF LIQUID FUEL MIXTURES

SUMMARY: *Moisture will not condense out of atmospheric air, even if the temperature of the air is much below the boiling point of water under atmospheric pressure.*

The amount of vapor that air can absorb and keep evaporated depends upon the temperature of the air, the pressure of the vapor in the air and the boiling point of the vapor under this pressure.

A method is here given to determine the amount of petroleum fuel that can be kept evaporated in air at various temperatures. It is found that for perfect evaporation of present-day gasoline a mixture temperature of about 110 to 130 deg. fahr. is necessary; while for kerosene the temperature must be raised somewhat over 200 deg. fahr.

Present-day gasoline shows an end point, that is, boiling point of the heaviest fractions, of about 430 to 460 deg. fahr. The end point of kerosene, (See page 12) may be close to 600 deg. fahr. One often meets the opinion that especially at low load the exhaust is not hot enough to evaporate such fuels, since the exhaust temperature may drop even below the end points mentioned.

Actually, a temperature much below these boiling points of the fuel is necessary for the complete evaporation thereof in a mixture of fuel and air. It is a popularly well-known fact that atmospheric air at all times carries considerable amounts of water vapor. Yet the temperature of the air does not reach anywhere near the boiling point of water, 212 deg. fahr. It is also well known that in order to ascertain the maximum possible pressure of water vapor in the air it is only necessary to look up in a steam table the pressure of saturated steam at the temperature of the air. As long as the "partial" pressure of the water vapor in the air is not equal to this saturation pressure, water can evaporate into the air. Only after the saturation pressure is reached, no further evaporation is possible, and water in contact with and at the temperature of the air must remain liquid.

Quite the same conditions apply to the air-fuel mixtures in the inlet manifolds of carbureting engines. If our fuels were simple chemical compounds such as benzol or alcohol, and if we knew exactly the boiling temperatures of these liquids under various pressures, then we could tell exactly how much fuel the inlet air could carry in an evaporated state at various temperatures. Unfortunately our petroleums are mixtures of a multiplicity of hydrocarbons, the exact chemical constitution of which we may not always even know. Nevertheless it is very instructive to estimate the order of magnitude of the temperatures necessary for the formation of so-called "dry", i. e., perfectly evaporated mixtures. It may be that such mixtures will not give the maximum engine power with four-cylinder engines. It may be that devices may be developed whereby mixtures of considerable wetness may be distributed satisfactorily to all cylinders of a multi-cylinder engine. Nevertheless, a perfectly evaporated mixture certainly is the safest mixture from the point of view of distribution, and a clear understanding of the conditions necessary for the formation of such a mixture would appear of considerable practical importance.

A presentation of the laws governing this matter will, therefore, here be given together with an estimate of the temperatures of evaporation of some representative fuel mixtures.

For this purpose, recourse will be had to an approximation which has been of the greatest value in the derivation of the laws of physical chemistry. This approximation is that vapors obey the laws of permanent gases. If this law is true, then it can be stated immediately what pressure a certain quantity of vapor will exert in any given volume at any given temperature. If the weight of the vapor is W, the volume V, and the temperature T, in degrees fahr. absolute (ordinary degrees fahr. + 460 degrees) then the state equation for gases is

$$(2) \quad pV = WRT$$

The letter p signifies pressure, in pounds per square foot; R is the so-called gas-constant. For air, the gas-constant is 53.34. For the various fuel hydrocarbons, the gas-constant has not been experimentally determined. It can, however, be estimated with considerable approximation from the fact that for a weight of any gas numerically equal to the "molecular weight" of the gas, it has the value 1544 in the units here employed. With molecular weight we mean a number indicating how many times heavier the molecule of a substance is than the hydrogen atom. (In this connection an atom of oxygen is taken to weigh exactly 16 times as much as the hydrogen atom used for reference, and all other substances are rated on the basis of this weight of the oxygen atom). The hydrogen molecule itself is made up of two hydrogen atoms and its molecular weight is consequently $= 2$. The gas-constant for hydrogen would be per pound $1544/2 = 772$. The molecular weight of oxygen, another "di-atomic" gas, is $2 \times 16 = 32$, and the gas-constant of oxygen is $1544/32 = 48.25$. The molecular weight of nitrogen, a gas made up of molecules containing two atoms of atomic weight 14 each, is $2 \times 14 = 28$. The gas-constant of nitrogen would be $1544/28 = 55.1$.[2] Air, a mixture of 23 per cent oxygen by weight, and 77 per cent nitrogen, with immaterial quantities of other chemically inactive gases, has an average molecular weight of 28.95, which gives the gas-constant indicated above.

For the purpose here in view there are given in Table 15. the names, chemical formulas, boiling points, molecular weights, and estimated gas constants of a series of normal hydrocarbons of the paraffin group. While all gasoline or kerosene, nowadays, is by no means made up of hydrocarbons of this character, yet the paraffins are the classical petroleum hydrocarbons, and other petroleum hydrocarbons, at present of increasing importance in America, — naphthenes, ethylenes, etc. — do not deviate enormously in their general behavior from these paraffins. For this reason an estimate made on the basis of the paraffins will be found to have practical value for petroleum hydrocarbons in general.

TABLE 15

PHYSICO-CHEMICAL DATA FOR PARAFFINS[3]

	Formula	Boiling Point deg. fahr.	Molecular Weight	Gas Constant
Pentane	C_5H_{12}	99	72	21.4
Hexane	C_6H_{14}	156	86	18.0
Heptane	C_7H_{16}	208	100	15.4
Octane	C_8H_{18}	257	114	13.5
Nonane	C_9H_{20}	302	128	12.0

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[2] The value given in Marks' Handbook is 54.99.

[3] From Smithsonian Tables, Landolt and Bornstein and Höfer.

	Formula	Boiling Point deg. fahr.	Molecular Weight	Gas Constant
Decane	$C_{10}H_{22}$	344	142	10.9
Undecane	$C_{11}H_{24}$	384	156	9.9
Dodecane	$C_{12}H_{26}$	418	170	9.1
Tridecane	$C_{13}H_{28}$	454	184	8.4
Tetradecane	$C_{14}H_{30}$	485	198	7.8
Pentadecane	$C_{15}H_{32}$	519	212	7.3
Hexadecane	$C_{16}H_{34}$	549	226	6.82
Heptadecane	$C_{17}H_{36}$	578	240	6.43
Octadecane	$C_{18}H_{38}$	605	254	6.08
Nonadecane	$C_{19}H_{40}$	626	268	5.76

Assume that there is in a certain volume V a gas mixture of total weight W, of absolute temperature T, of pressure p, and of average gas constant R. We then have for this quantity of mixture the relation given above

$$pV = WRT. \qquad \text{(a)}$$

Suppose there is in this mixture uniformly distributed throughout the same volume V a gas or vapor of weight G_1, and gas constant R_1. The temperature of this constituent of the mixture will be T, as for the mixture as a whole. It will contribute to the total pressure of the mixture a certain amount p_1, which is called the "partial" pressure of this gas or vapor. For this constituent we then also have the relation

$$p_1V = W_1R_1T. \qquad \text{(b)}$$

By division of equation (b) with equation (a) there is obtained the relation

$$\text{(3)} \qquad \frac{p_1}{p} = \frac{W_1R_1}{WR}.$$

or

$$\text{(4)} \qquad p_1 = \frac{pW_1R_1}{WR}$$

By means of this relation it is possible to determine the pressure of a given quantity of any gas or vapor in a gas mixture, provided the gas constants are known. As an example, what would be the pressure of heptane in a mixture of one pound of heptane to fifteen pounds of air? The total pressure of the mixture is atmospheric, i. e., 14.7 pounds per sq. in. The total weight of the mixture considered is $1 + 15 = 16$ lbs.

The average gas-constant for the mixture would be

$$\frac{1 \times 15.4 + 15 \times 53.34}{16} = 51.0$$

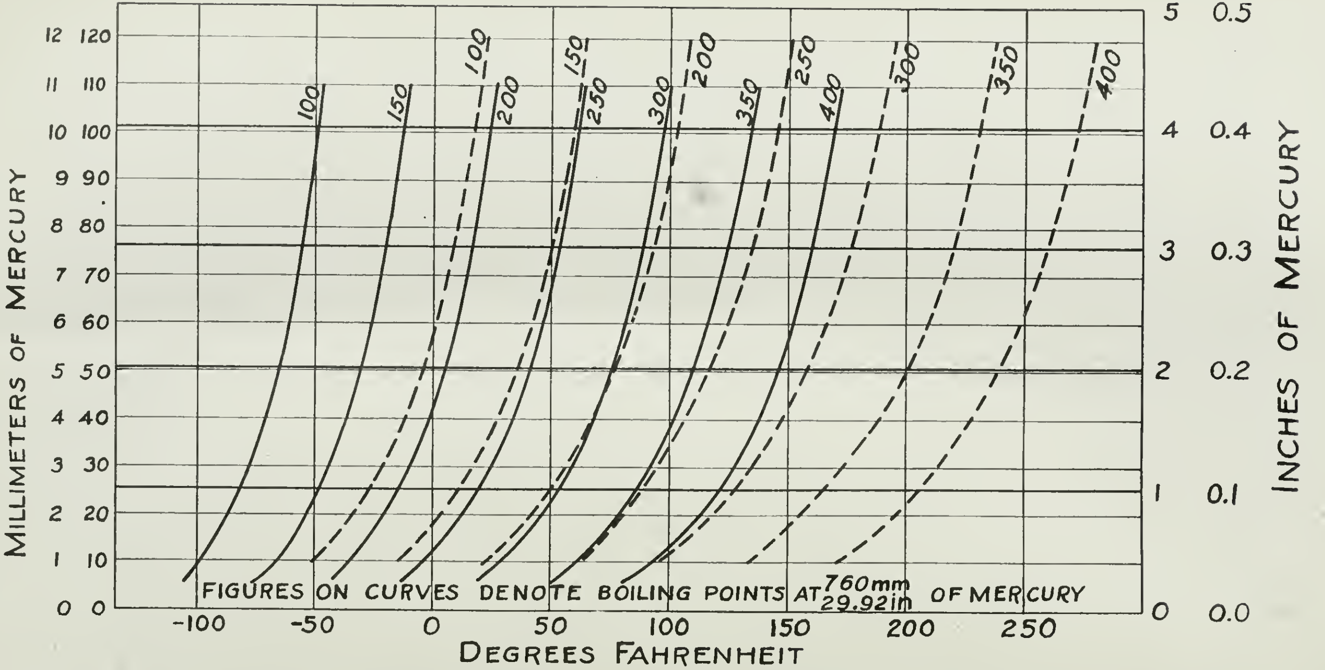
We have for the partial pressure of the heptane

$$p_1 = \frac{1 \times 15.4 \times 14.7}{16 \times 51.0} = 0.28 \text{ lbs. sq. in.}$$

If now the boiling point of heptane at a pressure of 0.28 pounds per sq. in. were known, then this boiling point would be the temperature necessary to keep one pound of heptane evaporated in a mixture with fifteen pounds of air. Actually such boiling points are known, not only for heptane, but also for pentane, hexane, and some other hydrocarbons. They are not known, however, for the majority of hydrocarbons occurring in gasoline and kerosene of today. Moreover, we never know which the hydrocarbons are that make up our commercial fuel distillates. All we know is what percentages of these fuels distill over below certain given temperatures. The question is: Does any possibility exist of obtaining an idea from the known boiling points of some hydrocarbons of the boiling points of other hydrocarbons under the same pressure?

Such a possibility exists on the basis of a fact established by Ramsay and Young, namely, that, at the same pressure, the absolute boiling temperatures of related chemical substances stand to each other in a nearly fixed ratio. Thus if pentane under atmospheric pressure boils at 100 deg. fahr. = 560 deg. fahr. absolute, then a hydrocarbon boiling under the same pressure at 200 deg. fahr. = 660 deg. fahr. absolute, would at all other pressures boil at absolute temperatures, standing to the absolute boiling points of pentane in the ratio 660:560.

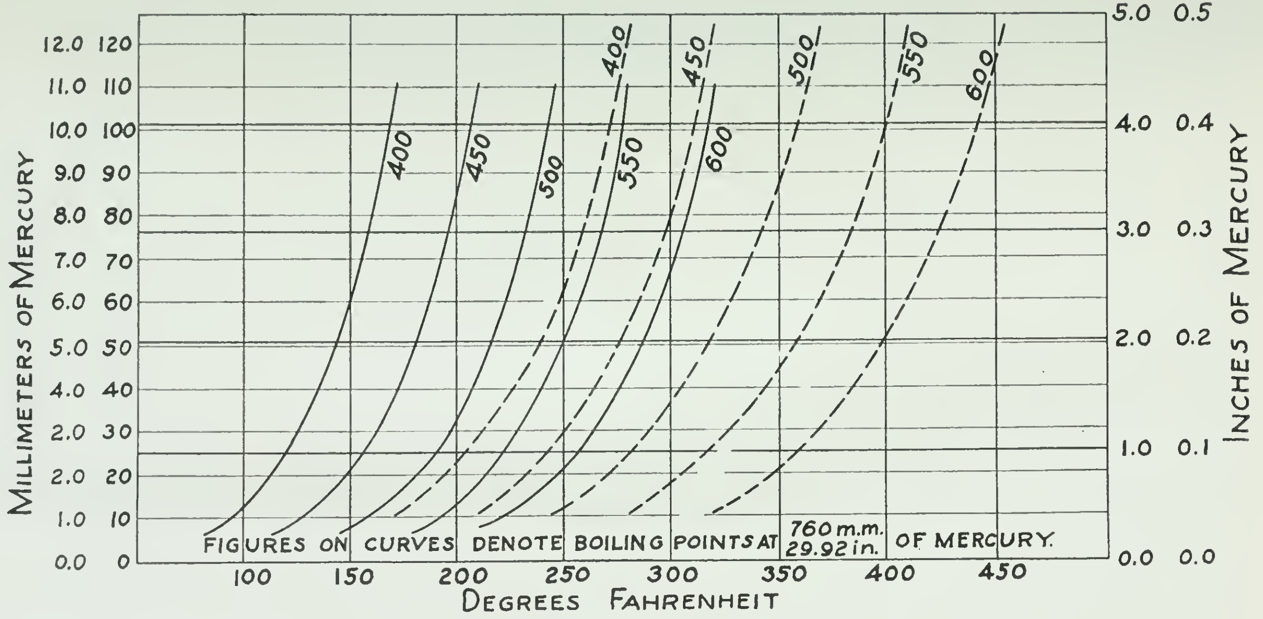
On the basis of this law, the author has in Figures 13 and 14 plotted curves, giving the boiling points of petroleum fractions which under atmospheric pressure distill over at the



VAPOR TENSION OF PARAFFINS

CURVES DERIVED BY MEANS OF RAMSAY'S AND YOUNG'S LAW FROM THOSE OF OCTANE AND DECANE
—— SCALE OF LOWER PRESSURES
– – – SCALE OF HIGHER PRESSURES

FIG. 13.



VAPOR TENSION OF PARAFFINS

CURVES DERIVED BY MEANS OF RAMSAY'S AND YOUNG'S LAW FROM THOSE OF OCTANE AND DECANE
—— SCALE OF LOWER PRESSURES
- - - SCALE OF HIGHER PRESSURES

FIG 14.

Fahrenheit temperatures marked on the curves.[4] As an explanation of the use of these curves, let us ask ourselves what would, under a pressure of 0.1 inches of mercury, be the boiling point of a fraction which, at atmospheric pressure, of 29.92 in. of mercury distills at 400 deg. fahr. In answering this question, greatest exactness is obtained by using the full line curves in Fig. 13, and the corresponding scale of lower pressures. We find that the curve marked 400 crosses the pressure line marked 0.1 at a temperature of about 120 deg. fahr. This, then, is the boiling point of the 400 deg. fahr. fraction, under a pressure of 0.1 inches of mercury. Conversely, if the partial pressure of this fraction in a fuel air mixture is only 0.1 inches of mercury, then a temperature of only 120 deg. fahr. is necessary to keep it completely evaporated.

An attempt will now be made with the aid of these curves to estimate what would be the necessary temperatures to keep perfectly evaporated kerosene or gasoline in air mixtures such as are used in carbureting engines.

Let us first see what temperature would be required to keep evaporated the heaviest 10 per cent of a commercial paraffin base gasoline in an over-rich mixture of twelve pounds of air to one of gasoline. To make sure that the temperature figured will not be too low, let us assume that all this gasoline boils at 450 deg. fahr. By reference to Table 15, it will be seen that such a gasoline would approximately correspond to tridecane, having a boiling point of 454 deg. fahr. The gas-constant would be 8.4. The average gas-constant of the whole body of gasoline might with sufficient accuracy be estimated at 12, corresponding to an average boiling point of about 300 deg. fahr. Under such circumstances, the average gas-constant of the whole fuel mixture would be

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[4] These curves are based mainly on those of octane and decane. Some of them were first published in the Automotive Industries, Feb. 27, 1919, where the actual location of points figured from experimental points for the mentioned hydrocarbons are shown. In the plotting of the curves here presented the author has in part had the benefit of the assistance of some of the younger engineers in Experimental Laboratory No. 5, of the International Harvester Co.

$$\frac{1 \times 12 + 53.34 \times 12}{13} = 50$$

Suppose the pressure of the whole mixture to be atmospheric, that is 760 mm. the millimeters being used simply because they lead to more round numbers with less uncertainty as to decimal points. The higher the pressure, the higher will be the boiling points of the liquid, the greater will be the temperature of the mixture necessary to keep the fuel evaporated. The assumption of atmospheric pressure, which certainly will be in excess of that actually obtaining in an inlet manifold will, therefore, lead to unnecessarily high figured air temperatures.

Calling the pressure of the gasoline fraction considered p_1, we have

$$p_1 = \frac{0.1 \times 8.4 \times 760}{13 \times 50} = 0.98 \text{ mm.}$$

This pressure as shown by the full line curve marked 450 in Fig. 14, corresponds to a boiling point of 130 deg. fahr. Consequently, this comparatively low temperature, with due regard to the conservative assumptions made above, is amply sufficient to keep completely evaporated the heaviest fraction even in somewhat over-rich gasoline mixtures. As this vaporization is on account of the very low partial pressure of the heaviest fraction, it might be well to estimate in addition what would be the temperature necessary to keep completely evaporated a paraffin of properties corresponding to the average properties of the gasoline. Assume this paraffin to have a boiling point of 300 deg. fahr., and a gas-constant of 12. The partial pressure p_g of the gasoline so averaged would in the mixture just considered be

$$p_g = \frac{1 \times 12 \times 760}{13 \times 50} = 14.0 \text{ mm.}$$

From Fig. 13, the corresponding boiling point, read from the dotted curve for 300 deg., would be about 106 deg. fahr.

There seems to be little question, then, that a mixture temperature of 110 to 130 deg. fahr. is all that is necessary to keep present-day gasoline completely evaporated. If this temperature is obtained by positively pre-heating the air going through the carburetor, there is little doubt but that complete evaporation can be secured. Owing to the cooling effect of the fuel, it is in this case necessary to heat the air to a temperature, say 60 deg. fahr. in excess of the temperature desired in the mixture. Such a heating means an air temperature of 160 to 190 deg. fahr. Similar calculations for kerosene show that for perfect evaporation, a mixture temperature of 200 to 220 deg. fahr. would be necessary, or an air temperature of close to 300 deg. fahr. Some tests seem to indicate that the best compromise for power, as well as economy may be to heat the inlet manifold rather than the air. In such cases the fuel first separates out from the air stream, and is then re-evaporated by the hot walls of the manifold. This evaporation may not be complete. In any case satisfactory operation has been obtained with mixture temperatures somewhat lower than the ones here estimated. (Say, for kerosene, 160 to 180 deg fahr.). It has been thought that these temperatures were so far below the end points of the fuel as to make it practically certain that the engines were operating on a wet mixture. Allowing for reduced manifold pressure, the calculations here carried out would tend to show that at least 180 deg. might not be far from the temperature necessary to keep kerosene in a fuel mixture completely evaporated.

Another interesting application of the method here developed might be to ascertain what portion of the commercial gasoline could be perfectly evaporated by unheated air in summer and in winter time. This question is of considerable interest in connection with the trouble of crankcase dilution of the lubricating oil.

Let us assume an air temperature of 70 deg. fahr. in summer time and 10 deg. fahr. in winter time. After the evaporation of the fuel the mixture temperature would then be about 20 deg. fahr. in summer time, and — 40 deg. in winter time. These seem enormously low temperatures, yet the coating of the carburetors with hoar-frost, even in summer time when volatile fuels are used, is a well-known phenomenon. Considering the fact that probably only the lighter fractions of the fuel will be evaporated, let us assume an average gas-constant of the fuel of about 17, and assume further that this is the gas-constant for the lighter half of the fuel in a mixture of one pound of fuel to twelve pounds of air. This means ½ pound of this lighter half, to 13 pounds of mixture. The partial pressure, p_1, of this half in a mixture of atmospheric pressure would be

$$p_1 = \frac{0.5 \times 17 \times 760}{13 \times 50} = 10 \text{ mm.}$$

This pressure would apply to a fuel with an average boiling point of perhaps 200 deg. fahr. Referring to the full line curve marked 200 in Fig. 13, we observe that at a pressure of 10 mm. a temperature of about 20 deg. would be necessary to keep this half of the fuel evaporated. Following the curve down to a point corresponding to a temperature of — 40 deg., we note that at this temperature the lighter half of the fuel would stay evaporated only under a pressure of less than 1 mm.

This rough investigation would then seem to indicate that even in summer time only about half of present-day commercial gasoline would be completely evaporated by cold air, while in winter time but a negligible fraction would be so evaporated. Since with the aid of the choke or other enriching arrangements considerable quantities of fuel are pumped into the cylinders during the starting period, a sufficient amount of this fuel will be soon enough evaporated by the heat of compression to give an ignitible mixture. In the meantime, however, no small amount of liquid fuel has accumulated in the cylinders, has flowed past the pistons, or has collected in puddles, so as to give rise to carbon formation found in the spark plugs, and to all those other troubles with which present-day gasoline has made us only too familiar.

The vanishingly small fraction of the fuel shown to be evaporated by a cold manifold in winter time gives emphasis to the suggestion accredited to Mr. C. F. Kettering in the Journal of the S. A. E., December, 1920, page 500, namely that auxiliary devices be installed to start engines on highly volatile substances, such as petroleum ether.

ARTICLE 16

THE EFFECT OF CHARGE HEATING ON ENGINE OUTPUT

SUMMARY: *Charge heating reduces the ratio of the absolute temperature after combustion to the absolute temperature before combustion, and hence reduces the pressure rise.*

The question here to be taken up is not the general one of the effect of temperature on the output of the engine. involving among other things, evaporation of the fuel, distribution of the charge to the various cylinders, acceleration of combustion. completeness of combustion, etc. The question here to be touched on is merely this: If all other conditions are the same, what effect will an increase in charge temperature as such have on the output?

In connection with carbureting engines it is well known that air heating has a tendency to reduce the test stand power. (See article 7, page 59). This reduction is often ascribed to the reduced weight of the charge entering the cylinder. Naturally, if the ratio of air to fuel remains constant, and the air is rarefied by heating, the amount of fuel energy per cubic inch of mixture admitted will be less. Hence, if everything else remains the same the power development of the engine per stroke should be less. The question, is, however: How is this brought about? The piston of the engine feels nothing else than pressure and if this pressure is the same, there is no difference in the power output of the engine no matter whether the pressure is brought about by steam or by water or by any other medium. Somehow, if the power output of the engine is reduced by air heating, it is necessary that the mean pressure on the piston is reduced.

The way in which this is effected is best illustarted by an example.

It is a well-known fact that the pressure of a quantity of gas enclosed in a fixed volume varies directly as the absolute temperature. Neglect the change in the composition of the gas during combustion, as can be done for rough approximation, call the pressure at the end of the compression stroke p_1, that after combustion p_2, the corresponding absolute temperature T_1 and T_2, and suppose the combustion to take place entirely in the clearance space, i. e., at constant volume. We then have

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

Let us consider two cases. In the first case, assume the mixture to be taken in at atmospheric temperature. but to be heated by the cylinder walls and the hot residue to 200 deg. fahr. before the beginning of the compression stroke. This temperature is equal to $200 + 460 = 660$ deg. fahr. absolute. By the compression, this temperature will be increased about 50 per cent, i. e., to 990 deg. fahr. absolute.

Suppose in another case the mixture to be drawn in heated so that the temperature at the beginning of the compression stroke is 200 deg. fahr. higher. The absolute temperature is then 860 deg. fahr. absolute. This temperature will also by compression be increased, though perhaps, due to wall cooling, somewhat less, say 40 per cent only. The temperature at the end of compression is then 1200 deg. fahr. absolute.

Suppose now that the combustion in both cases raises the temperature 2000 deg. fahr. In the first case, the "explosion" temperature will be 2990 deg. fahr. absolute; in the second case, 3200 deg. fahr. absolute.

The ratio of the explosion pressure to the compression pressure will be as the explosion temperature to the compression temperature. In the first case, we have the pressure ratio

$$\frac{2990}{990} = 3.03;$$

in the second case

$$\frac{3200}{1200} = 2.68.$$

If the compression pressure is the same in both cases, the explosion pressure will be about 12 per cent lower with mixture heating than without it. The loss in power will be very much of the same order of magnitude. This computed loss is in good agreement with the loss shown by experiments (See curves in Fig. 3 on page 59).

ARTICLE 17

THE HEAT DEVELOPMENT IN COMBUSTION ENGINES

SUMMARY: *The purpose of this article is to investigate the heat development with various mixtures. For this purpose the nature of the chemical reactions is first discussed, and then the heat derivable from each one of the reactions ascertained.*

The heats of reaction vary both with the temperature and the pressure; but with the possible exception of the combustion of carbon to carbon monoxide, the variation is not too great to allow of the use of average values for practical calculations. Dissociation plays an important part in limiting the maximum temperatures attained in all engines running on mixtures rich enough to be fired in a carbureting engine, but yet admits of momentary temperatures above 4000 deg. fahr. The heat development is greater with mixtures containing the theoretical quantity of air and is less both for over-rich and over-lean mixtures.

The heat development in combustion engines depends on the chemical reactions taking place, on the pressure and volume changes, and finally on the cooling losses during combustion.

1. *The Chemical Reactions.* The chemical reactions occurring depend not only on the character of the fuel and on the amount of air present; but they depend also on the temperature and pressure obtaining.

Messrs. H. T. Tizard and D. R. Pye at the Cardiff meeting of the British Association for the Advancement of Science, on August 25, 1920, presented a most valuable paper on this subject. At this writing the author has seen only abstracts of the paper and shall therefore have to content himself mainly with discussions based on investigations of his own extending over a number of years. He would, however, advise a thorough study of Messrs. Tizard's and Pye's paper as soon as complete copies can be secured (for abstracts see London Engineering, September 3, 1920, and Automotive Industries, October 14, 1920).

In a general way it may be said that petroleum hydrocarbons "gravitate towards" methane.[5]

At increased temperature methane in turn is increasingly dissociated into carbon and hydrogen. At atmospheric pressure this dissociation according to Pollitzer, is 50 per cent complete at red heat, and at 3600 deg. fahr. it is practically complete.

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[5] For investigations of such processes see, for instance, Pollitzer, "Die Berechnung Chemischer Affinitäten," Stuttgart, 1912, p. 106.

Chemical analysis at combustion temperatures is generally not possible. The investigator is, therefore, reduced to theoretical deductions, or estimates, from established physico-chemical facts.

Pressure, it is true, has a tendency to counteract this dissociation.[6]

None the less, Messrs. Fieldner, Straub and Jones, (Bureau of Mines Report No. 187-B-41), found both methane and hydrogen regularly in the exhaust gases of a considerable number of automotive vehicles investigated by them.[7] There was, however, in general, much more hydrogen than methane.

Methane is composed of one atom of carbon and four of hydrogen. The chemical formula is CH_4.

The liquid paraffin hydrocarbons given on page 115 are all more complex. Consider one of the simplest ones, *hexane*. It still is made up of 6 carbon atoms and 14 hydrogen atoms, the chemical formula being C_6H_{14}.

In this complex there is enough hydrogen for only three and a half methane atoms, but sufficient carbon for six. In consequence, when hexane is converted into methane, either some carbon must separate out, or some more complex "polymerization" products must be formed along with the methane.

Both the gravitation of petroleum hydrocarbon towards methane and the subsequent dissociation of methane into hydrogen and carbon have then a strong tendency to "precipitate" carbon. If air throughout the charge is intimately mixed with the fuel, then the carbon the instant it is separated out, is pounced upon by the oxygen and turned into carbon dioxide or carbon monoxide. There will then be no solid accretions of carbon glowing in the gas. The flame will be blue and barely visible. If the mixture is not so intimate, or if knocking takes place, the carbon precipitated has time to form accretions. These become incandescent in the gas. A yellow luminous flame results. For boiler firing this is rather an advantage, provided only that there is sufficient oxygen present and sufficient space for final consumption of even the glowing carbon. The incandescent particles radiate heat to the boiler. the heating surface of which is able in this way to absorb and transmit much more heat than it would by simple contact with blue combustion gas. In combustion engine cylinders the luminous flame is wholly objectionable. As a rule, the combustion space is there too small, and the combustion period too short, to allow complete combustion of carbon once aggregated into incandescent particles. Carbon deposits and smoky exhaust result, likewise great heat losses to the walls on account of radiation, and finally fuel waste due to this cooling loss and to incomplete combustion.

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[6] The general law is that every chemical reaction leading to increased volume is counteracted by pressure; every reaction leading to decreased volume is favored by pressure. When methane is dissociated (chemical equation $CH_4 = C + 2H_2$) there are formed two volumes of hydrogen for every volume of methane. The solid carbon simultaneously separated out is of too small volume to count.

[7] Under winter conditions hydrogen in the exhaust gases from twelve passenger cars and five trucks was found to vary between 2.0 per cent and 4.4 per cent, methane between 0.5 per cent and 2.2 per cent.

When a pound of hexane, C_6H_{14}, is formed from the elements, there results a molecular agitation equal to about 1200 B. t. u. From this pound of hexane there may be formed 0.65 pounds of methane having a "heat of formation" of about 1400 B. t. u. The difference is not very great, and may be neglected. For the further disruption of methane into hydrogen and carbon an agitation equal to 2100 B. t. u. per pound of methane is necessary. This heat must be taken from the heat of combustion.

The measured lowering heating value (See article 4, Part II) of methane at constant volume is 21400 B. t. u. per pound and covers the heat of disruption. It will be observed that the heat of disruption constitutes about 10 per cent of this value.

The lower heating value of a pound of hydrogen at constant volume is 51500 B. t. u.; that of a pound of carbon burned to carbon dioxide is 14600 B. t. u.

Methane contains 75 per cent carbon and 25 per cent hydrogen. Without any allowance for heat of disruption, its heat value ought to be

$$0.75 \times 14600 + 0.25 \times 51500 = 23830 \text{ B. t. u.}$$

This value is 2430 B. t. u. larger than the measured one for methane. The agreement with the heat of disruption of methane above given is practically within the limits of error in measurements of this character. Heats of combustion figured under the assumption of the burning of free carbon and free hydrogen instead of the chemical compounds actually used may then be about 10 per cent greater than the actual heat development.

With this understanding it is of value for the elucidation of the occurrences in a combustion engine cylinder to scrutinize the conditions on the assumption of carbon and hydrogen as the burning substances.

2. *Combustion of Carbon and Hydrogen.* In the presence of sufficient oxygen carbon will burn to carbon dioxide and hydrogen to water vapor. In carbureting engines somewhat over-rich mixtures, i. e., deficiency of oxygen, are the rule rather than the exception.

Under such circumstances two reactions can take place. Either carbon may burn partly to carbon monoxide and the hydrogen completely to water vapor, or the carbon may burn to carbon dioxide and some of the hydrogen may escape unburned. This latter reaction takes place at lower temperature, the former at higher. According to investigations by Hahn,[8] the chemical equilibrium at a temperature of 1200 to 1300 deg. fahr. calls for about twice as much carbon dioxide and hydrogen in the combustion gas as carbon monoxide and water vapor. At 2000 deg. fahr. there is, however, about twice as much carbon monoxide and water vapor as there is carbon dioxide and hydrogen.

In Table 15 is given a number of analyses of combustion gases from an automobile engine operated with mixtures of various richnesses (From Bureau of Mines Report No. 187-B-41, by Messrs. Fieldner, Straub and Jones).

TABLE 15

Pounds air per pound gasoline	Exhaust Gas Analysis, per cent.					
	CO_2	O_2	CO	CH_4	H_2	N_2
14.5	13.4	1.7	1.2	0.2	0.0	83.5
14.2	12.0	1.4	2.0	1.1	0.0	83.5
11.8	10.2	0.3	6.4	0.8	2.4	79.9
9.9	6.5	1.2	11.6	1.0	6.4	73.3

The mixtures in this table all contain less oxygen than needed for complete combustion. The appearance of free oxygen in the exhaust gas indicates, therefore, imperfect mixing of the fuel with the air. In the first two samples containing a nearly sufficient amount of air there is no free hydrogen, although there is some carbon monoxide. The richer the mixture is, the more free hydrogen is formed simultaneously with carbon dioxide. Such a condition indicates the occurrence of the so-called "water gas" reaction ($CO + H_2O = CO_2 + H_2$). The presence of methane may be accounted for by imperfect mixing, or by the occurrence of a reaction $CO + 3H_2 = CH_4 + H_2O$, converting carbon monoxide and hydrogen into methane.

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[8] Original article in *Zeitschrift für physikalische Chemie*, v. 44, p. 513 and v. 48, p. 735; here quoted from Pollitzer, Berechnung chemischer Affinitäten, p. 73

At atmospheric pressure this last reaction is possible in quantity only at comparatively low temperature, say 1200 deg. fahr. and less. The reaction is very much favored by pressure; yet at temperatures over 2000 deg. fahr. it is all the other way: Methane in the presence of water vapor is converted into carbon monoxide and hydrogen.

This is a new reminder of the fact that exhaust gas analysis is no guide to the actual chemical happenings in a combustion engine cylinder. A good portion of the final combustion products is formed only after the exhaust has cooled down. As shown by the diagrams in Figures 24 and 25, pages 152-153, and by calculations yet to follow, the temperature during combustion may reach considerably over 3000 deg. fahr., possibly over 4000 deg. fahr., and the temperature even at the end of the expansion stroke will, as a rule, be over 2000 deg. fahr.

At these temperatures the primary combustion products formed with insufficient oxygen are carbon monoxide and water vapor. The more oxygen there is present, the larger is the percentage of carbon monoxide which will be further converted into carbon dioxide.

For an estimate of the heat development and the temperatures attained, the best approximation is, therefore, to assume chemical reactions along this scheme.

3. *Dissociation.* Carbon monoxide, while dissociable to a considerable extent at temperatures below 1800 deg. fahr., dissociates only to a vanishingly small extent even at the highest temperature occurring in combustion engines.[9] Carbon dioxide

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[9] The dissociation of pure carbon monoxide into carbon and carbon dioxide is complete at 450 deg. c., 842 deg. fahr., but decreases rapidly as the temperature is raised until at 1000 deg. c., 1832 deg. fahr., the dissociation is zero. See Damour, Industrial Furnaces and Methods of Control, First ed., p. 35.

and water vapor dissociate considerably. Curves showing this dissociation as given by Nernst ("Theoretische Chemie", page 715) are plotted in Figures 15 and 16. These data have been

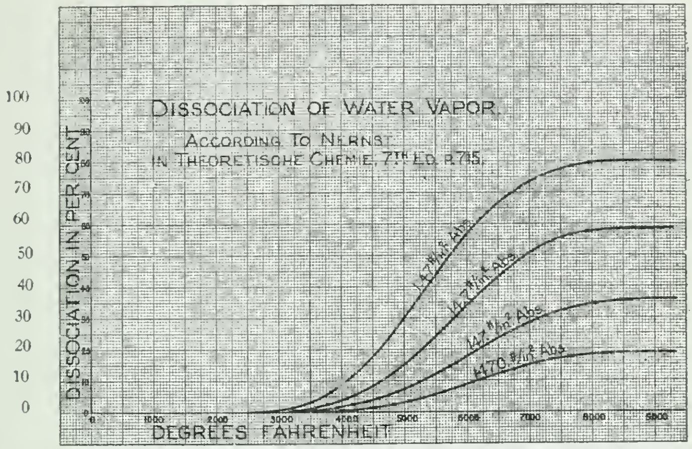

Fig. 15.

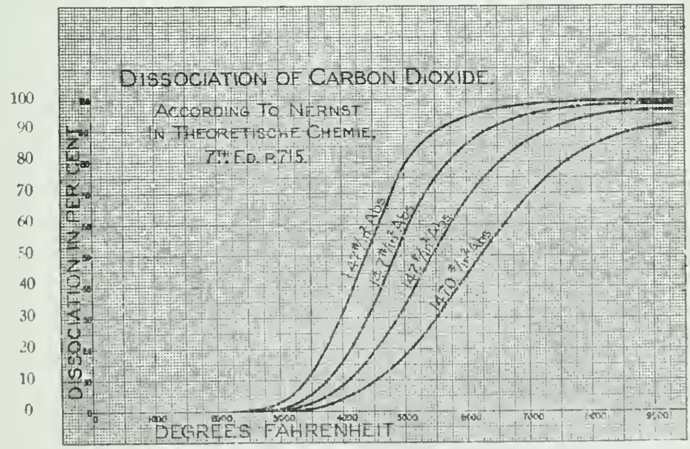

Fig. 16.

criticized, but are nevertheless accepted also by English investigators as the best available information on the subject.

From these curves one would conclude that at pressures obtaining in combustion engine cylinders dissociation plays an unimportant part in the case both of water vapor, and of carbon dioxide for the temperatures up to 3500 deg. fahr. Above this temperature, dissociation becomes pronounced especially in the case of carbon dioxide.

The question is: How much heat is required to raise the temperature of the combustion gases to this lower limit of dissociation? Does the combustion furnish this amount of heat?

In Figures 17 and 18 are given the specific heats of combustion gases from a fuel assumed to consist of 86 per cent carbon and 14 per cent hydrogen. These specific heats have been figured on the basis of the data underlying the entropy chart, described in the succeeding article, but apply to a pound of gas.* In applying such data for the computation of temperature, the usual assumption is that the combustion first takes place and that the heat thereby developed is used to raise the temperature of the combustion gas.

Let us assume that the fuel has a lower heat value of only 18,000 B. t. u. per pound, which corresponds to that for light gasoline. Let us assume further that the compression carries the temperature of the mixture to 600 deg. fahr. Consider four mixtures, containing 10, 12, 15 and 18 pounds of air per pound of fuel respectively. The first will be near the limit of ignitibility on account of richness, the last near the same limit on account of leanness.

The temperatures arrived at are given in Table 17.

TABLE 17

Pounds of air per pound of fuel	10	12	15	18
Specific heat c_v at 600 degrees fahr......	.203	.199	.193	.191
Specific heat c_v at 3500 degrees fahr.....	.272	.276	.277	.269
Average specific heat between 600 degrees fahr. and 3500 degrees fahr...	.238	.238	.235	.230
Heat available per pound of combustion gas (From Table 21)............	870	1000	1120	950
Figured temperature rise, degrees fahr..	3660	4200	4750	4130
Figured maximum temperature, degrees fahr.	4260	4800	5350	4730

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* (In the computation of these curves the author has had the assistance of Mr. Van B. Teach, graduate student at the Ohio State University).

These temperatures all fall in the region of appreciable dissociation, and are therefore higher than those actually obtained. Nevertheless it is now, especially in England, becoming generally recognized that temperatures in excess of 4000 deg. fahr. do occur for short intervals in engines running on readily volatilized and readily combustible fuels. Actually Hopkinson with a closed

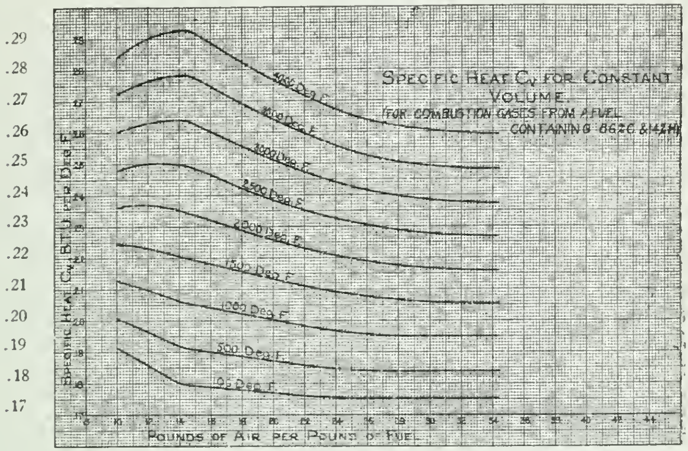

FIG. 17.

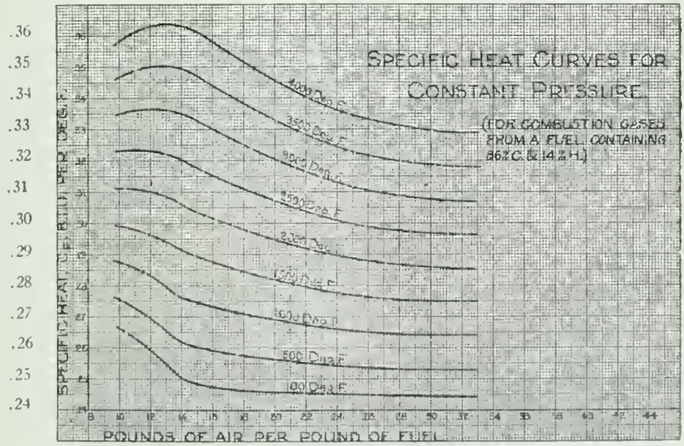

FIG. 18.

explosion vessel found that combustible gas mixtures on exploding melted platinum — i. e., reached temperatures at least in excess of 3200 deg. fahr. Yet these mixtures were ignited at atmospheric temperature without compression. (See Dugald Clerk, Gas, Petrol and Oil Engines, 1909 Edition, Vol. I, page 189).

While such high temperatures may be attained under the conditions mentioned, there are nevertheless factors tending, as a rule, to keep the peaks of temperature down. These factors are heat loss to the water cooled walls and combustion continuing during the expansion. If the combustion is complete before exhaust takes place, all the heat developed will, at least in some measure, go towards power development. For an estimate of the efficiency of the engine a knowledge of the heat actually available is the factor of first importance. To estimate this will be our next attempt.

4. *Influence of Temperature on Heats of Reaction.* The impossibility of perpetual motion demands that the difference in energy between two chemical states must be the same, no matter by what way the second state is arrived at from the first. Say that a chemical reaction is first carried out at ordinary temperature and that this reaction produces the amount of heat Q_o. Suppose then that the same initial substances are heated to a temperature T by expenditure of the heat q; that the reaction takes place at this temperature, developing the heat Q_T; finally that the products of reaction are cooled back to ordinary temperatures yielding up on cooling the amount of heat q_2. Then by the above principle

$$(5) \qquad Q_o = Q_T + q_2 - q_1, \text{ or}$$

$$(6) \qquad Q_T = Q_o + q_1 - q_2.$$

Supposing for simplicity the specific heats of the initial substances to be constant and equal to C_1, and those of the final products to be also constant and equal to C_2; then, if with the sign Σ we denote the sum of a number of terms, we have

$$(7) \quad Q_T = Q_o + \Sigma\, C_1\, (T - T_o) - \Sigma\, C_2\, (T - T_0).$$

Take as an illustration the combustion of hydrogen to water vapor:

$$(8) \qquad 2H_2 + O_2 = 2H_2O,$$

which also might be written

$$(9) \qquad 2H_2 + O_2 - 2H_2O = O \text{ (zero)}.$$

For this reaction we would have

$$Q_T = Q_o + 2C_{H^2}\,(T - T_o) + Co_2\,(T - T_o) - 2C_{H_2O}\,(T - T_o)$$

Understanding summations algebraically, i. e. to involve subtraction as well as addition, dependent upon sign, we might write simply

$$(10) \qquad Q_T = Q_o + \Sigma\, C\,(T - T_o).$$

The summation must be taken with the signs obtained from the reaction equation written in the form (9).

Should the specific heats not be constant, but vary with the temperature then the equation (10) takes the form

$$(11) \qquad Q_T = Q_o + \Sigma \int C dT$$

Especially if the specific heat is expressible in the form used in the entropy chart

$$C = a + bT,$$

then

$$(12) \qquad Q_T = Q_o + \Sigma\, a\,(T - T_o) + \Sigma \frac{b}{2}\,(T^2 - T^2{}_o).$$

In this equation the quantities are introduced in pound molecules or mols. The specific heats used apply to one mol of substance, and the heats of reaction to the number of mols indicated by the chemical equation.

The inherent reason hereof will be somewhat further discussed in the next article. Suffice it here to say that one pound molecule, or mol, is a number of pounds equal to the molecular weight of the substance.

These molecular weights are

for carbon, C..............	12
for hydrogen, H_2	2
for oxygen, O_2	32
for nitrogen, N_2	28
for carbon monoxide, CO....	28
for carbon dioxide, CO_2.....	44

Take, as an example, the chemical equation describing the combustion of hydrogen to water vapor:

$$2H_2 + O_2 = 2H_2O + 208{,}000 \text{ B. t. u.}$$

This means that 2 × 2 pounds of hydrogen with 32 pounds of oxygen burn to form 2 × 18 pounds of water vapor and that this combustion is accompanied by a heat of reaction equal to 208,000 B. t. u.

One mol of hydrogen, 2 pounds, would then produce 104,000 B. t. u., and 1 pound of hydrogen 52,000 B. t. u.

The values of the specific heats per mol are given in the next article.

5. *Heats of Reaction at Constant Pressure and at Constant Volume.* Consider the combustion of carbon to carbon monoxide

$$2C + O_2 = 2CO.$$

Carbon is in the solid form and occupies but a negligible volume. Oxygen is gaseous and one molecule of oxygen takes part in the reaction. Two molecules of carbon monoxide are formed. Remembering that equal volumes of gases contain equal numbers of molecules, we observe that one volume of oxygen gives two volumes of carbon monoxide. If the reaction has taken place in a calorimeter at constant volume, the crowding of two volumes into the space of one results in the products of combustion being left, after cooling, at a pressure twice as high as that of the original substances. If now the enclosed gases are allowed to expand down to the original pressure, this expansion means displacement work done, and the energy necessary for this work is taken out of the gases. In other words, the gases will cool down and after equality of temperature has been established between the gases and the measuring water in the calorimeter, this water will register a smaller total temperature rise than it did at constant volume. Referring the heat of reaction to an unvarying atmospheric temperature, T_o, the difference will be the displacement work for one mol of gas at this temperature.

Calling Q_v the heat of reaction per pound molecule at constant volume and Q_p that at constant pressure we have in the case of the combustion of carbon to carbon monoxide

$$(13) \qquad Q_p = Q_v - pV_m/E = Q_v - R_mT_o/E,$$

where V_m is the volume per pound molecule, R_m is the gas-constant per pound molecule and E is the mechanical equivalent of heat.

Writing the reaction equation in the form

$$2C + O_2 - 2CO = O \text{ (Zero)}. \tag{14}$$

and summing the volumes of gases, or molecules, algebraically we find

$$\Sigma \text{ mols} = 1 - 2 = -1$$

From this we arrive at the general rule

$$Q_p = Q_b + \Sigma R_m T/E.$$

Here $\Sigma R_m T$ means that $R_m T$ is to be multiplied by the number obtained by summing the pound molecules of gases according to the reaction equation written as in (9) or (14).

(Be it particularly noted that the summation is to extend only to gaseous, and not to solid or liquid substances).

For an illustration let us consider the three remaining chemical reactions of greatest importance in combustion engineering:

(a) *Combustion of hydrogen to water vapor*

$$2H_2 + O_2 - 2H_2O = O \text{ (Zero)}$$
$$\Sigma \text{ mols} = 2 + 1 - 2 = +1$$
$$Q_p = Q_v + R_m T/E$$

The heat of reaction at constant pressure is greater than the heat of reaction at constant volume. At ordinary temperature this difference is not great; at the temperature in combustion engines it becomes more noticeable.

(b) *Combustion of carbon to carbon dioxide*

$$C + O_2 - CO_2 = O \text{ (Zero)}.$$

The sum of the pound molecules of gases

$$1 - 1 = O.$$

In this case the heat of combustion at constant volume is equal to the heat at constant pressure.

(c) *Combustion of carbon monoxide to carbon dioxide.*

$$2CO + O_2 - 2CO_2 = O \text{ (Zero)}.$$
$$\Sigma \text{ mols} = 2 + 1 - 2 = + 1.$$
$$Q_p = Q_v + R_m T/E.$$

The heat of combustion at constant pressure is greater than that at constant volume.

(6) *Numerical Values of (Lower) Heats of Combustion*

On the basis of the rules just given the numerical values of the lower heats of combustion at various temperatures have been computed.

(a) *Combustion of Hydrogen.*

The heat of combustion at constant pressure of one mol (2 pounds) of hydrogen at ordinary temperature (according to Pollitzer, 63 deg. fahr.) can be taken equal to 104,000 B. t. u.

The specific heat values used are those given by Nernst in his Theoretische Chemie, 7th Ed., page 267. A more detailed discussion of these values is given in the article on the Entropy chart. The linear formulas for the specific heats of simple gases and of water vapor there given are in this case a rather accurate representation of Nernst's values.

The heats of combustion arrived at are given in Table 18.

TABLE 18

HEATS OF COMBUSTION OF HYDROGEN

Temperature degrees fahr.		540	1540	2540	3540	(4540)
Temperature degrees fahr. abs.		1000	2000	3000	4000	(5000)[10]
Heat of combustion per mol, (2H) B. t. u.	At constant pressure	105,205	106,890	107.910	107,810	(106.310)
Heat of combustion per pound, B. t. u.		52,600	53,445	53,955	53.905	(53.150)
Heat of combustion per mol, (2H) B. t. u.	At constant volume	104,205	104,890	104.900	103 810	(101,310)
Heat of combustion per pound, B. t. u.		52,100	52.445	52,450	51,905	(50.655)

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[10] The bracketed values fall in the region of considerable dissociation and do therefore not apply.

In figuring these values dissociation has been neglected. The values for 4540 deg. fahr. where dissociation begins to become an appreciable factor are, therefore, put within brackets.

The variation of the heat of combustion within the range of 1500 to 3500 deg. fahr. will be observed to be of the order of magnitude of only 1 per cent. The heat at constant volume even at 3500 deg. fahr. is only 4 per cent less than that at constant pressure.

For some computations regarding the heat development in constant volume engines, given later, the author has for hydrogen assumed an average heat of combustion at constant volume of 52,000 B. t. u. per pound (104,000 B. t. u. per mol). For constant pressure an average value of 53,500 or 54,000 B. t. u. per pound might be assumed.

(b) *Combustion of Carbon to Carbon Dioxide*

In a combustion engine the question is not about the combustion of carbon as such, but of carbon the instant it is torn out of the hydrocarbon molecules. It is questionable whether under these conditions the carbon will behave as a solid body. The uncertainty hereof and also of the value of the specific heat of carbon at increased temperature makes it hardly worth while to figure closely the variation of the heat of combustion with the temperature. It has already been stated that for this reaction the heat of combustion at constant volume is the same as that at constant pressure.

Following Pollitzer we may for both assume an average value of 176,000 B. t. u. per mol. or 14,600 B. t. u. per pound for all conditions.

(c) *Combustion of Carbon to Carbon Monoxide.*

What was said under (b) about the uncertainty regarding the specific heat and the state of carbon applies also here.

Pollitzer gets around this difficulty by using an approximate equation, which in B. t. u. and deg. fahr. becomes

$$Q_T = 54{,}000 - 1.75\,T + 0.0008\,T^2. \tag{15}$$

Here Q_T is the heat of combustion at constant pressure per mol, and T the absolute temperature. The heat at constant volume is greater than that at constant pressure. Values for both are given in Table 19.

TABLE 19

HEAT OF COMBUSTION OF CARBON TO CARBON MONOXIDE

Temperature, degrees Fahrenheit		540	1540	2540	3540	(4540)
Temperature, degrees Fahrenheit, absolute		1000	2000	3000	4000	(5000)[11]
Heat of combustion, B. t. u. per mol.	At constant pressure	53,050	53,700	55,950	59,750	(65,250)
Heat of combustion, B. t. u. per pound		4,430	4,480	4,660	4,970	(5,430)
Heat of combustion, B. t. u. per mol.	At constant volume	54,050	55,700	58,950	63,750	(70,250)
Heat of combustion, B. t. u. per pound		4,530	4,650	4,920	5,330	(5,850)

The heat values in this case grow considerably with the temperature. For average estimates under conditions in combustion engine cylinders, a heat value per pound of about 4600 B. t. u. may be assumed at constant pressure and one of 5000 B. t. u. at constant volume.

(d) *The Combustion of Carbon Monoxide to Carbon Dioxide.*

The combustion of carbon monoxide to carbon dioxide does not figure in the discussion here to follow and is of subordinate importance in connection with liquid fuel. It is, however, of considerable importance in connection with gaseous fuels.

(The variation of the heat value with the temperature may be computed with tolerable accuracy with the aid of the linear specific heat expressions underlying the entropy chart, since in the formula used the specific heat is multiplied by a temperature difference covering the range from atmospheric to combustion temperature.)

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[11] See Foot Note to Table 18.

The chemical equation is for one mol of CO

$$CO + 1/2\,O_2 - CO_2 = O. \quad \text{(Zero)}$$

and the heat of reaction at ordinary temperature and constant pressure is 53,300 B. t. u. per mol.

The values found are given in Table 20.

TABLE 20

COMBUSTION OF CARBON MONOXIDE TO CARBON DIOXIDE

Temperature degrees fahr.		540	1540	2540	3540	(4540)
Absolute temperature degrees fahr. absolute		1000	2000	3000	4000	(5000)[12]
Heat of combustion per mol, B. t. u.	At constant pressure	123,420	123,940	122,600	119,380	(114,200)
Heat of combustion per pound, B. t. u.		4,410	4,420	4,380	4,260	(4,080)
Heat of combustion per mol, B. t. u.	At constant volume	122,420	121,940	119,000	115,380	(109,200)
Heat of combustion per pound, B. t. u.		4,380	4,350	4,260	4,130	(3,800)

The sum of the heats of combustion of carbon to carbon monoxide and carbon monoxide to carbon dioxide ought per mol to be equal to that of carbon to carbon dioxide. This last heat has in the preceding discussion been taken equal to 176,000 B. t. u. per mol and practically constant at all temperatures.

An addition of the values from Tables 19 and 20 will show a sum increasing somewhat with the temperature, yet in no case exceeding 179,130 B. t. u. In view of the approximate character of the formula underlying the combustion of carbon to monoxide and the uncertainty of the specific heat values both for solid carbon and for carbon dioxide, this is an agreement which must be considered sufficiently close.

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[12] See Foot Note to Table 18.

7. *Heat Development with Oil Fuel and Mixtures of Varying Richness.* For the purpose of obtaining an approximate idea about the heat development in an engine cylinder with mixtures of varying richness we may assume an oil fuel containing 86 per cent carbon and 14 per cent hydrogen, and suppose this to burn first to water vapor and carbon monoxide, then further to carbon dioxide, if sufficient oxygen is available.

The conditions will then be as shown in Table 21. In figuring this table the following average values for the heat derivable from the various reactions occurring have been assumed as follows:

Heat per pound of water vapor formed	5,800 B. t. u.
Heat per pound of carbon dioxide formed	4,000 B. t. u.
Heat per pound of carbon monoxide formed....	2,100 B. t. u.

The final heat values obtained have been corrected for the heat of disruption of methane, or of the original hydrocarbons, by subtracting 2,000 B. t. u. per pound of fuel. Quite unintentionally this turns out to give an even 18,000 B. t. u. per pound in the case of complete combustion to carbon dioxide.

The interesting fact brought out is now that the heat available per pound of combustion products is greatest at 14 to 15 pounds of air per pound of fuel, that is to say, is greatest with the theoretically required amount of air. It is less both for over-rich mixtures and over-lean mixtures and the actual heat available is about the same with 10 pounds of air per pound of fuel as with 20 pounds of air per pound of fuel.

Owing to dissociation, the full heat development here figured can in no case be attained in the combustion space. An after-burning must take place during the expansion stroke, where the temperature increase that would result from combustion is counterbalanced by cooling caused by expansion and by heat losses to the walls. The greater the heat available per pound of combustion gas, the greater necessarily will be this after-burning, and the greater will also be the cooling loss. This is fully borne out by the curves reproduced in Fig. 19 from a paper by

TABLE 21

HEAT DEVELOPMENT WITH VARYING RICHNESS OF MIXTURE

(FUEL CONTAINING 86% C. AND 14% H.)

Pounds of air per pound of fuel	10	11	12	13	14	15	16	17	18	19	20
Pounds combustion products per pound of fuel.......	11	12	13	14	15	16	17	18	19	20	21
Per cent of H_2O in combustion products (by weight)	11.5	10.4	9.7	9.0	8.4	7.9	7.4	7.0	6.6	6.3	6.0
Per cent CO in combustion products	18.2	12.8	8.7	5.1	2.1	0	0	0	0	0	0
Per cent CO_2 in combustion products	0	6.2	10.6	14.3	17.6	19.8	18.6	17.5	16.6	15.8	15.0
Per cent N_2 in combustion products	70	70.8	71.1	71.5	72.0	72.2	72.6	72.9	73.0	73.1	73.3
Per cent O_2.................	0	0	0	0	0	1	1.45	2.6	3.8	4.8	5.7
B. t. u. per pound combustion products	870	960	1,000	1,060	1,090	1,120	1,050	1,000	950	900	855
B. t. u. per pound of fuel....	9,500	11,460	13,170	14,790	16,270	18,000	18,000	18,000	18,000	18,000	18,000

C. A. Norman and B. Stockfleth, (Journal S. A. E., February, 1920, page 105).

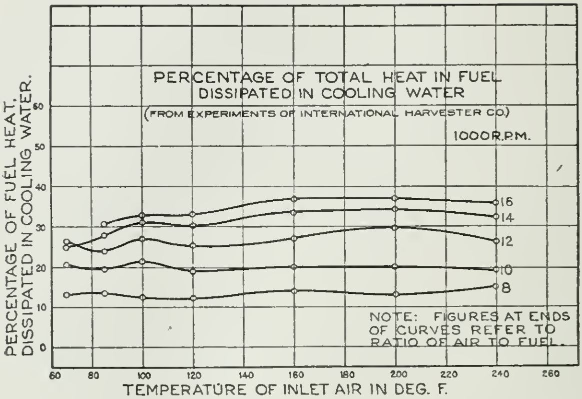


Fig. 19.

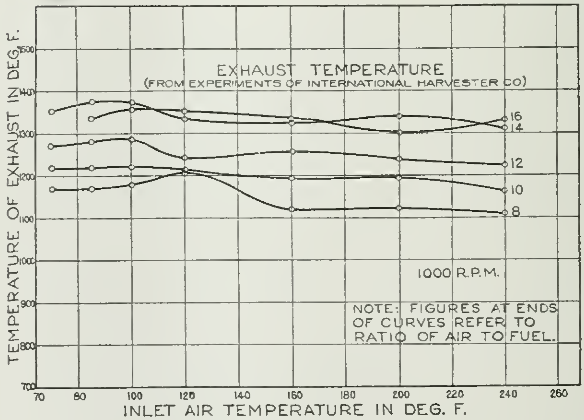
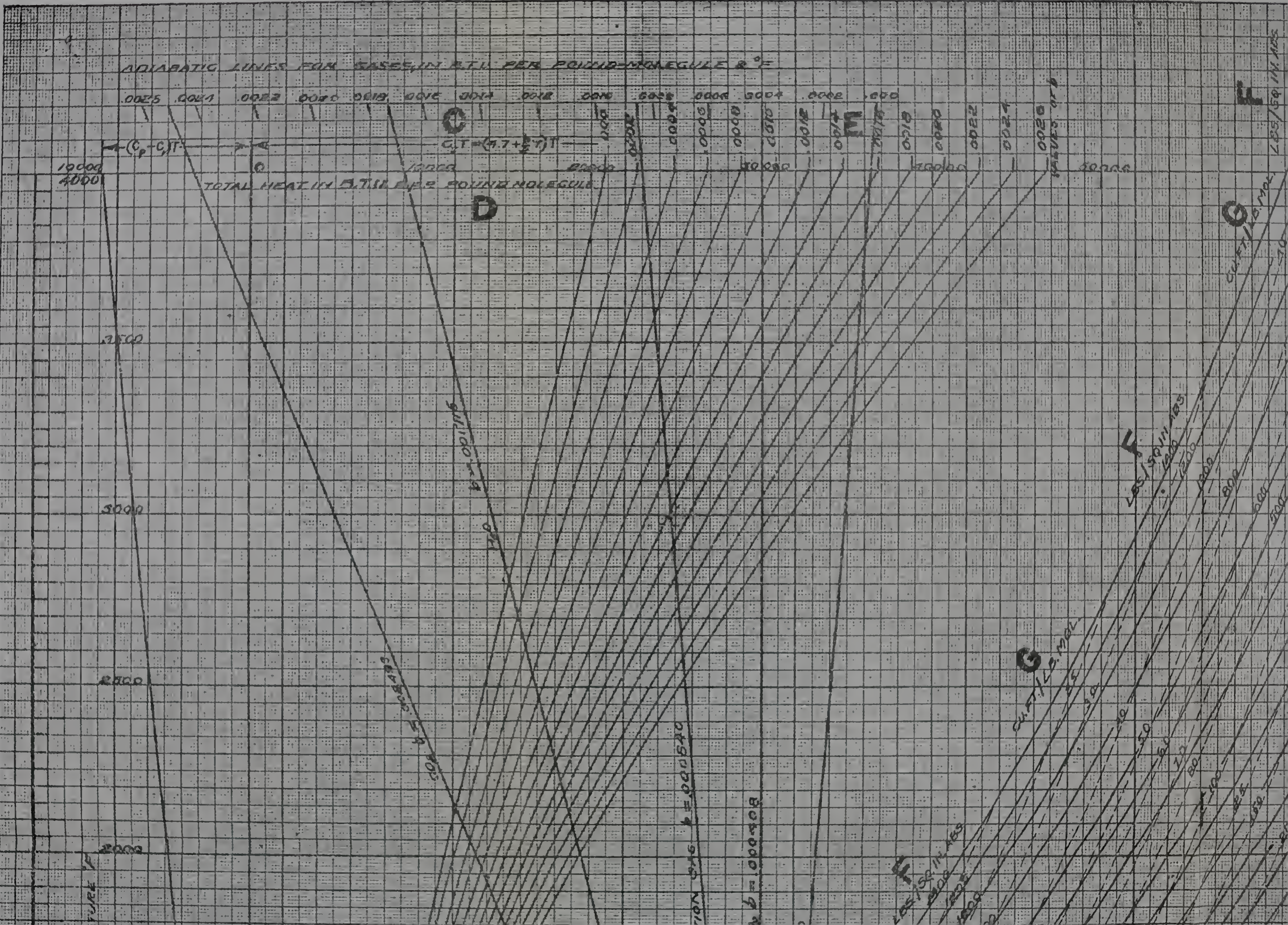
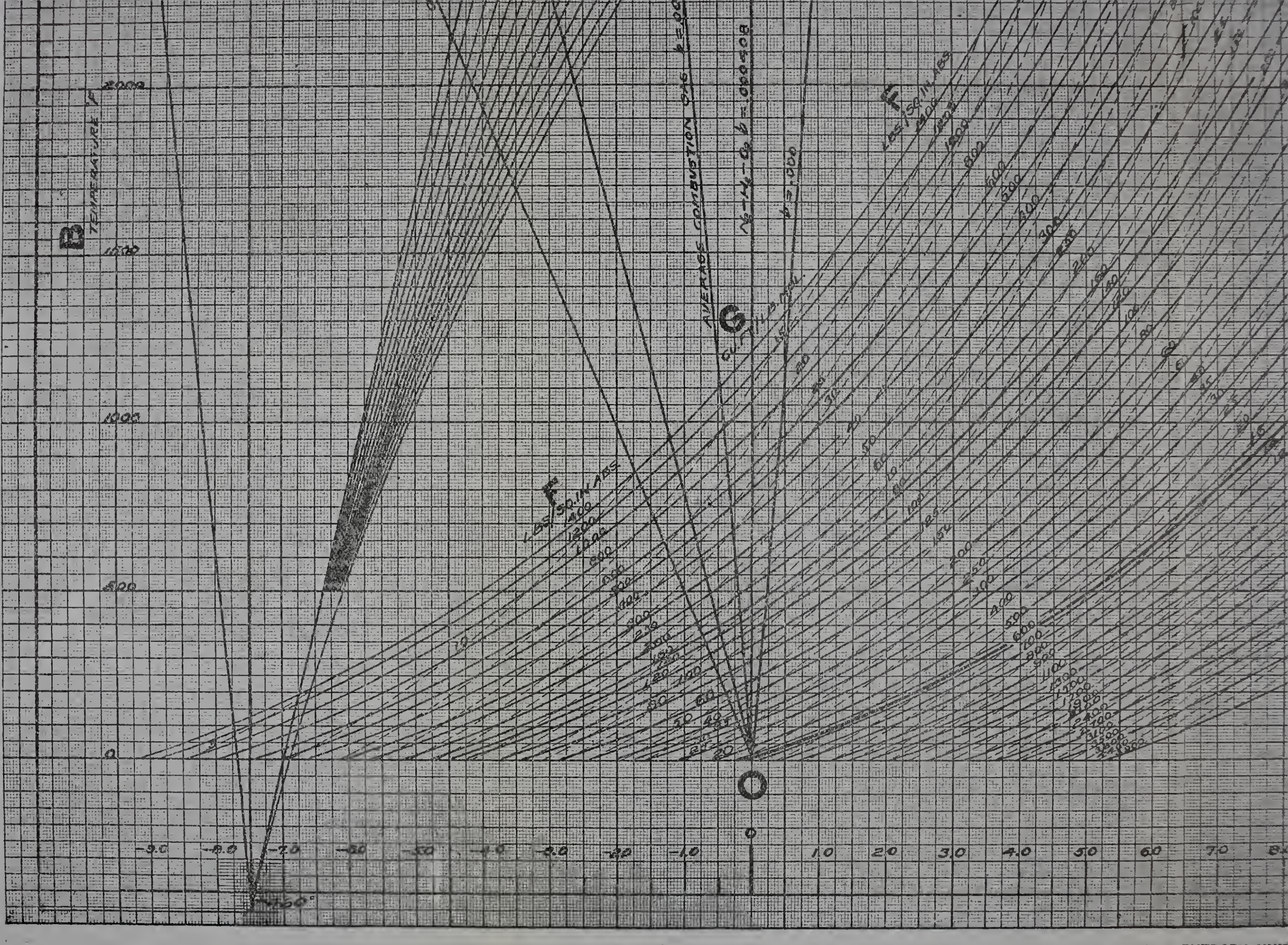


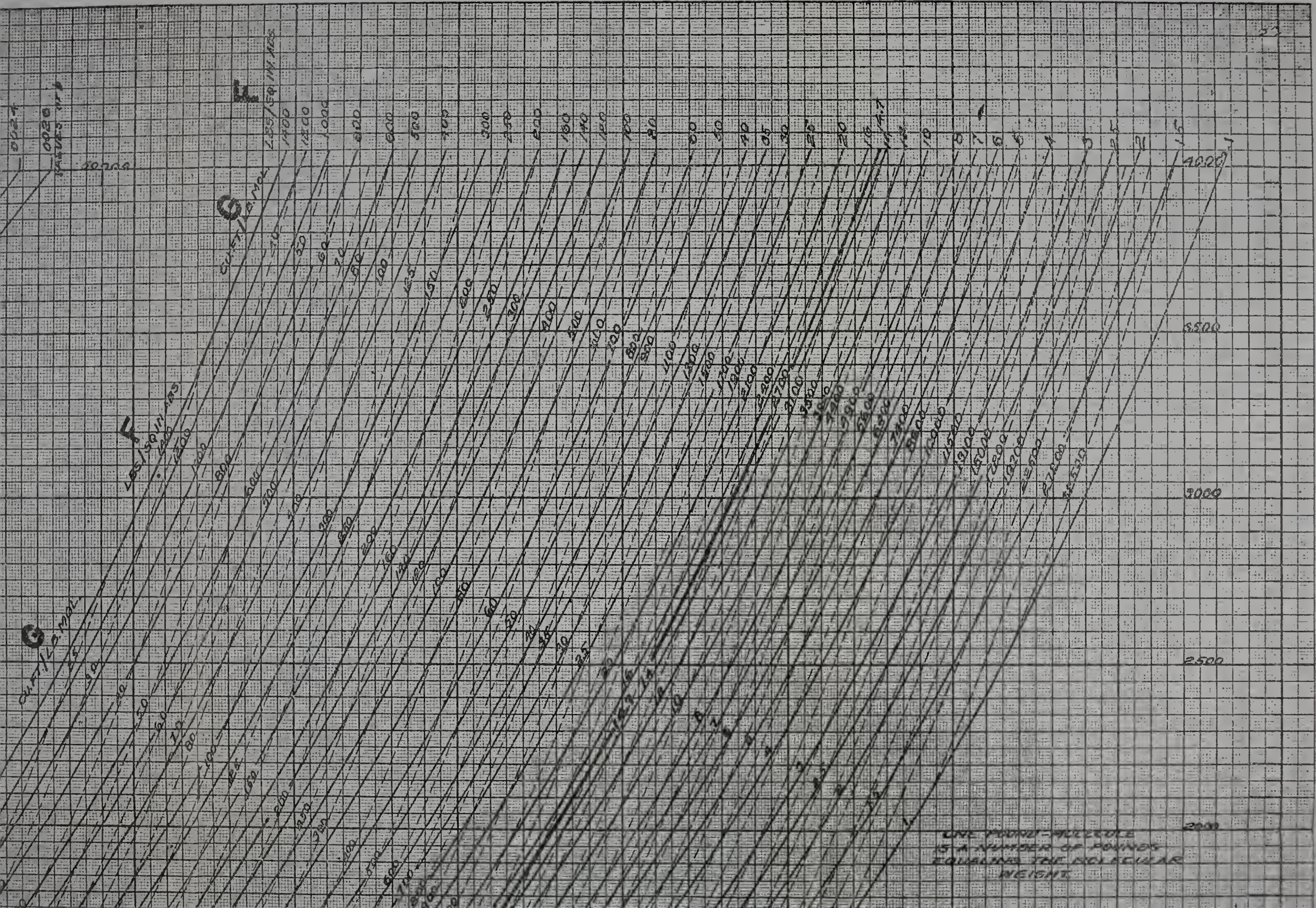
Fig. 20.

From the curves in Fig. 20 taken from the same paper it may be observed, that also the exhaust temperature is greater with the leaner mixture.

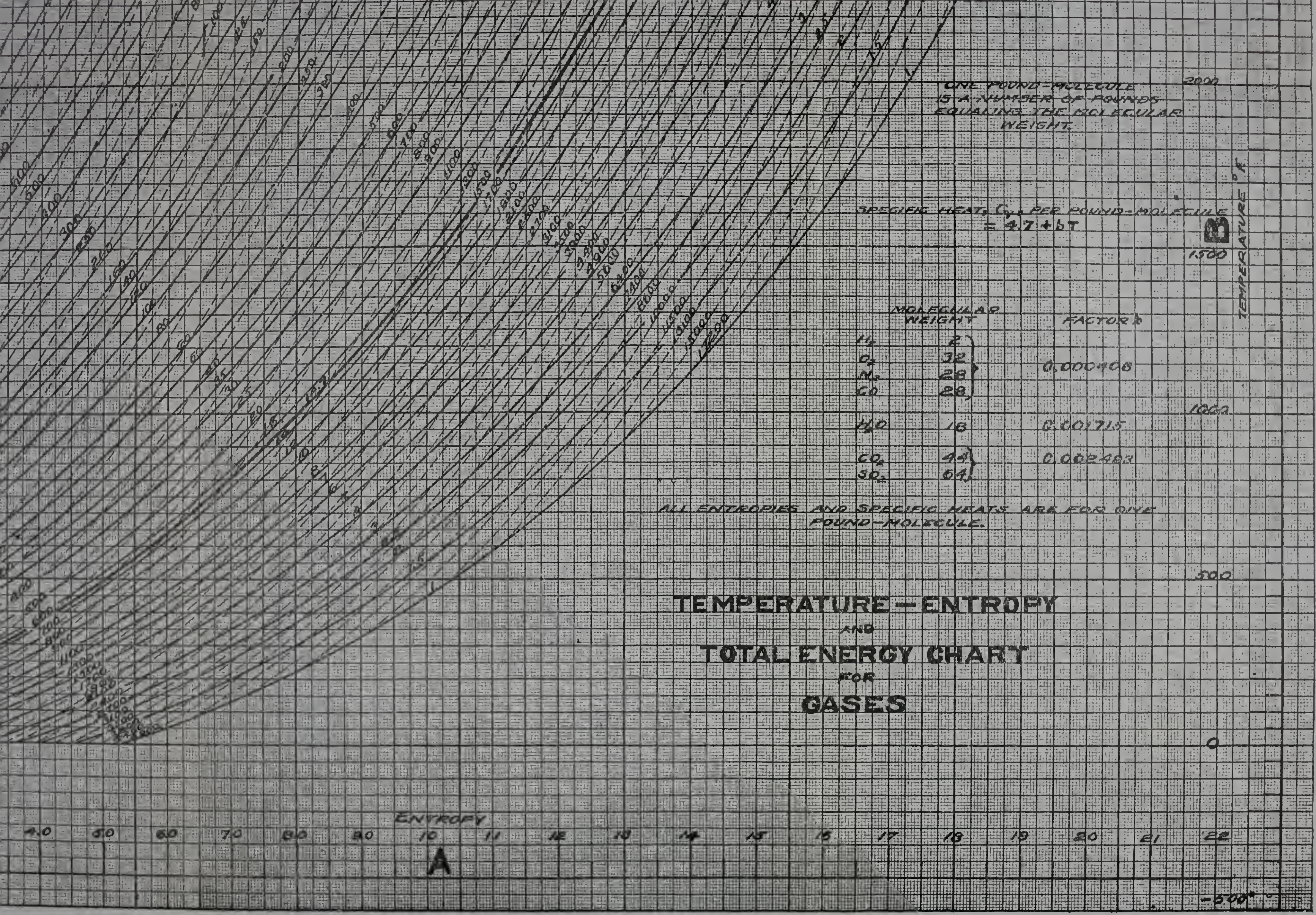

ADIABATIC LINES FOR GASES, IN B.T.U. PER POUND-MOLECULE & °F
.0025
.0024
.0022
.0020
.0018
.0016
.0014
.0012
.0010
.0008
.0006
.0004
.0002
.000
C
E
F
D
G
$C_v T = (4.7 + \frac{B}{2}T)T$
$(C_p - C_v)T$
0
10000
4000
10000
20000
30000
40000
50000
TOTAL HEAT IN B.T.U. PER POUND MOLECULE
VALUES OF b
3500
3000
2500
2000
LBS./SQ.IN. ABS.
CU.FT./LB.MOL.
b = .0011176
b = .000493
b = .000520
b = .000408



SCALE: ½ INCH = 1 ENTROPY UNI



ONE POUND-MOLECULE IS A NUMBER OF POUNDS EQUALING THE MOLECULAR WEIGHT.



ONE POUND-MOLECULE IS A NUMBER OF POUNDS EQUALING THE MOLECULAR WEIGHT.

SPECIFIC HEAT, C_v, PER POUND-MOLECULE = 4.7 + bT

	MOLECULAR WEIGHT	FACTOR b
H_2	2	0.000408
O_2	32	
N_2	28	
CO	28	
H_2O	18	0.001715
CO_2	44	0.002483
SO_2	64	

ALL ENTROPIES AND SPECIFIC HEATS ARE FOR ONE POUND-MOLECULE.

TEMPERATURE—ENTROPY AND TOTAL ENERGY CHART FOR GASES

SCALE: ½ INCH = 1 ENTROPY UNIT

ARTICLE 18

ENTROPY CHART.[13]

(By C. A. Norman and W. F. Borgerd.)

SUMMARY: *The entropy chart makes possible the presentation of diagrams showing immediately the heat and temperature variation in engines. It, also, with the addition of an energy chart, makes possible the computation of probable engine performance with allowance for actual composition of combustion products, for variation in specific heats with temperature, etc. A chart based on recent specific heat values is presented and sample diagrams and calculations are given. These examples cover both carbureting and injection engines, as well as gas turbines.*

An entropy chart of the kind here presented may serve two purposes. First, it can be used for analysis and investigation of the heat production and the heat exchanges generally occurring in combustion engines with the aid of an ordinary indicator diagram only. Secondly, it can be used for the precomputation of the performance and heat utilization to be expected from heat motors even of new and untried types.

For the first purpose the entropy charts is, of course, not the only resource available. When a certain quantity of gas is either expanded, or compressed, the changes in pressure and volume taking place can in general be mathematically covered by the equation

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[13] The chart and calculations here presented are based on methods first suggested by Professor A. Stodola of Zurich in 1898. (Zeitschrift des Vereines Deutscher Ingenieure. 1898, p. 1045), and subsequently applied by him to gas entropy charts, the last of which, to the author's knowledge, was published as an addition to the fourth edition of his Dampfturbinen in 1910. This gas chart was in metric units and was based on specific heat values which may not be quite as up to date as those given by Nernst in the seventh edition of his Theoretische Chemie, page 267. The author of this bulletin, therefore, first refigured the Stodola chart in English and in absolute units with Nernst's specific heat values. The chart herewith presented is, however, based on recalculations undertaken entirely in English units in the Gas Power Engineering Department of the International Harvester Company in Chicago, under the direction of the author of this bulletin, but at every step with the cooperation of Mr. W. F. Borgerd, Assistant Chief Engineer on Stationary Engines in this department. The actual work both of computing and drawing the chart, was carried out with the aid of several younger draftsmen, but special thanks are due to Mr. Cecil K. Wright.

$$(16) \qquad pv^n = \text{constant.}$$

Here p is the pressure, v is the volume and n is the so-called exponent of compression or expansion. If n equals unity, the change takes place at constant temperature and is called isothermal. If n is greater than unity, the expansion is accompanied by a drop in temperature, and compression is accompanied by an increase in temperature. Should, during the compression or expansion, no heat be either added, or subtracted from the gas, except in the form of compression or expansion work, then the change is called adiabatic, and the exponent is called the exponent of adiabatic change.[14]

If n is greater than the adiabatic exponent, then the expansion proceeds under loss of heat to the outside, and the compression under absorption of heat. If n is less than the adiabatic value, then expansion proceeds under heating and compression under cooling. It is thus, with the aid of the compression or expansion exponent, possible to say what kind of heat exchanges take place during compression or expansion, provided only that the adiabatic value for the gases is known. For air this value at ordinary temperature is about 1.4 and for combustion gases very much diluted with air or with nitrogen, it may not be very far from this value. Unfortunately, for gases in general and for combustion gases like carbon dioxide and water vapor in particular, the adiabatic exponent does not depend on chemical composition only, but changes also with the temperature. If, therefore, we succeed in determining the expansion exponent applicable to the working of a combustion engine, this is not enough to tell us where heating by combustion preponderates and where cooling by the jacket preponderates unless we know also the value of the adiabatic exponent at every point of the stroke. No really convenient way of plotting this change together with the expansion curve has been suggested. By means of the entropy chart as introduced by Stodola, a very easy way to allow for this change is given and heating and cooling effects on the gases are shown to the eye in the most simple and lucid way possible. How this can be done will now first be described.

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[14] It is to be understood that the gas may be heated not only by the direct application of heat from an outside source, but also by friction and turbulence leading finally to increased molecular agitation, that is to say, to heat.

Preliminary Explanation of the Chart.

Consider, to begin with, this chart as an accomplished fact, such as herewith presented. The theoretical justifications for the make-up of the chart will be given later. It will be observed that the horizontal scale on the chart is one giving "*entropy*", the vertical scale, one giving *temperature*. The physical meaning of entropy may for the moment be neglected. Suffice it to say that the product of temperature and entropy change means heat given up or absorbed in the form of heat, as the gas passes from one state to another. Across the chart from lower left to upper right are drawn lines representing constant pressure, or constant volume. Suppose, now, for instance, that a gas expands from the pressure p_1 to the pressure p_2, the temperature remaining constant at the value T_A. In Fig. 21 this expansion is represented by the straight line A B. The entropy increases from the value N_1 to value N_2. The heat absorbed during this expansion is equal to $T_A (N_2 - N_1)$ and this expansion is manifestly represented by the area of the rectangle N_1 ABN_2. If the entropy remains constant, there is no heat either absorbed or given up. On the chart, then, the gas might expand from the pressure p_1 to the pressure p_2, at constant entropy N_1, reaching

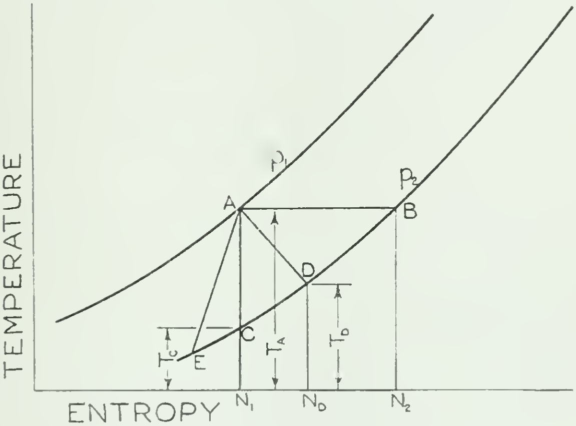

FIG. 21.

the p_2 line at the point C. This point C gives immediately the temperature T_C attained by the gas at the end of such expansion. As has been stated already, an expansion of this character is called adiabatic. On the chart the isothermal expansion is shown by horizontal lines, adiabatic expansion by lines for constant entropy, which to begin with we may take to be vertical. A change deviating to the *right* from the constant entropy line is a change involving heat absorption. A change deviating to the *left* means a change involving loss of heat to the outside. Thus an expansion of the gas from the pressure p_1 to the pressure p_2 along the line A D is an expansion involving absorption of heat, and the amount of the heat absorbed is given by the area N_1ADN_D. If AD is a straight line, the numerical value of the area is equal to

$$\frac{T_A + T_D}{2}(N_D - N_1).$$

Conversely, a compression of the gas from the pressure p_2 to pressure p_1, p_1 being higher than p_2, deviates to the left from the constant entropy line through D and consequently involves heat loss. The heat loss is again represented by the area N_DDAN_1, and is mathematically expressed by the quantity just given. An expansion from p_1 to p_2 beginning at A and ending at E deviates to the left of the entropy line through A and consequently proceeds under cooling. A compression from p_2 to p_1 beginning at E and ending at A deviates to the right from the constant entropy line through E and proceeds with heat absorption.

It will then be seen that in order to read immediately on the chart not only qualitively whether a certain compression or expansion proceeds with heat gain or heat loss, but beyond this, to determine quantitatively how much heat is gained or lost during this change, it is only necessary to lay off the change graphically on the entropy chart, that is to say, to identify on the chart the pressures, temperatures and volumes attained by the gas during the change. As the chart shows lines both for constant pressure and for constant volume, an ordinary indicator diagram may readily be transferred to the chart. For this purpose it is only necessary that the volumes given by the indicator diagram be changed in the ratio of the weight of gas covered by the chart to the weight of gas trapped in the engine cylinder.

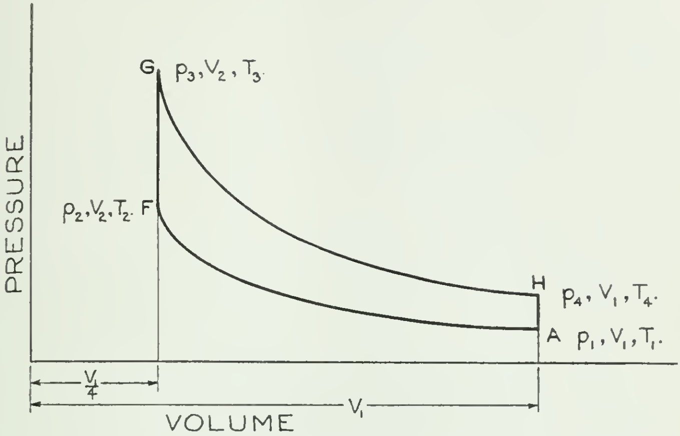

Fig. 22.

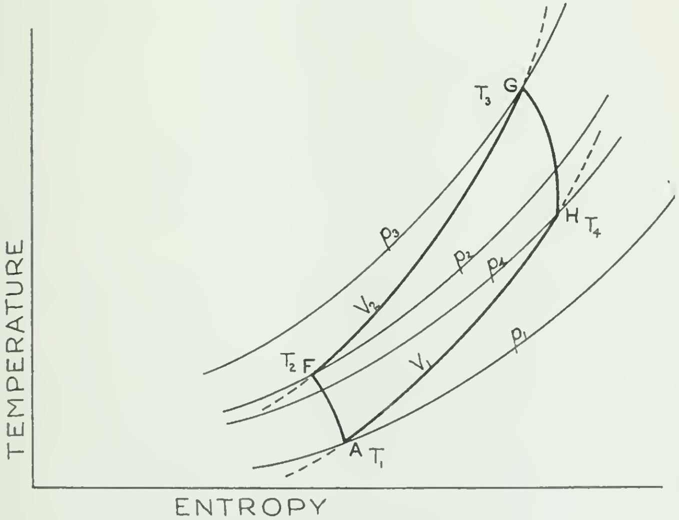

Fig. 22 a.

Transfer of Indicator Diagrams to the Entropy Diagram.

Consider the indicator diagram depicted in Fig. 22. Suppose that the temperature at the beginning of compression, that is to say, at the point A, is known. Call this temperature T_1, the pressure p_1, and the volume v_1. On the corresponding entropy diagram in Fig. 22a the line for the same pressure p_1 reaches the temperature T_1 at the point A. The constant volume line through this point is V_1. The volume v_1 on the indicator diagram then corresponds to V_1 on the entropy diagram. With reference again to the indicator diagram the pressure at the end of compression may be p_2, and the volume v_2. Suppose that v_2 is ¼ of v_1. On the entropy liagram the line for a volume ¼ of V_1 is V_2. This volume line intersects the line for the pressure p_2 at the point F. This point then corresponds to the point F on the indicator diagram. It gives immediately the temperature T_2 obtaining at the end of compression. Since the point F occurs to the left of the constant entropy line through A, the compression has proceeded on the whole with a cooling loss to the jacket. Reverting again to the indicator diagram, a combustion at the constant volume v_2 takes place, raising the pressure to p_3. On the entropy diagram the intersection of the V_2 and the p_3 line is at G from which the temperature T_3 is found immediately. The combustion in the clearance space is followed by an expansion. During the first part thereof an after-burning takes place. The expansion line deviates to the right of the constant entropy line. Then cooling preponderates and the expansion becomes apparently more nearly adiabatic. Finally, the original volume v_1 on the indicator diagram, and and V_1 on the entropy diagram is reached and the diagram closed along the V_1 line.

In the actual use of the chart it should be noted that the constant entropy lines run vertically only in the case of air and other simple gases like nitrogen, hydrogen and oxygen, and also carbon monoxide. This direction is given by a vertical line through O on the chart and the chemical signs for these gases are therein given. For all other gases the lines for constant entropy are inclined. The inclination for CO_2 is the extreme one to the left. All changes in a quantity of carbon dioxide involving no loss or gain of heat in the form of heat run parallel to this line. Similarly all adiabatic changes taking place in superheated water vapor run paralled to the line marked H_2O. Finally, there is a line drawn on the chart marked average combustion gas. The inclination of this line corresponds roughly to a gas composition derivable from the combustion of oil fuel or carbon with the theoretical amount of air. For rough and ready reference, it may serve also for other compositions.

Without explaining at this place the reason for this inclination of the entropy lines, let us rather consider the application of the entropy chart to two actual problems. In Fig. 23 is shown the indicator diagram from an ordinary explosion gas engine. In Fig. 24 is shown this same indicator diagram transferred to the entropy chart. The compression commences at A at a pressure of about 13 pounds absolute. Through this point is drawn the constant entropy line for pure air. (To be absolutely exact, the inclination of the line should allow also for the presence of a certain amount of more or less vaporized fuel in the air). The compression proceeds from A along a curve to the line for 60 pounds absolute pressure. The pressure first deviates to the right from the constant entropy line, showing heating by the walls of the cylinder, then changes its course to the left of the constant entropy direction, showing that when a pressure of about 30 pounds has been reached the cooling effect of the jacket preponderates. At about 60 pounds pressure ignition takes place and the diagram line now shoots sharply off to the right and reaches at the point M a pressure close to 300 pounds absolute. This is the maximum pressure attained. Heat addition evidently takes place during this whole period, that is to say, the combus-

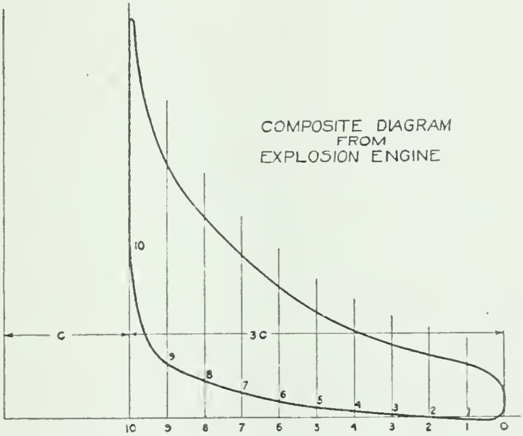

Fig. 23.

tion goes on. Through M is drawn a line parallel to the line for average combustion gas given on the main chart. It will be seen that at the instant expansion sets in, not only does the temperature drop, but the expansion line deviates sharply to the left from the constant entropy line, indicating a considerable cooling effect down to a temperature of about 3,000 deg. fahr. Below 3,000 deg. fahr. the curve becomes more nearly parallel to the constant entropy line. Since, however, the jacket must have a certain cooling effect on the gases at this high temperature, we may legitimately assume that a certain amount of after-burning takes place. The sharp peak of the diagram carrying the temperature up to something like 4,000 deg. fahr. and the rapid cooling following might indicate that this part of the diagram is

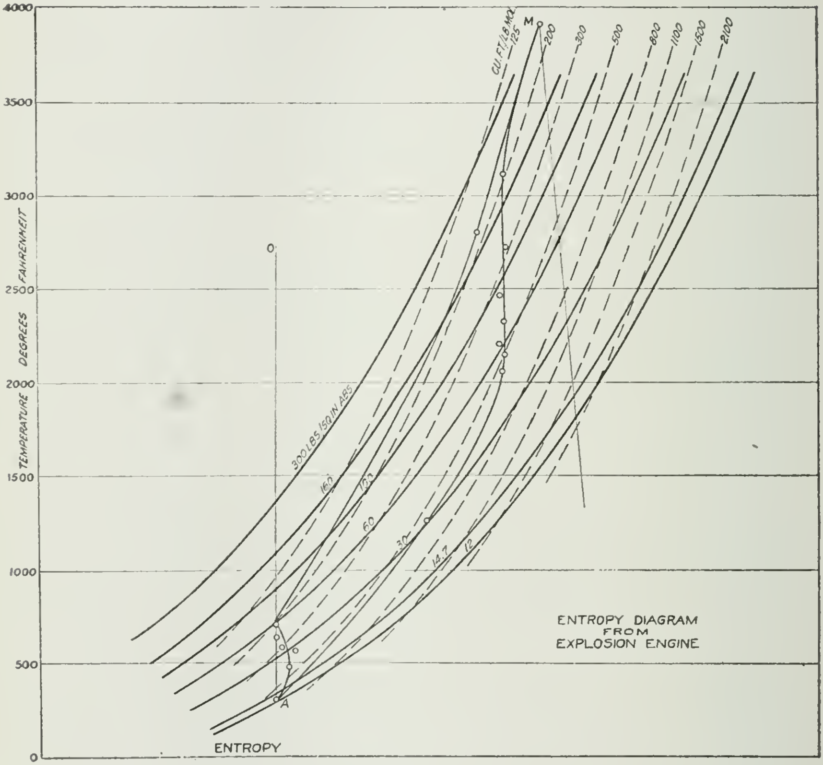

Fig. 24.

caused by knock.[15] It is well known that when the engine is knocking, the heat loss to the jacket is increased; this is due possibly to the fact that incandescent carbon separates out and transmits great quantities of heat to the wall by direct radiation.

It is to be observed that the absolute values of the temperature as given by the entropy diagram are subject to an error equal to that in the temperature estimate at the starting point, say in this case A. This starting temperature at A has here after considerable study been taken somewhat less than 300 deg. fahr, which should be fairly correct, in view of the mixing of the entering air with the hot residue. The high maximum temperature reached might seem a surprise to some engineers. Actually,

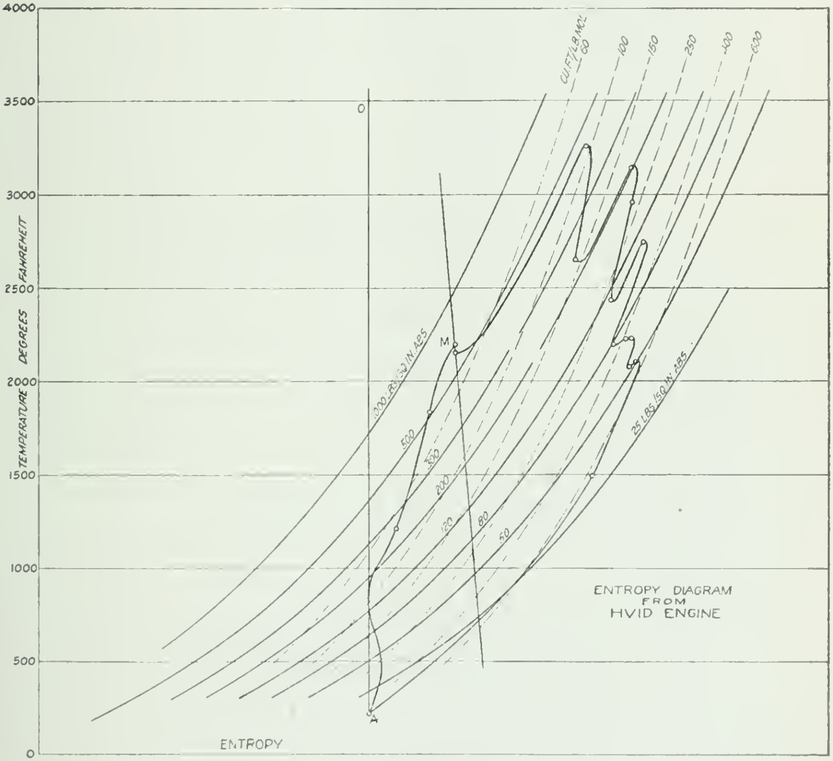

FIG. 25.

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[15] Simple "jumping" of the indicator arm may also be responsible.

however, even this high temperature is not quite enough to explain the power development of the engine. The fact that it occurs only for a short moment renders it impossible to measure it by any thermo-electric, or other devices, which will give only the average temperature during a considerable period near the peak.

For comparison there is in Fig. 25 given the entropy diagram from a so-called Hvid engine. The indicator diagram is given in Fig. 26. The combustion is here started partly by an explosion in the so-called ignition cup, partly by heat of compression of the main charge. The compression reaches very much higher values, but runs first with heating and then with cooling, just as in the former case. At about 160 pounds pressure combustion has manifestly set in, and the curve carries explosively to the right. A maximum pressure of about 650 pounds is reached at the point M. The expansion then begins, but interesting to note, with a further very considerable increase in temperature, in no wise brought out by the indicator diagram. Moreover, even after the maximum temperature of about 3250 degrees is reached, the curve still continues deviating to the right from the constant entropy direction, as given by the line for average combustion gas through the point M. In fact, combustion continues unquestionably until the pressure has dropped below 50 pounds, at which point exhaust commences. The area within the diagram denotes the amount of heat converted into

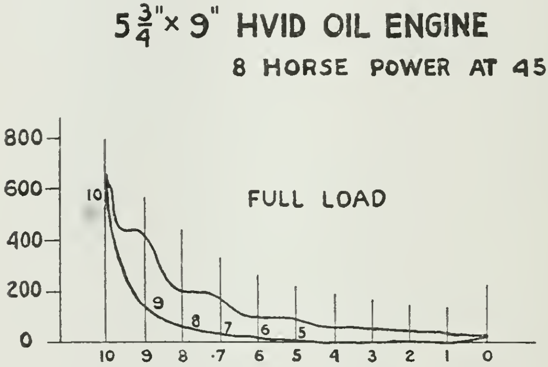

Fig. 26.

work per unit weight of combustion gas. It will be seen that this area is very much greater for the Hvid engine than for the carbureting engine, this in spite of the fact that very likely the mixture in the carbureting engine was very much richer than in the Hvid engine. The reason lies in the far greater expansion in the Hvid engine, the main cause for the greater width of the diagram. If it were not for dissociation and cool ing losses at the high temperatures reached, even a better heat utilization, however, would have been obtained had the combustion been entirely completed in the clearance space instead of dragging out through the whole expansion stroke. This, however, would involve pressure and temperature rises which it would be well nigh mechanically impossible to handle.

Having thus in a general way illustrated the use of the entropy chart for the analysis of combustion engine processes with the aid of the indicator diagram, it may be well to scrutinize to some extent the theoretical foundations of the chart, the first step of which will be to explain how the actual composition of the gases is allowed for and how the exact slope of the constant entropy lines for various gas compositions is arrived at.

Allowance for Gas Composition, Inclination of Entropy Lines.

Pressure and temperature as usually measured do not involve any notion of the quantity of gas involved. Volume, however, must refer to a certain specific quantity. Entropy likewise, being heat addition divided by temperature, must also refer to a definite weight of gas. The weight of gas to which the chart applies is what is known as a *pound molecule* or briefly a *mol.* This is a quantity very generally used in all the computations of chemistry and physical chemistry, but not very frequently met with in ordinary computations of mechanical engineering. The reason it has here been introduced is that by so doing the behavior of all gases and gas mixtures can be covered by the same chart. According to the well-known hypothesis of Avogadro equal volumes of gases at the same pressure and temperature contain the same number of molecules. This hypothesis, although of course not confirmable by actual count of molecules, has now been borne out so completely by all its measurable consequences, that we regard it as a basic scientific truth. If this is so, then the weight of equal volumes of gases at the same pressure and temperature must stand to each other in the same ratio as the weights of the molecules themselves. From chemical investigations the relative weights of the molecules are known with a great degree of accuracy. If the weight of the oxygen atom is taken equal to 16, then the weight of the hydrogen atom is approximately 1, the weight of the nitrogen atom 14, the weight of the carbon atom 12, the weight of the sulphur atom 32, etc. The molecules of most simple gases contain two atoms. The molecular weight of hydrogen (chemical formula H_2) is consequently 2, of oxygen (O_2) 32, of nitrogen (N_2) 28. The molecular weight of carbon monoxide (CO), made up of one atom of carbon and one of oxygen, $12 + 16 = 28$. The weight of carbon dioxide (CO_2), made up of one atom of carbon and two of oxygen, is $12 + 2 \times 16 = 44$. The molecular weight of water vapor, made up of two atoms of hydrogen and one of oxygen, is $2 \times 1 + 16 = 18$, etc. Equal volumes of all these gases at the same temperature and pressure now stand to each other in the same relation as these molecular weights. Hence, on the other hand, weights of these gases numerically equal to the molecular weights occupy the same volume at the same temperature and pressure. In other words, 2 pounds of hydrogen, 32 pounds of oxygen, 28 pounds of nitrogen, or carbon monoxide, 44 pounds of carbon dioxide, 18 pounds of water vapor, at the same temperature and pressure, all occupy the same volume.

These weights are called *pound molecules* or *mols.* One pound molecule or mol of any gas taken at the same pressure and temperature occupies the same volume. Mathematically this is expressed by the equation

$$pV_m = R_mT. \tag{17}$$

Here p is the pressure in pounds absolute per square foot, V_m the volume per mol, T is the absolute temperature in deg. fahr. abs., and R is a universal constant having the same value for all gases. This value is 1544 foot pounds per deg. fahr.

Thus, as the same equation between pressure, volume, and temperature, when applied to a mol of gas, covers the behavior of all gases quantitatively, it can readily be seen that as far as relations between pressure, temperature and volume are concerned, the same chart, if applied to one mol of gas, will cover the behavior of all gases. In order to use such a chart not only for simple gases but also for gas mixtures it is only necessary to determine what the average molecular weight is, that is to say, the weight of the mol for these gas mixtures. This is the weight to which the chart applies. The method for determining this average molecular weight either from volumetric analysis of the gas mixtures or from a calculated composition by weight will be given shortly.

Unfortunately, however, the conditions are not quite as simple, if in addition to relations between pressure, temperature and volume, heat exchanges are also to be considered. Heat exchanges involve the specific heat of the gas, that is to say, the amount of heat required to heat a unit weight of the gas one deg. fahr. Now it has been found that this specific heat varies with the temperature and varies from gas to gas even if it is applied to a pound molecule of gas. Nevertheless, for gases of approximately similar constitution there is a fairly close agreement between the specific heat values per mol. Thus, in fact, it will be found possible to represent the specific heat for hydrogen, oxygen, nitrogen and for carbon monoxide by the same mathematical expression.*

For gases with molecules containing 3 or more atoms the variation of specific heat value both from gas to gas and from temperature to temperature becomes much more complicated and irregular.

Now, as Stodola has pointed out, if in spite of all these variations, it will prove possible to represent the specific heat of all gases per mol by the expression

$$C_v = a + bT, \qquad (18)$$

where *a* always has the same value and only *b* varies from gas to gas, then it is possible to cover the heat exchanges of all gases by means of the same chart in a fairly simple manner. Stodola carried out his suggestions and presented a gas chart based on specific heats furnished mainly by Langen. (Stodola, Dampfturbinen, 4th Ed. 1910). Shortly afterwards Nernst in the 7th edition of his Theoretische Cremie, p. 267, presented a set of specific heat values determined by Pier and others in his laboratory and checked against those of earlier investigators. While some criticism has been heard of these heat values, nevertheless they seem to be the most comprehensive compilation of such heat values involving consideration of the latest investigations of the subject. These specific heat values are plotted in Fig. 27. It will be seen that in the case of the diatomic gases, as well as of water vapor, they may, within the limits of accuracy of the points themselves, be fairly represented by straight lines running through a common point on the C_v axis. This common point gives the value of a in the linear expression above. The value of a so obtained is 4.7.

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*For gases the molecules of which consist of only one atom, such as argon or mercury vapor, the relations are exceedingly simple. For these the specific heat per mol is the same at all temperatures and has a constant value of about 3.

In the case of carbon dioxide (and sulphur dioxide) no such linear representation is possible without straining the case very considerably. Without such a representation the Stodola gas chart is an impossibility. In view of the important service such a chart can render in the analysis of combustion motor phenomena, it may therefore be worth while to inquire as to what errors are introduced, if in spite of all, a rectilinear representation for the specific heat of carbon dioxide is adopted as shown on the chart. Be it said at once that at a temperature of 3600 deg. fahr. the error for carbon dioxide alone is of the order of magnitude of 36 per cent of the true value. Specific heat, however, serves for the computation of heat exchanges over ranges of temperature. As the line is drawn, the errors are both positive and negative and would tend to cancel, for instance, for the temperature rise in the clearance space of a combustion engine. Moreover, combustion gases are always mainly made up of nitrogen. The maximum carbon dioxide that can occur even from the combustion of pure carbon is 21 per cent by volume. From the combustion of hydrocarbons there is generally formed at least an equal number of volumes of water vapor and of carbon dioxide. Very much the same can be said about combustion from water gas and natural gas. In view of all these circumstances the maximum error introduced in the analysis of combustion engine processes pertaining to these liquid and gaseous fuels will be of the order of magnitude of 4 per cent at any one point and not more than half of this over greater ranges of temperatures. This error is, however, altogether negligible in comparison with the uncertainty about the duration of combustion, the amount of dissociation, and the actual chemical processes going on at each stage, not to speak of the inaccuracy in all measurements and estimates of such things as friction losses, cooling losses, throttling losses, etc. A glance at Fig. 27 will also show that this error is very small indeed in comparison with that introduced by the assumption that the specific heat is constant, or that it is the same for combustion gas as for air.

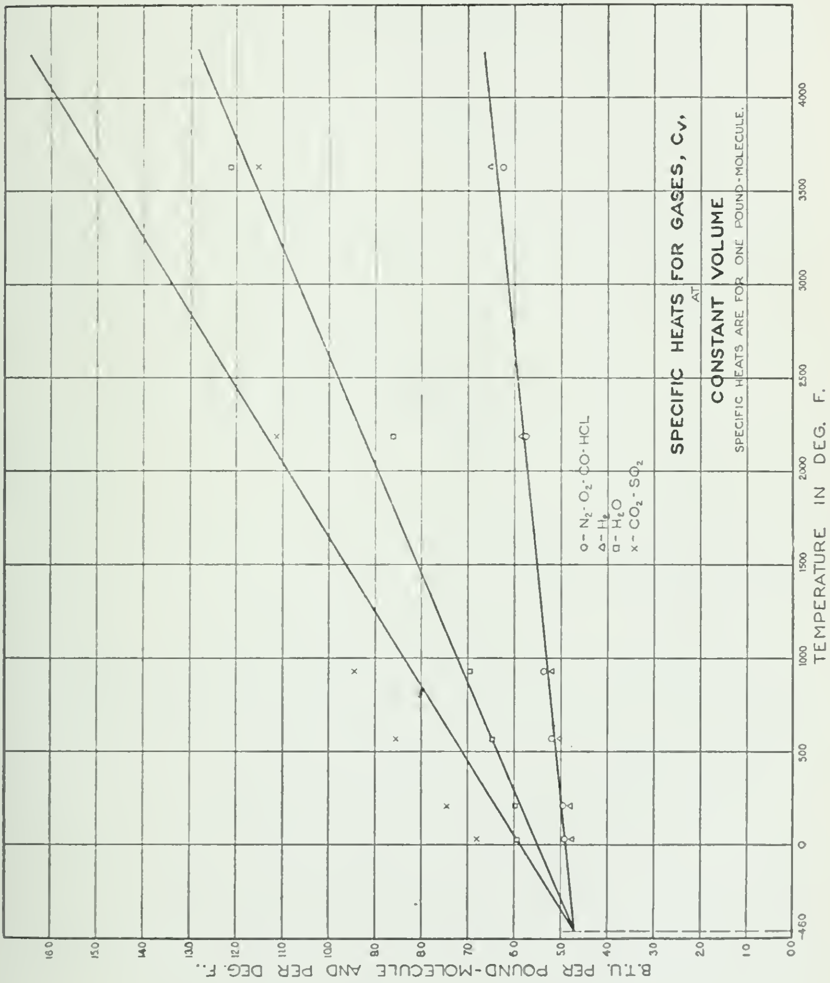

Fig. 27.

There is no question, then, that very much added light can be thrown on the actual occurrences in combustion motors by the application of a chart based on the linear representations of Nernst's heat values as given in Fig. 27. Actually, as the author has had occasion to convince himself, mysterious discrepancies between expected and actual performance can in this manner be cleared up completely and fair agreement between pre-calculated performance and actual performance secured. This is hardly possible by any other method. In view thereof, we may now proceed with the explanation of the chart on the basis of the specific heat values derived from the plot in Fig. 27. These heat values are given on the chart. They are in all cases represented by the equation

$$C_v = 4.7 + bT. \tag{19}$$

The *b* values given on the chart are

for H_2, N_2, O_2, CO
$b = 0.000408.$

for H_2O
$b = 0.001715.$

for CO_2 (and SO_2)
$b = 0.002493.$

(The number of places to which these *b* values are given may not be warranted by the accuracy of the material, but is given as obtained by simple inspection from the inclination of the lines in Fig. 27.)

Referring now to the chart itself, there appears at the top to the left a scale C under the heading "Adiabatic Lines for Gases, b in B. t. u. per pound molecule and degree Fahrenheit." Suppose b for a gas mixture has been figured. Find on this scale C the corresponding value for b. Connect this point with the origin O by means of a straight line. The inclination of this straight line gives the inclination for the lines of constant entropy, that is to say, the inclination of lines denoting expansion or compression without gain or loss of heat. The lines for CO_2, H_2O and simple gases have actually been drawn in.

The formulas for the determination of the average molecular weight for gas mixtures and for the b-values of gas mixtures will now be given. Suppose first that the composition of the gas is known from an ordinary gas analysis giving per cent by volume. Say that these percentages are V_a for a gas of molecular weight a, V_b of a gas molecular weight b, V_c of a gas of molecular weight c, etc. The average molecular weight of the gas mixture will then be

$$(20) \qquad M = \frac{V_a a + V_b b + V_c c + \ldots}{V_a + V_b + V_c + \ldots\ldots}$$

In case the composition is given by weight, for instance, thus: W_a pounds or per cent of gas of molecular weight a, W_b of the molecular weight b, W_c of the molecular weight c, etc., then the formula for the average molecular weight will be

$$(21) \qquad M = \frac{W_a + W_b + W_c + \ldots}{\frac{W_a}{a} + \frac{W_b}{b} + \frac{W_c}{c} + \ldots}$$

The average molecular weight so found will be the number of pounds contained in a mol of the gas mixture, and will be the weight of gas to which the chart applies. To find the average b-value for the gas mixture it should be remembered that b is a constant pertaining to pound molecules of gas. In other words, to find the average b for the gas mixture it is necessary to ascertain the number of mols of every gas contained in the mixture and to weight the b-values with these numbers of mols. The volume percentages are, however, directly proportional to the number of mols present, and can be substituted for these. On the other hand, if weights are given, these weights must be divided by the molecular weight in order to give the number of mols corresponding to these weights. We have then the following formulas for the average b:

(1) Per cent by volume as in formula (20)

$$(22) \qquad b_{av} = \frac{V_a b_a + V_b b_b + V_c c_c + \ldots}{V_a + V_b + V_c + \ldots\ldots\ldots}$$

Here b_a, b_b, b_c, etc., are the b-values for the constituent gases.

(2) Per cent weight as in formula (21)

$$(23) \qquad b_{av} = \frac{\dfrac{W_a b_a}{a} + \dfrac{W_b b_b}{b} + \dfrac{W_c b_c}{c} + \cdots}{\dfrac{W_a}{a} + \dfrac{W_b}{b} + \dfrac{W_c}{c} + \cdots\cdots}$$

As an example of the use of these formulas, figure first the average molecular weight of atmospheric air which contains by volume 21 per cent oxygen and 79 per cent nitrogen, neglecting secondary gases. The average molecular weight would be

$$\frac{21 \times 32 + 79 \times 28}{21 + 79 = 100} = 28.84$$

Figuring, on the other hand, that air contains by weight 23 per cent of oxygen and 77 per cent nitrogen, we obtain

$$\frac{23 + 77}{\dfrac{23}{32} + \dfrac{77}{28}} = 28.8.$$

Actually, on account of the presence of small quantities of Argon, Neon, Krypton, etc., most of which are heavier than oxygen, the apparent molecular weight of air is 28.95. The *b* value for air, neglecting the secondary gases above mentioned, will be the same as for the simple gases, viz., 0.000408. What would, however, be the *b* value for a combustion gas from pure carbon with the theoretical amount of air, containing by volume 21 per cent carbon dioxide and 79 per cent nitrogen? Using Formula 11 we find

$$b_{av} = \frac{21 \times .002493 + 79 \times .000408}{21 + 79 = 100} = .000842.$$

The gas would in this case contain by weight 29.4 per cent CO_2 and 70.6 per cent N. Using Formula 12 to obtain the average *b*-value from these weights percentages we find

$$b_{av} = \frac{\dfrac{29.4}{44} \times .002493 + \dfrac{70.6}{28} \times .000408}{\dfrac{29.4}{44} + \dfrac{70.6}{28}} = .000842$$[16]

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[16] For those not very familiar with combustion computations it may be well at this point to figure an example in order to demonstrate how, from a given composition of the fuel and a given air to fuel ratio, the final composition of the combustion gases is arrived at. Assume, for instance, a fuel containing 85 per cent of carbon and 15 per cent of hydrogen, to be burned with only 12 pounds of air. By referring to what has been statedq in Article 4 of Part II, one pound of carbon requires for complete combustion to carbon dioxide 8/3 pounds of oxygen, or

$$\frac{8}{3 \times .23} = 11.6 \text{ pounds of air.}$$

One pound of hydrogen requires for combustion to water vapor

$$\frac{8}{.23} = 34.8 \text{ pounds of air.}$$

Consequently, one pound of the fuel considered requires for complete combustion to CO_2 and H_2O

$$.85 \times 11.6 + .15 \times 34.8 = 15.07 \text{ pounds of air.}$$

Actually only 12 pounds are supplied. In consequence, part of the fuel may be assumed to be burned to carbon dioxide. Every pound of carbon requires for combustion to carbon monoxide

$$\frac{32}{2 \times 12 \times 0.23} = 5.8 \text{ pounds of air.}$$

It may be assumed that all hydrocarbon burns to water vapor and all carbon first to CO. If then there is any oxygen left, some CO will burn further to CO_2.

.15 pounds H_2 requires $.15 \times 34.8 = 5.22$ pounds of air
.85 pounds C burnt to CO requires
$.85 \times 5.8 = 4.93$ pounds of air
Total 10.15 pounds of air

Of 12 pounds of air available, there remain

$$12 - 10.15 = 1.85 \text{ pounds of air.}$$

There is enough to burn $1.85 \times .23 \times \frac{12}{16} = .32$ pounds of carbon, from carbon monoxide to dioxide (Equation $CO + O = CO_2$). For the combustion gas we have finally

$$.32 \times \frac{44}{12} = 1.17 \text{ pounds } CO_2$$

$$.53 \times \frac{28}{12} = 1.23 \text{ pounds } CO$$

$$.15 \times \frac{18}{2} = 1.35 \text{ pounds } H_2O$$

$$.77 \times 12 = 9.25 \text{ pounds } N_2$$

Figuring in per cent by weight and remembering that there are 13 pounds of combustion gas for every pound of fuel, we have

$$CO_2 = \frac{1.17}{13} \times 100 = 9.00 \text{ per cent}$$

$$CO = \frac{1.23}{13} \times 100 = 9.45 \text{ per cent}$$

$$H_2O = \frac{1.35}{13} \times 100 = 10.42 \text{ per cent}$$

$$N_2 = \frac{9.25}{13} \times 100 = 71.13 \text{ per cent}$$

$$\text{Total} \quad \overline{100.00} \text{ per cent}$$

In case it should be desired for some purpose to obtain per cent by volume one need only remember that equal volumes contain equal numbers of molecules To obtain the molecules corresponding to the weights of the various constituents given above it is necessary to divide these weights by the molecular weights. We find

No. mols $CO_2 = 1.17/44 = 0.0266$
" " $CO = 1.23/28 = 0.044$
" " $H_2O = 1.35/18 = 0.075$
" " $N_2 = 9.25/28 = 0.330$

Total number of mols 0.4756

The per cent by volume will then be as follows:

$$CO_2 = \frac{.0266}{.4756} \times 100 = \ \ 5.58 \text{ per cent}$$

$$CO = \frac{.044}{.4756} \times 100 = \ \ 9.24 \text{ per cent}$$

$$H_2O = \frac{.075}{.4756} \times 100 = 15.75 \text{ per cent}$$

$$N_2 = \frac{.330}{.4756} \times 100 = \underline{69.43} \text{ per cent}$$

$$\text{Total} \qquad 100.00 \text{ per cent}$$

Inicdentally, it will be observed that the average molecular weight of this mixture will be $13/.4756 = 27.4$, and that the factor b in the formula for the specific heat will be

$$\frac{5.58 \times 0.002493 + (9.24 + 69.43)\ 0.000408 + 15.75 \times 0.001715}{100}$$
$$= 0.000731.$$

Theoretical Basis of Entropy Chart. It has alreadv been observed that the heat absorbed, or given up, by a gas in passing from one state to another is expressed by the difference in entropy times the average temperature during the change. If the change as depicted on the gas chart does not proceed along a straight line, then the average temperature is not strictly equal to the mean of the initial and the final temperature and the heat exchanges, while expressed exactly on the chart by the area under the line of change, can mathematically be expressed only by an integral. Introducing differential calculus, a very small heat exchange would be denoted by dQ, a very small change in entropy by dN. During this very small heat change the absolute temperature can be assumed as constant, and equal to T. The basic definition of entropy is given in terms of these quantities and is

$$(24) \qquad dN = \frac{dQ}{T};$$

hence

$$(25) \qquad dQ = TdN,$$

and the whole quantity of heat Q absorbed or given up by the gas in passing from a state 1 to a state 2 is equal to

$$(26) \qquad Q = \int_1^2 TdN.$$

On the other hand, we have also

$$(27) \qquad N_2 - N_1 = \int_1^2 \frac{dQ}{T}.$$

Now heat can be absorbed only for two purposes: first, to increase molecular agitation; second, to cover work done by the gas. If the volume of the gas does not increase, then the gas does no outside work and all the heat goes to increase of molecular energy. If C_v is the specific heat at constant volume and the increase in temperature is equal to dT, then the heat gone to energy increase is per mol C_vdT. If, on the other hand, the volume per mol increases the amount dV while the pressure is p, then the work done by the gas is equal to pdV. We then have

$$(26) \qquad dQ = C_vdT + pdV.$$

It is to be noted, however, that C_v is usually given in B. t. u. per degree Fahrenheit, while pdV is given in foot-pounds. In order to reduce the foot-pounds to B. t. u. it is necessary to divide the B. t. u. by the mechanical equivalent of heat, $E = 778$. For numerical purposes the equation is then

$$(27) \qquad dQ = C_vdT + \frac{1}{E}pdV.$$

Between p, V and T there obtains the relation

$$(28) \qquad pV = R_mT$$

From this relation we derive

$$(29) \qquad p = \frac{R_mT}{V} \quad dV ,$$

hence

$$(30) \qquad dQ = C_v dT + \frac{R_m T}{E} \frac{dV}{V},$$

and

$$\frac{dQ}{T} = dN = \frac{C_v dT}{T} + \frac{R_m}{E} \frac{dV}{V}.$$

If C_v is a constant, this integrated gives

$$(31) \qquad N_2 - N_1 = C_v \log_e \frac{T_2}{T_1} + \frac{R_m}{E} \log_e \frac{V_2}{V_1}.$$

If, however, C_v, instead of being a constant, is given by the expression

$$C_v = a + bT$$

then, as can readily be shown, the expression for the entropy increase will be

$$(32) \quad N_2 - N_1 = a \log_e \frac{T_2}{T_1} + b\,(T_2 - T_1) + \frac{R_m}{E} \log_e \frac{V_2}{V_1}.$$

The expression (32) differs from the expression (31) only by the term $b(T_2 - T_1)$. This is a term linearly proportional to the temperature increase. Suppose entropy is counted from a certain zero value, which on the chart is taken at 14.7 pounds per square inch absolute pressure and zero Fahrenheit. Suppose further that the entropy chart is constructed covering, to begin with, a gas for which the specific heat is equal to a and for which b is equal to zero. To obtain the entropy values for any other b-value it is only necessary to add to the entropy values from the chart so constructed the quantity $b\ t$, where t is the Fahrenheit temperature. This can be done by counting the entropy not from the line for $b = 0$, but from another line so inclined that the horizontal intercepts between these lines and the line for $b = 0$ are equal to $b\ t$. Some such lines are shown on the chart, notably the line for simple gases corresponding to $b = 0.000408$; for average combustion gas with $b = 0.000840$, etc. These inclined lines correspond to the entropy value $N = 0$ for the gases they pertain to. Lines for any other entropy value will manifestly run parallel to these lines. In other words, changes taking place at constant entropy, i. e. adiabatic changes, run along lines parallel to these lines for zero entropy. This is the explanation of the directions given above for finding whether a change proceeds with heat gain or with heat loss.

The expressions (31) and (32) connect the entropy change with volume changes. An equation connecting entropy with pressure may be of interest.

We have

$$V = \frac{R_m T}{p};$$

consequently

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} \; \frac{p_1}{p_2}$$

and $\log_e\left(\frac{V_2}{V_1}\right) = \log_e\left(\frac{T_2}{T_1}\right) + \log_e\left(\frac{p_1}{p_2}\right) = \log_e\left(\frac{T_2}{T_1}\right) - \log_e\left(\frac{p_2}{p_1}\right)$

Hence

$$N_2 - N_1 = \left(a + \frac{R_m}{E}\right) \log_e \left(\frac{T_2}{T_1}\right) + b(T_2 - T_1) - \frac{R_m}{E} \log_e \left(\frac{p_2}{p_1}\right) \tag{33}$$

Applying the equation to one mol, we have

$$\frac{R_m}{E} = \frac{1544}{778} = 1.985$$

With $a = 4.7$ the equation 15 for the purposes of the chart becomes

$$N_2 - N_1 = 6.685 \log_e \frac{T_2}{T_1} + b\,(T_2 - T_1) - 1.985 \log \frac{p_2}{p_1}. \tag{34}$$

The Energy Chart. The entropy chart as described so far can be used to derive temperature changes from known pressure and volume changes and can also depict heat exchanges quantitatively by means of areas. The calculation of these areas, is, however, laborious and some method is desirable whereby the exact amounts of heat gained or lost during the various pressure, volume, or temperature changes may be readily ascertained. A graphical means for this purpose is given to the left on the chart. It is applicable both to the calculation of heat exchanges in ordinary combustion engines operating on the so-called constant volume cycle, as well as to the heat exchanges in motors operating with constant pressure processes. Let us first examine the conditions obtaining in *constant volume engines.*

In these the addition of heat is theoretically supposed to take place entirely in the clearance space at unvarying volume. When the volume of a gas does not increase, the gas does no outside work, and all the heat applied to it goes to increase its internal molecular or *"intrinsic"* energy, that is to say, to raise the temperature. If the specific heat does not vary when the temperature changes from T_1 to T_2, then the heat absorbed or given up, is

$$Q = C_v(T_2 - T_1). \qquad (35)$$

Conceiving the gas to be capable of maintaining its gaseous state and its characteristic properties unchanged even down to absolute zero, the total intrinsic heat in the gas at the temperature T is per mol equal to

$$I = C_vT \qquad (36)$$

If, however, the specific heat is not a constant, but is represented by an expression

$$C_v = a + bT,$$

then the total heat is no longer equal to

$$C_v\ T$$

but, as may readily be shown by integration

$$I = aT + \frac{bT^2}{2}[17] \qquad (37)$$

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[17] For those who find the thinking in terms of integrals difficult, this formula can easily be explained by noting that the average specific heat between zero absolute and T is equal to

$$\text{mean}\ \ C_v = a + \frac{bT}{2}$$

This multiplied by T gives

$$I = \left(a + \frac{bT}{2} \right) T = aT + \frac{bT^2}{2}$$

In Fig. 28 as well as on the chart the way is indicated in which the total intrinsic heat I may be obtained graphically from the chart at any temperature. The quantity aT is laid off from a zero line ZO, to a straight line ZM. The quantity

$$\frac{bT^2}{2}$$

is laid off from ZM to certain curves ZN, ZO, ZP, etc., each curve corresponding to a certain b-value, which is given on scale E at the top of the chart. The heat which is to be added to a gas at constant volume in order to increase the temperature from a value T_1 to the value T_2 is equal to

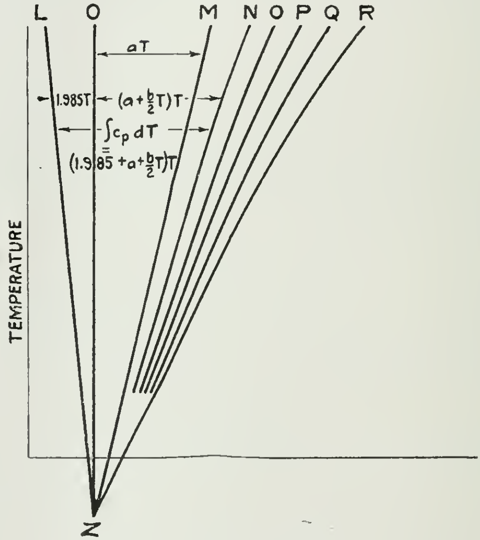

Fig. 28.

$$Q_v = I_2 - I_1$$

where I_2 and I_1 are measured directly on the chart. The same heat is obtainable from the gas if it is cooled from the temperature T_2 to the temperature T_1. This heat is also derivable in the form of work from a mol of gas, if it expands in a constant volume engine without gain or loss of heat, so that the temperature drops from T_2 to T_1.

The way to compute the heat exchanges and the work obtainable from a constant volume engine is now as follows: With reference to Fig. 29 assume the compression to start at a certain pressure p_1 and proceed along the more or less inclined line to the compression pressure p_2, the inclination of the line being determined by the greater or smaller heat loss and by the composition of the charge. Suppose now the combustion to take place, and a combustion gas to be formed with a *b*-value determined as above indicated. The intrinsic heat per mol at the pressure p_2 is obtained by projecting a horizontal line from the point p_2 to the line OZ. Call this intrinsic heat I_2. From combustion there may be derived an amount of heat Q_v carrying the intrinsic heat of the gas to a certain value I_3 equal to $I_2 + Q_v$. If this combustion has taken place at constant volume, it proceeds along the constant volume line V_2 to a point obtained by projecting the line for I_3 over to the point p_3. On the line for V_2 this point marks the intersection of this constant volume line with the constant pressure line for p_3. Draw through p_3 a constant entropy line corresponding to *b* for the combustion gas. If the expansion is supposed to proceed with cooling, it deviates to the left from this line. If it proceeds with heating, it deviates

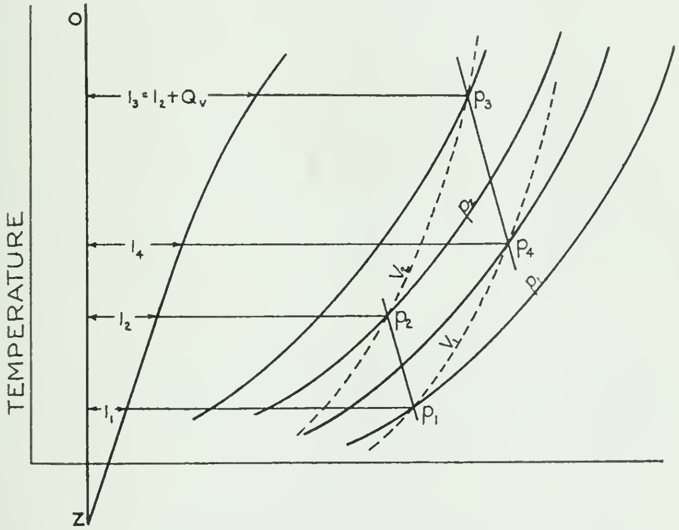

FIG. 29.

to the right. The latter can be the case only if considerable after-burning takes place, in which case, of course, the *b* value changes, and the constant entropy line should, strictly speaking, be a curve. For most purposes, however, it is sufficiently accurate to assume either an average *b* value, or in the case of most carbureting engines, the *b* value for the final combustion product. The expansion carries, or continues, down to the constant volume line V_1 through the point p_1, except that early opening of the exhaust valve may cause a slight rounding of the line before this pressure is reached. The final pressure of expansion is equal to p_4. In agreement with the indicator diagram corresponding to this cycle we may now complete the diagram by following the constant volume line for V_1 back to the initial pressure p_1. In case it may be assumed that the expansion follows exactly the constant entropy line from p_3 to p_4, which would mean in a real engine that after-burning exactly balances the cooling loss, then the heat converted into work during the expansion is found by projecting over the point p_4 to the OZ line and there obtaining the intercept I_4. The expansion work per mol is

$$W_e = I_3 - I_4.$$

The compression work may equally be found by projecting over the point p_1 and obtaining the intercept I_1. The compression work per mol of mixture is

$$W_c = I_2 - I_1.$$

The net work is $W_e - W_c$. If the total heat in the fuel per pound of combustion gas is Q_c, the indicated thermal efficiency of the engine is

$$e_1 = \frac{W_e - W_c}{Q_c}, \tag{37}$$

and the actual fuel utilization of the engine is obtained by multiplying this quantity by the mechanical efficiency, which may be called e_m.

The computation here presented is only an approximation. However, in view of the uncertainty of the actual amount of the after-burning and the cooling losses, it is an approximation close enough for practical estimates. In any case, the chart gives a ready means for ascertaining with a high degree of exactness the maximum efficiency obtainable from an engine, if neither cooling losses, nor after-burning is allowed for. An estimate of this character will be given in the succeeding article and will serve as an example in the use of the chart.

Application to Constant Pressure Engines. The diagram for a Diesel engine, in which a combustion takes place at constant pressure, would vary somewhat from the one just given.

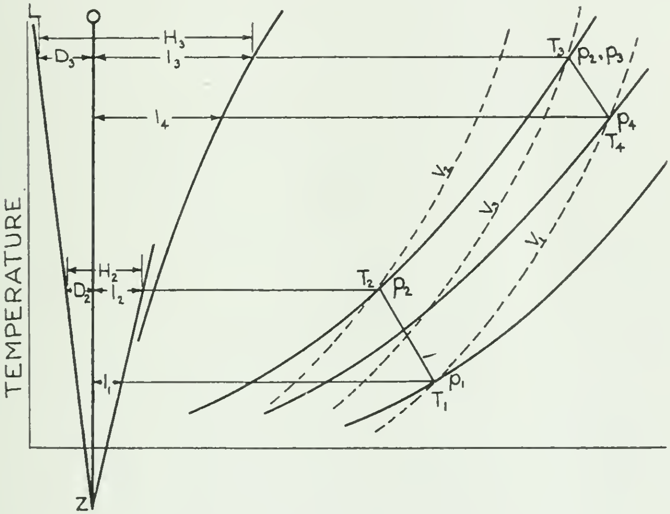

Fig. 30.

With reference to Fig. 30 the combustion would take place along the constant pressure line through p_2 so that at the beginning of expansion at the point p_2, p_3 the pressure p_2 would still obtain. The corresponding indicator diagram is shown in Fig. 31. From there on the expansion would, however, proceed as before to the constant volume line V_1, reached at the pressure p_4. To obtain the amount of heat necessary to cover the combustion at constant pressure it must be considered that the heat applied does not go to increased temperature alone, but also to the covering of the expansion work while the piston moves out at the constant pressure p_2. This expansion work is per mol equal to

$$p_2(V_3 - V_2) = R_m(T_3 - T_2), \tag{38}$$

where T_3 is the temperature corresponding to the volume V_3 and T_2 the temperature coresponding to the volume V_2. This expression gives the work in foot-pounds. To convert it into B. t. u. it is necessary to divide with the mechanical equivalent of heat, 778 foot-pounds per B. t. u. Supposing for a moment the specific heat to be constant, the amount of heat per mol necessary to raise the temperature of the gas from T_2 to T_3 at constant volume is equal to

$$C_v = (T_3 - T_2).$$

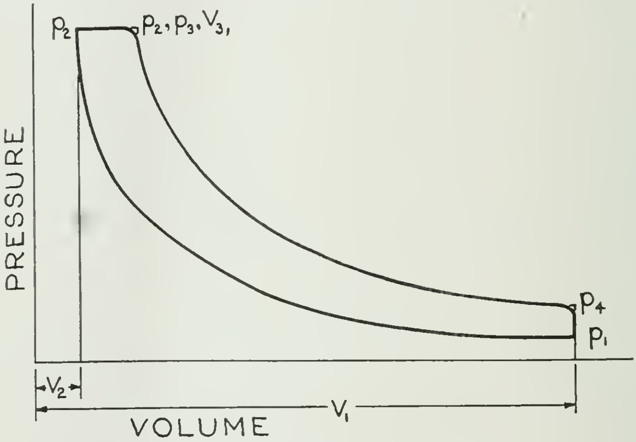

FIG. 31.

This quantity of heat has formerly been called Q_v. Call Q_p the quantity of heat necessary to raise the temperature of one mol of gas from T_2 to T_3 *at constant pressure.* Then by introducing a specific heat C_p, at constant pressure, we should have

$$Q_p = C_p(T_3 - T_2).$$

The difference between Q_v and Q_p is the displacement work. Hence

$$Q_p - Q_v = (C_p - C_v)\ (T_3 - T_2) = \frac{R_m}{E}(T_3 - T_2). \tag{39}$$

Here E is equal to the mechanical equivalent of heat, 778 foot-pounds per B. t. u. From this expression is observed the interesting fact that

$$\frac{R_m}{E} = C_p - C_v \tag{40}$$

Hence, since R_m is equal to 1544 for all gases per mol,

$$C_p - C_v = \frac{1544}{778} = 1.985 = \text{very nearly } 2. \tag{41}$$

This relation applies also if the specific heats are not constant, but vary with the temperature.

With *total heat* of a gas at the absolute temperature T is understood the heat necessary to raise its temperature from absolute zero to T, with unvarying pressure. This quantity is called H. From what has just been said it is evident that

$$H = I + \frac{R_m}{E} T \tag{42}$$

and for one mol

$$H = I + 1.985T,$$

where I as before is the "intrinsic heat."

As illustrated in Fig. 28, the quantity 1.985T may be added to I graphically simply by drawing through the point Z a line ZL so that the intercepts between ZL and ZO are equal to 1.985T. H is then scaled from ZL to the proper *b* curve, just as I is scaled from ZO.

If then the quantity of heat developed per mol of combustion gas in a Diesel engine is known, the total heat per mol of combustion gas is the heat from combustion plus the heat present in the air before combustion per mol of combustion gas. If 1 pound of fuel is burnt with *m* pounds of air, then there are (1 + m) pounds of combustion gas per pound of fuel, and for every pound of combustion gas there is

$$\frac{m}{1 + m}$$

pounds of air. To be exact, then, in order to obtain the total heat per pound of combustion gas after combustion, obtain, first from the diagram the total heat per pound of air at the end of compression, and multiply this by the fraction

$$\frac{m}{1+m}$$

Say that the total heat in the air per mol, as scaled from the chart is H_2, and that the molecular weight of the air is M_a. The total heat per pound of the air is, then,

$$\frac{H_2}{M_a}$$

and the total heat per pound of combustion gas

$$\frac{m}{1+m} \quad \frac{H_2}{M_a}$$

The heat per mol of combustion gas, if the molecular weight of the gas is M_c, is

$$\frac{m}{1+m} \quad \frac{H_2 M_c}{M_a}$$

If the heat from combustion is Q_p, the total heat after combustion will be

$$H_3 = Q_p + \frac{m}{1+m} \quad \frac{H_2 M_c}{M_a}. \tag{43}$$

Usually M_c/M_a is so nearly equal to unity that it need not be introduced into the formula.

By laying off this H_3 on the energy chart from the ZL line to the b-curve corresponding to the actual composition of the combustion gas, the temperature attained may be determined, and hence by projection, as shown in Fig. 30, the volume V_3 obtaining after combustion. From here on the expansion proceeds with more or less after-burning or cooling, as the case may be, along the expansion line in Fig. 30 down to the final pressure p_4. The work obtainable from this expansion is, as before, equal to

$$I_3 - I_4,$$

which quantity may be scaled on the energy diagram as indicated.

In addition there is, however, also the expansion work 1.985 $(T_3 - T_2)$, so that the total expansion work is equal to

$$(44) \qquad W_e = I_3 - I_4 + 1.985 \ (T_3 - T_2).$$

The last term also can be scaled on the energy chart, and is equal to $(D_3 - D_2)$ on Fig. 30.

If the compression work per pound of air is equal to W_c, then this compression work referred to a pound of combustion gas will be

$$\frac{m}{1 + m} W_c,$$

and referred to a mol of combustion gas

$$\frac{M_c m W_c}{1 + m}.$$

The net work per mol finally is

$$W_n = W_e - \frac{M_c m W_c}{1 + m}.$$

The indicated thermal efficiency of the engine will be

$$\frac{W_n}{Q_p}$$

These statements apply on the assumption that the heat available in the fuel is completely developed by combustion. Should this not be the case, then to obtain the indicated thermal efficiency, it is necessary to substitute in the efficiency formula for Q_p, the heat actually developed, the quantity Q_f, that is to say, the heat available in the fuel.

As Diesel engines very often run on very lean mixtures, the weight added to the air by the introduction and combustion of the fuel is very slight, and might for approximate purposes be neglected. As a result, the formulas can be correspondingly simplified.

Thermal Efficiency of a Gas Turbine.

As a further illustration of the use of the chart, the indicated thermal efficiency or heat utilization to be expected from a gas turbine will here be computed. As gas turbines are expensive and difficult things to experiment with, it is desirable that the best fuel economy to be secured by these motors should be ascertained, as far as possible, before such experiments are indulged in. In the case of turbine machinery generally the performance can be calculated in advance with considerable accuracy, and experiments indicate that predictions of fair accuracy can be assumed also in the case of gas turbines.

The importance of the gas turbine as a possible prime mover, at least for stationary purposes, has been enhanced by recent news of improvements in the Holzwarth gas turbine in Germany. It appears that this turbine has been brought to a stage of development where its future commercial usefulness seems to be admitted by responsible engineers.[18] The Holzwarth turbine is a so-called explosion machine. The turbine here to be considered as an example will be rather of the "continuous combustion" type. In this kind of prime mover, air is compressed in a compressor of some type, and delivered in a compressed state into a combustion chamber. Into this combustion chamber the fuel, which we are supposing to be liquid, is also injected. The combustion takes place at a constant pressure determined by the air compressor. For every cubic foot of air pumped into the combustion chamber, the combustion, however, gives 3 to 5 cubic feet of hot products of combustion. Disregarding losses, the expansion work obtainable from a cubic foot of such combustion gas is of the same order of magnitude as the compression work to be spent on a cubic foot of air. Hence, in the ideal case only $\frac{1}{3}$ to $\frac{1}{5}$ of the expansion work derivable from the combustion gases is necessary to cover the compression work. A theoretical margin of $\frac{2}{3}$ to $\frac{4}{5}$ remains for useful work. Very high expansion ratios are possible in turbines and hence the conversion of the heat into expansion work might be very high. Very good possible fuel utilizations have been computed by scientifically trained inventors. This type of turbine becomes especially interesting in view of the fact that some of the heat imparted to the air after compression may be taken from the exhaust,* an arrangement which makes possible the utilization of exhaust energy directly in the motor itself.

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[18] See for instance Zeitschrift des Vereines deutscher Ingenieure. Feb. 28, 1920, p. 197.

For the purpose of our example assume a petroleum fuel containing 85 per cent carbon, 14 per cent hydrogen, and 1 per cent oxygen. The lower heating value is 19,000 B. t. u. per pound. Suppose 18 pounds of air per pound of fuel. The theoretical air requirement is only 14.76 pounds. The air is compressed into the combustion chamber by means of a centrifugal compressor of an efficiency equal to 65 per cent "isothermal." This means that the actual work required by the compressor stands to the theoretical work necessary for a compression without increase in temperature as 100 to 65. Thirty-five per cent of the power delivered to the compressor is dissipated into other things than compression work. Of this 35 per cent assume, however, 30 to have been absorbed in turbulence of the air going through the compressor. At the end of the compression, this turbulence will then be present in the form of heat.

The fuel is injected into the combustion chamber. The pressure therein maintained is 150 pounds per square inch absolute The turbine is a one-stage impulse machine expanding the gases to atmospheric pressure. Its shaft efficiency, allowing for all losses due to friction, turbulence, and cooling, may be assumed equal to 55 per cent.

The work in B. t. u. required to compress one pound of air isothermally from the pressure p_0 to the pressure p_1, when the absolute initial temperature is T_0, is expressed by

$$L = \frac{(C_p - C_v)}{M_{air}} T_0 \log_e \frac{p_1}{p_0}$$

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* There are so-called "continuous combustion" engines of reciprocating type in which such utilization of the exhaust is likewise possible. These engines work on the same cycle as the gas turbine here to be calculated. In ordinary internal combustion engines the utilization of the exhaust heat in the motor itself is not possible — unless, perhaps, by the use of exhaust generated steam on one side of the piston. This latter scheme is used in the Still engine. (For description see, for instance, Motorship, July, 1919).

The molecular weight of air, M_{air}, is 28.95. Introducing for $(C_p - C_v)$ the value 1.985, we have, with an initial air temperature of 70 deg. F. = 530 deg. F. abs.,

$$L = \frac{1.985}{28.95} \times 530 \log_e \frac{150}{14.7} = 84.5 \text{ B. t u.}$$

With an efficiency of 65 per cent the actual work required is

$$84.5/.65 = 130 \text{ B. t. u.}$$

Since 30/35 of the loss appears as heat, the air leaves the compressor with a temperature of

$$530 + \frac{(130 - 84.5) \times 30}{35 \times c_p}$$

The factor c_p is the specific heat of air per pound, at constant pressure. Using our linear equations for the specific heat and assuming the average temperature in the compressor to be 600 deg. F. abs., we have

$$c_p = \frac{1.985 + 4.7 + 600 \times .000408}{28.95} = 0.24.$$

Introducing this value for c_p we find the temperature of the air entering the combustion chamber equal to 693 deg. fahr. abs. = 233 deg. fahr.

Every pound of the fuel burns with 18 pounds of air. The composition of the combustion products per pound of fuel is

$$CO_2 = .85 \times \frac{44}{12} = 3.12 \text{ lbs., or } \frac{3.12}{44} = 0.071 \text{ pound mols.}$$

$$H_2O = .14 \times \frac{18}{2} = 1.26 \text{ lbs., or } \frac{1.26}{18} = 0.070 \text{ pound mols.}$$

$$N_2 = .77 \times 18 = 13.86 \text{ lbs., or } \frac{13.85}{28} = 0.495 \text{ pound mols.}$$

$$O_2 = (18 - 14.76).0.23 + .01 = .76 \text{ lbs., or } \frac{.76}{32} = 0.024 \text{ pound mols.}$$

Total......... 19.00 lbs., or 0.660 pound mols.

The apparent molar weight of the combustion gas is

$$\frac{19}{0.660} = 28.8$$

The average b-value of the specific heat is

$$\frac{.071 \times .002493 + .070 \times .001715 + (.495 + .024) \times .000408}{.660} = 0.000770$$

The heat added by combustion per mol of gas is

$$\frac{19.000 \times 28.8}{19} = 28{,}800 \text{ B. t. u.}$$

The total heat present in the air at 233 deg. fahr. and imparted to the gas may be estimated from the chart at 4700 B. t. u. per mol of gas.

Assuming a 10 per cent cooling loss in the refractory lined combustion chamber, we find the total heat per pound mol after combustion

$$0.9 \times 28{,}800 + 4.700 = 30{,}600 \text{ B. t. u.}$$

The corresponding flame temperature is obtained by scaling 30,600 B. t. u. between the ZL line and the curve for b = 0.000770. The temperature so found is 3320 deg. fahr.

The gas enters the turbine nozzles at this temperature and expanding, gains jet velocity. This jet velocity is turned into shaft power by the turbine wheel. Following the procedure usual in turbine calculations, we locate on the entropy chart the point corresponding to 150 lbs. pressure and a temperature of 3320 deg. fahr. Through this point we draw an adiabatic line of an inclination corresponding to b = 0.000770. This line intersects the 14.7 pound pressure line at a temperature of 1800 deg. fahr. The total heat left in the gas at this temperature is 17,000 B. t. u. per mol. The theoretical amount of heat turned into work is

$$30{,}600 - 17{,}000 = 13{,}600 \text{ B. t. u. per mol., or } \frac{13{,}600}{28.8} = 472$$

B. t. u. per pound.

Of this amount the turbine turns into shaft work

$$0.55 \times 472 = 260 \text{ B. t. u. per pound of gas, or } 260 \times \frac{19}{18} = 274$$

B. t. u. per pound of air.

The compressor consumes 130 B. t. u. per pound of air.

Net work per pound of air

$$274 - 130 = 144 \text{ B. t. u.}$$

$$\text{Heat from fuel per pound of air } \frac{19{,}000}{18} = 1055 \text{ B. t. u.}$$

The heat utilization, or thermal efficiency is consequently

$$\frac{144}{1055} = 13.6 \text{ per cent.}$$

This heat utilization is not very high and in general it is hardly to be expected that great improvements in fuel economy will be brought about by the introduction of gas turbines. The type of turbine here considered, moreover, possesses the characteristic of being extremely sensitive to inefficiencies, both of the compressor and of the turbine part. Losses in these decrease the margin of net work from two sides. It is to be said, however, that the experiments with the Holzwarth turbine in Germany seem to have shown performances rather better than expected. The continuous combustion type of gas turbine, besides, is able to use the heat of the exhaust to relieve the combustion of part of its duty in raising the temperature of the compressed working fluid. If this is done, a not inappreciable improvement in fuel economy may be expected. In view of the growing success of the Holzwarth turbine, it would naturally appear desirable that research with the gas turbine type of prime mover should also be taken up in America. For large power outputs, it possesses decided advantages from the point of view of simplicity and ease of operation over the reciprocating types of prime movers running on heavy liquid fuel. The Holzwarth turbine has been spoken of in Germany in connection with the proposed building of enormous electric power stations in lignite and coal districts. These power stations would use low grades of solid fuel. Gasification and by-products recovery would eliminate the transportation of this type of fuel, and introduce economies from large scale systematization and utilization of natural resources such as are now aimed at in America by the "superpower" development in the East.

ARTICLE 19

EFFICIENCIES ATTAINABLE IN INTERNAL COMBUSTION ENGINES

SUMMARY: *The maximum amount of work producible by a chemical reaction may be greater or less than the heat of reaction. In the case of combustion processes it is in most cases very nearly equal to the heat of reaction. By utilizing the reaction in electro-chemical primary batteries, very high factors of utilization may be attained and have experimentally been attained. If the enormous expansion ratios common in steam turbines were possible in internal combustion engines, then this class of engines might also, in theory, turn perhaps 75 per cent of the heat energy into power.*

As such expansions are not manageable with present types of engines, even ideally perfect engines of this kind could attain only very much lower heat utilizations. What these ideal standards are with actual fuels and actual combustion processes has been investigated on the theoretical side, among others, by Tizard and Pye, and on the practical side, among others, by Ricardo. A sample calculation of such an ideal efficiency is given. These efficiencies are much lower than the so-called "air-standard" efficiencies. In fact, the margin of possible improvement of present-day engines at their best, is less than one might expect.

For this reason, the desirability of maintaining a receptive attitude towards possible new types of prime movers is emphasized.

The ultimate goal to strive for in heat engine performance would, of course, appear to be the total conversion of the fuel energy into power. This would be 100 per cent utilization. Strictly speaking, however, the maximum available fuel energy is not exactly the same as the measured heat of combustion. Without going too far into this subject, be it stated that the relation between heat of combustion and maximum available work doing power of the combustion process is mathematically expressed by the equation

$$W_{max} = Q + T\frac{dW_{max}}{dT} \qquad (46)$$

Here W_{max} is the maximum available work, Q the heat of reaction, and T the absolute temperature. This relation obtains both at constant pressure and at constant volume. It states that the difference between the heat of reaction and the maximum available work is equal to the temperature coefficient of the maximum work multiplied by the absolute temperature. If this temperature coefficient is positive, that is to say, if the maximum work increases with the temperature, then the maximum work is greater than the heat of reaction. If, on the other hand, the maximum work decreases with the temperature then the maximum work is smaller than the heat of reaction. Both cases are met with in dealing with our ordinary fuels. In the case of the combustion of hydrogen to water vapor and carbon monoxide to carbon dioxide, the maximum available work is smaller than the heat of combustion. According to Pollitzer, (Die Berchnung Chemischer Affinitäten, page 129), W_{max} would at ordinary temperature be 94 per cent of Q for the combustion of hydrogen and 98 per cent of Q for the combustion of carbon monoxide. In the case of the combustion of carbon to carbon dioxide W_{max} and Q are practically identical and vary but little with the temperature. In the case of the combustion of carbon to carbon monoxide the very interesting case of a maximum work greater than the heat of combustion is met with. At ordinary temperature W_{max} is about 21 per cent greater than Q and at 3100 deg. fahr. it is, according to Pollitzer, as much as 118 per cent greater. What this means is that the reaction, starting at the temperature of the surroundings, would end at a temperature very much below that of the surroundings. Hence the reaction products would be able to absorb heat from the surroundings and this heat would enable these products to perform outside work.

In internal combustion engines such processes could hardly be realized. The combustion processes can, however, be utilized as the energy-producing reaction in an electrical primary cell. It is found that the electro-motive force of such a cell can be accurately calculated from the maximum work of the reaction. Such a cell, if operating on a process where the maximum work is greater than the heat of reaction, will actually cool itself below the atmosphere.

The maximum work from any chemical process is obtainable when the process is performed isothermally, that is to say, at constant temperature. This would mean that in our combustion engines we would have to operate at a temperature at any rate not exceeding the end temperature of compression, a manifest impossibility. The combustion always leads to appreciable temperature rises and a considerable portion of these temperature rises remain unutilized in the exhaust.[19]

Owing to the possibility of performing the isothermal process in a primary battery and of directly converting it into work by means of the electric current, attempts have already been made to develop so-called combustion batteries. For some interesting advances actually made in the solution of this problem, and for some enormous heat utilizations, at least for brief periods, the work of Baur, Taitelbaum and Ehrenberg in Germany might be referred to. (Zeitschrift für Elektrochemie, 1910, p. 286, 1912, p. 1002).

Leaving aside these interesting, although yet distant, possibilities, we may here turn to the far more practical question: What is a reasonable maximum efficiency to be looked for in connection with combustion motors of types not radically dissimilar from those known today; in other words, what is the standard of perfection to strive for in connection with our combustion engines; how far are we from this standard; what are the means by which the standard can be approached, and what are the reasons that the standard has not yet been approached, if this should be the case? The latter questions have in some measure been dealt with before. The question of standards of perfection will here be taken up to some extent.

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[19] Nevertheless, by introducing in combustion motors enormous expansion ratios such as those possible in steam turbines, it has already been noted (Article 5, Part II, p. 62), that heat utilizations as high as 75 per cent would be possible for a perfect engine.

(1) *The Air Cycle Efficiency.*

In England the so-called air cycle efficiency has been much used in discussions of engine performance. The writer is of the opinion that in many cases it has tended to becloud the issue rather than to clear it. Even a perfect combustion engine without friction losses and cooling losses, but operating on actual combustion gas and at high temperature could not approach the air cycle efficiency.

Nevertheless, the formula for the air cycle efficiency is easy to derive, and as the derivation throws some light on the fundamental factors affecting efficiency, the derivation will here be given.

The basic assumptions of the air cycle are:

(a) A working medium consisting of an ideal gas, with unvarying specific heat and unvarying adiabatic exponent of compression and expansion, n. This exponent n is taken equal to that of air at ordinary temperature — approximately 1.4.

(b) Heating of this ideal gas, not by combustion involving additions of extraneous substances to the gas and change of the chemical composition of the gas, but by the application of pure weightless heat.

(c) No cooling losses, delayed combustions, friction losses, etc., are assumed.

For constant volume engines the additional assumption is made that the compression ratio is exactly the same as the expansion ratio. Call it r, where r is greater than unity.

For a constant volume engine with the compression beginning at the absolute temperature T_1 and ending at the temperature T_2, we have under these conditions for the compression work per pound (or mol)

(47)
$$W_c = C_v(T_2 - T_1) = C_vT_2\,(1 - \Bigl(\frac{T_1}{T_2} = C_vT_2\left[1 - \left(\frac{1}{r}\right)^{n-1}\right]$$

The heat addition at constant volume carries the temperature from T_2 to T_3, and the heat consumed is

$$(48) \qquad Q_v = C_v\,(T_3 - T_2)$$

The expansion to temperature T_4 produces the work

$$(49) \qquad W_e = C_v(T_3 - T_4) = C_vT_3\left[1 - \frac{T_4}{T_3}\right] = C_vT_3\left[1 - \left[\frac{1}{r}\right]^{n-1}\right]$$

The indicated thermal efficiency or heat utilization is

$$(50) \qquad \frac{W_e - W_c}{Q_v} = \frac{C_vT_3\left(1 - \left[\frac{1}{r}\right]^{n-1}\right) - C_vT_2\left[1 - \left[\frac{1}{r}\right]^{n-1}\right]}{C_v(T_3 - T_2)}$$

$$= 1 - \left[\frac{1}{r}\right]^{n-1},$$

or introducing the air value for n and calling this efficiency e_a

$$(51) \qquad e_a = 1 - \left[\frac{1}{r}\right]^{0.4}$$

The air cycle efficiency then appears independent of temperature and dependent on volumetric compression ratio only.

To approach the air cycle efficiency it is necessary not only to reduce cooling losses and friction losses to a minimum and attain complete combustion in the clearance space, but it is also necessary to run on extremely lean mixtures so as to obtain a working medium as nearly as possible akin to pure air at ordinary temperature. As the expansion work stands to the compression work in the ratio of the absolute temperatures T_3 to T_2, (or T_4 to T_1), low temperatures during expansion mean small margins of net work, large engine dimensions, and low mechanical efficiency. Probably 50 per cent of the air cycle efficiency as a brake thermal efficiency would be a high standard to strive for.

Lean mixtures and comparatively low temperatures may be readily attained in injection engines, and also by stratification, or subdivision of the charge in carbureting engines. In view of the growing importance of both injection and stratification a somewhat greater interest may also be given to the air cycle efficiency. In Table 22 this efficiency is given for a number of expansion ratios. It is now of interest to ascertain the influence on the thermal efficiency of deviations from the assumptions underlying the air cycle efficiency.

TABLE 22

Expansion Ratio	Air cycle efficiency $1-\left[\frac{1}{r}\right]^{.4}$	Tizard & Pye efficiency $1-\left[\frac{1}{r}\right]^{.295}$	Ricardo efficiency $1-\left[\frac{1}{r}\right]^{.25}$	Observed results with experimental engine
4.0	0.426	0.336	0.296	0.277
4.5	0.452	0.359	0.314	0.297
5.0	0.475	0.378	0.332	0.316
5.5	0.494	0.396	0.348	0.332
6.0	0.512	0.411	0.361	0.346
6.5	0.527	0.424	0.375	0.360
7.0	0.540	0.437	0.386	0.372
7.5	0.553	0.449	0.396	0.383
8.0	0.565	0.460	0.406	

Influence of Variable Specific Heat on Air Standard Efficiency. The specific heats for combustion gases from mixtures of varying richness have been given in Figs. 17 and 18 of Article 17, page 133.

It will be noticed that the specific heats increase very considerably with the temperature and somewhat less with the richness of the mixture — (except that at high temperatures and over-rich mixtures the replacement of carbon dioxide with carbon monoxide causes a downward deflection in the curve).

The exponent n for adiabatic expansion is equal to the quotient of the specific heats for constant pressure and for constant volume

$$n = C_p/C_v$$

This exponent then also varies greatly with the temperature, the actual variation being given in Fig. 32.

To allow accurately for these variations it is necessary to use the entropy chart. An example of such a computation will shortly be given. For the purpose of a general estimate of how variations in the specific heats within the range noted affect the attainable efficiency, e_i, we may here, however, resort to the unabridged formula (50) for the air cycle efficiency above derived, but introduce for C_v and n, C_{v2}, n_2, during the expansion stroke and C_{v1} n_1 during the compression stroke. These are the *average* values during the stroke.

Call the average specific heat during the temperature rise in the clearance space C_{v3}.

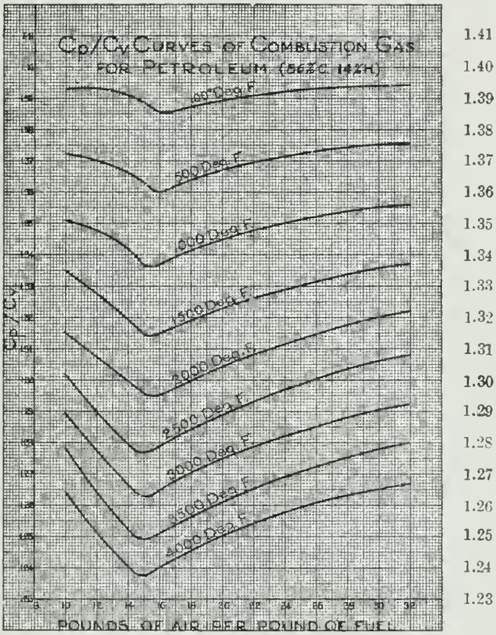

Fig. 32.

We have

$$e_i = \frac{C_{v2}T_3\left(1 - \left[\frac{1}{r}\right]^{n_2-1}\right) - C_{v1}T_2\left(1 - \left[\frac{1}{r}\right]^{n_1-1}\right)}{C_{v3}(T_3 - T_2)}$$

In carbureting engines, even without any allowance for cooling loss, turbulence and leakage, the compression exponent n_1 on account of the admixture of fuel and residue, will be less than the exponent for air. In the case of injection engines this statement is not true. Yet for rough approximation such as is here attempted, it will be permissible to put $n_1 = n_2$. We then have for the ideal efficiency

$$e_1 = \frac{\left[1 - \left(\frac{1}{r}\right)^{n_2 - 1}\right]\left[1 - \frac{C_{v1}T_{:}}{C_{v2}T_{:}}\right]}{\frac{C_{v3}}{C_{v2}}\left\{1 - \frac{T_2}{T_3}\right\}}$$

From this formula it will be seen that the indicated thermal efficiency varies in a rather complicated manner with the temperature and the specific heats. In general it will be found that the efficiency drops with increase in temperature, although the drop will be somewhat less than if the exponent n_2 were the only quantity affected.

Tizard's and Pye's and Ricardo's Work.

Messrs. Tizard and Pye in their paper before the British Association for the Advancement of Science (Cardiff meeting, August 25, 1920) give an approximate formula for the maximum attainable efficiency which allows both for dissociation and for the variation of the specific heat. This formula is

$$e_i = 1 - \left(\frac{1}{r}\right)^{0.293}$$

Mr. Ricardo in his paper reproduced in the Automotive Industries for Feb. 24, 1921, gives a table containing the ideal efficiencies computed by this formula, and in addition publishes certain maximum efficiencies which he considers the highest thinkable for engines of present-day type. These efficiencies presuppose

(1) A combustion chamber designed to allow of the minimum possible heat loss;

(2) A cylinder of comparatively large capacity;

(3) A speed of not less than 1500 r. p. m. so as to reduce heat loss;

(4) Perfect carburetion and distribution;

(5) Equal compression and expansion ratio;

(6) A homogeneous mixture of the most economical strength.

The formula Mr. Ricardo considers applicable to this case is

$$e_i = 1 - \left(\frac{1}{r}\right)^{0.25}$$

His table also contains the air cycle efficiency and observed results with an experimental engine of variable compression ratio and designed to give best possible performance at any cost. These data are reproduced in Table 22, which is Mr. Ricardo's identical table with slight changes only in the number of places.

It will be observed that while the results actually attained remain, and for reasons above explained must remain, very remote from the air cycle efficiency, they come quite close to Mr. Ricardo's practical ideal efficiency and do not remain immeasurably below even Tizard's and Pye's highest theoretical efficiency.

These figures, however, apply to present day carbureting engines only, and the author for one is not entirely in sympathy with setting up a standard of perfection based on such a narrowly circumscribed and well-nigh fossilized field of possibilities. He is also not in favor of expressing the standard in the form of mathematical formulas which do not show the limitations on which they are based, and the influence of important factors such as temperature. Is it not better to give a standard taking in as wide a field as possible, and at the same time give the technical means for ascertaining the exact influence of the factors which make for or against the attainment of this standard? The advent of stratification and injection in the automotive engine; the knowledge that certain other nations are following new lines of development in the prime mover field, which may lead to possibilities of an entirely unsuspected nature, makes one hesitate to set up any lower standards than that of the maximum possible energy utilization by any means whatever.

Sample Computation of Ideal Efficiency

In view hereof, and of the thoroughness of Messrs. Tizard's and Pye's as well as Mr. Ricardo's work in the field it covers, the author will here confine himself simply to an example of the use of the entropy chart in establishing the maximum attainable efficiency for a constant volume injection engine. The volumetric compression ratio may be 8, the ratio of air to fuel, 30; the composition of the fuel 86 per cent carbon and 14 per cent hydrogen.

There will be 31 pounds of combustion gas per pound of fuel and this amount of gas will contain

$$1.26 \text{ pounds } H_2O = \frac{1.26}{18} = 0.07 \text{ mols.}$$

$$3.16 \text{ pounds } CO_2 = \frac{3.16}{44} = 0.072 \text{ mols.}$$

$$23.1 \text{ pounds } N_2 = \frac{23.1}{28} = 0.823 \text{ mols.}$$

$$3.48 \text{ pounds } O_2 = \frac{3.48}{32} = 0.109 \text{ mols.}$$

$$31.00 \text{ pounds} \qquad 1.074 \text{ mols.}$$

The average molar weight will be

$$\frac{31}{1.074} = 28.8$$

The average b value in the specific heat will be

$$b = \frac{0.07 \times 0.001715 + 0.072 \times 0.002493 + (.823 + .109).000408}{1.074}$$
$$= 0.000634.$$

The initial temperature of the air before compression may be taken equal to 100 deg. fahr., or 560 deg. fahr. absolute.

The compression, to be strictly theoretical, ought to proceed with an exponent of 1.4, since air only is compressed. In actual engines it is often found that a lower exponent, provided it is due to *cooling,* not to leakage, means lower compression work and more net work. Let us here assume the exponent equal to 1.3.

The temperature ratio corresponding to this exponent may be read from the curve in Fig. 4, page 62. It is

$$\frac{1}{0.53} = 1.89.$$

This means an end temperature of

$560 \times 1.89 = 1060$ deg. fahr. absolute $= 600$ deg. fahr.

The compression work in B. t. u. may be figured by the formula

$$W_c = \frac{RT_1\left(\dfrac{T_2}{T_1} - 1\right)}{(n-1)\,778}$$

R is the gas constant for air per pound $= 53.3$, T_1 is the temperature before compression, T_2 after compression, n the compression exponent.

We have per pound of air

$$C = \frac{53.3 \times 560}{.3 \times 778}(1.89 - 1) = 114 \text{ B. t. u.}$$

The heat available per pound of fuel may be taken equal to 18000 B. t. u.

The heat per pound of combustion gas is $18000/31 = 581$ B. t. u. and per mol $581 \times 28.8 = 16{,}700$ B. t. u.

To obtain the intrinsic heat and the temperature after combustion, it is necessary to add to this available heat the heat in the air before combustion, reduced to mol of combustion gas. The heat per mol of air may be scaled on the chart to the curve for air ($b = 0.000408$), or it may be figured:

$$I_1 = \left(4.7 + \frac{0.000408 \times 1060}{2}\right) 1060 = 5210 \text{ B. t. u.}$$

Per mol of combustion gas this becomes

$$5210 \times \frac{30}{31} \times \frac{28.8}{28.95} = 5020 \text{ B. t. u.}$$

The total intrinsic heat in the combustion gas after combustion is consequently

$$16{,}700 + 5020 = 21{,}720 \text{ B. t. u. per mol.}$$

The corresponding temperature found on the chart by scaling to a curve corresponding to $b = 0.000634$ is

3260 deg. fahr.

Dissociation at this temperature may be neglected.

To obtain the expansion work it is necessary to find the volume per pound, or the pressure, corresponding to this temperature.

Supposing the compression to commence at 14 pounds absolute, then, with a compression exponent of 1.3 it would end at 212 pounds absolute. This pressure at a temperature of 1060 deg. fahr. absolute corresponds to a volume of 53.6 cubic feet per mol of air.

$$\frac{1544 \times 1060}{212 \times 144} = 53.6.$$

The combustion is assumed to take place entirely at constant volume. The volume per mol of air at the end of compression could without much error be regarded as the volume per mol of combustion gas at the beginning of expansion. It would be more exact to allow for the crowding in of substance from one pound of fuel for every 30 pounds of air, and also for the volume change resulting from the chemical reaction. This volume change is pictured in the change of molecular weight — equal number of mols corresponding to equal volumes. In this case, 28.8 pounds of combustion gas will at the same temperature and pressure occupy as much space as 28.95 pounds of air; or in the same space, the pressure of the combustion gas would at the same temperature be higher than that of air in the ratio of 28.95 to 28.8. Similarly, neglecting change in chemical composition, the pressure will be raised in the ratio of 31 to 30 by crowding 31 pounds of combustion gas into the space occupied by only 30 pounds of air. The temperature of the air at the end of compression was 1060 deg. fahr. absolute. After combustion, the temperature of the gas is $3260 + 460 = 3720$ deg. fahr. absolute. The pressure rise, disregarding all other factors, would be in the ratio of these temperatures.

Allowing for all factors concerned, the pressure after combustion will be

$$\frac{212 \times 31 \times 28.95 \times 3720}{30 \times 28.8 \times 1060} = 770 \text{ lbs. per sq. in. absolute.}$$

The corresponding volume per mol of gas is 51.7 cubic feet rather than 53.6. The volume after expansion will be $51.7 \times 8 = 414$ cubic feet.

Drawing on the chart a constant entropy line corresponding to $b = 0.000634$ from the point for 770 pounds and 3260 deg. fahr. to the line for a volume of 414, we find the temperature after expansion = 1560 deg. fahr.

From the energy chart this corresponds to an intrinsic energy of

10800 B. t. u. per mol.

The heat turned into work by expansion is then

$$21720 - 10800 = 10920 \text{ B. t. u.}$$

This is per pound of air

$$\frac{10920 \times 31}{28.8 \times 30} = 392 \text{ B. t. u.}$$

The net work per pound of air is

$$392 - 114 = 278 \text{ B. t. u.}$$

The heat in the fuel per pound of air is

$$\frac{18000}{30} = 600 \text{ B. t. u.}$$

The indicated thermal efficiency is consequently

$$\frac{278}{600} = 46.3 \text{ per cent.}$$

For the same compression ratio Tizard and Pye's formula gives an ideal efficiency of 46 per cent. This applies to carbureting engines at most economical mixture ratio. The close agreement with the efficiency computed for an injection the engine on a very lean mixture is, of course, accidental. Nevertheless it bears out the fact that heat utilizations already experimentally attained with both carbureting engines and with injection engines of present-day type are not immeasurably below the maximum ideal efficiencies, considering the actual nature of the combustion process and of the working medium.

GENERAL CONCLUSIONS

There are available in this country, and in the world at large, of petroleum, or liquid fuel closely akin to petroleum, reserves great enough to last for generations. In order to avoid temporary and unnecessary shortages, the one thing necessary is economy and statesmanlike foresight in production and use.

The production and distribution of petroleum today seem to involve serious waste. This phase of the liquid fuel problem must be left to other publications.

The waste in utilization of oil fuel consists partly in the application of the fuel to purposes such as furnace work or steam raising in stationary boiler plants, where coal would do as well, and consists partly in the inefficiency of the motors used.

Experiments carried out in Germany suggest the possibility of electrical combustion batteries capable of thermal efficiencies very much in excess of those within the range of combustion engines. The possibility of as radical innovations as this should not be overlooked by organizations possessing ample facilities for experimentation.

Nevertheless the immediate object must be the improvement of the efficiency of existing types of prime movers. The poorest performance is shown by present-day automobile engines of throttling, carbureting type under ordinary conditions of driving, that is to say, at reduced load. These engines, moreover, require a peculiarly volatile kind of fuel, a fuel of which there is an insufficient supply naturally available in the crude oil.

Two ways present themselves to meet this difficulty:

(1) To adapt steam engines, or still better injection engines, to automotive uses. Both these types of engines can be made to handle any kind of liquid fuel, and in addition, the

injection engine shows a higher degree of utilization than does any other type of prime mover.

(2) To try to change the fuel so as to make it more suitable for carbureting engines, and simultaneously try to improve the performance of this engine at reduced load.

The second way appears to be the more difficult and uncertain one, but experimentation with large means may show unsuspected possibilities, such as, on the fuel side, the dopes or "compressionizers" of the General Motors Research Corporation, or fuel conversion with small loss and valuable by-products; and on the engine side, new stratification devices, improved carburetion and ignition devices, etc.

In this bulletin the attempt has been made to review impartially all suggestions and possibilities, and to furnish, in addition, certain new data and means of analysis by scientifically accepted methods.

The author hopes that adherence to an existing engine type may not be the fetish which will prevent courageous and broadgage engineering development in the prime mover field in this country. He feels certain that a small fraction of the effort now put into the modification of the fuel, or of the carbureting engine, would, for instance, produce a workable automotive injection engine, if such an engine has not been produced already. With its tremendous resources and the native mechanical intelligence of its people, this country can do more than lead in production methods and quantity production. A strong awakening is now apparent in the automotive engineering field. Let us hope that this awakening will lead not only to continued masterly production and business achievement in the domestic field, to that brilliant detail inventiveness always characteristic of American engineering, but also to long-range commercial outlook, to quantitative scientific scrutiny and perspective — in short, to systematic scientific onward march, and this in the line of business statesmanship, no less than in the line of appreciation and application of science in the narrower sense of the word.

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THERMAL, ELECTRICAL AND MAGNETIC PROPERTIES OF ALLOYS

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THERMAL, ELECTRICAL AND MAGNETIC PROPERTIES OF ALLOYS.*

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ALPHEUS W. SMITH, Ph.D.

Ohio State University.

INTRODUCTION.

THE physical properties of alloys including hardness, conductivity for heat and electricity, thermoelectric power, magnetic susceptibility, rate of change of resistance with the temperature, etc., are intimately connected with the constitution of the alloys. In the absence of intermetallic compounds the physical properties are in general continuous functions of the composition for any given series of alloys. The physical property may be a linear function of the concentration as is ordinarily the case in conglomerates or it may pass through a maximum or a minimum as in alloys formed of metals which are mutually soluble in each other in all proportions. Discontinuities may occur in cases of limited solubility in the solid state and the curve which represents the variation of the physical property with the composition then shows an abrupt change in direction at the concentration at which one metal ceases to be soluble in the other. In case the metals entering into the alloys form one or more intermetallic compounds the alloys will have a new set of physical properties at the point at which the concentration of the compound is a maximum.

For the purposes of this study of the relation between the thermal, electrical and magnetic properties of alloys and their constitution alloys may be divided into five groups.

I. The two components are not soluble in each other and form no chemical compounds with each other. A metallographic study shows that the alloys in this case are mechanical mixtures of the two components. The lead-cadmium series is an example of this type of alloys. In such alloys the physical property is usually a linear function of the concentration of one of the components. For example, the electrical conductivity of a series of such conglomerates is represented by Curve I of Fig. 1.

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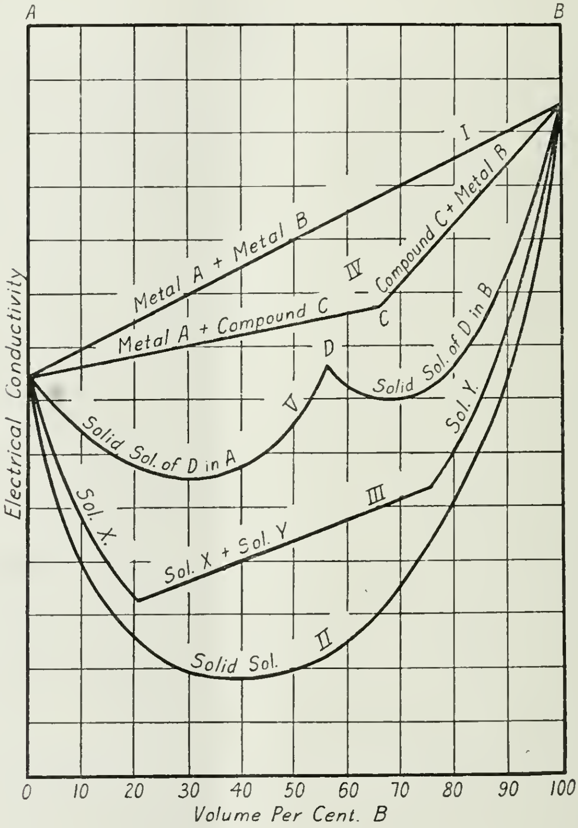
II. The two components are soluble in each other in all proportions, *i.e.*, they form by varying the concentration of one of the components an unbroken series of mixed crystals. Alloys of palladium with silver or nickel with copper belong to this group. A curve representing the physical property as a function of the concentration of one of the components is characterized by a pronounced maximum or minimum. The electrical conductivity when plotted as a function of the concentration of one of the components gives in this case a curve of the form of Curve II of Fig. 1.

FIG. 1.



III. Each of the components is soluble in the other to a limited extent. The alloys of this series will then consist of three parts, solid solutions of the first component in the second, solid solutions of the second component in the first and mechanical mixtures of saturated solutions of the first component in the second with saturated solutions of the second component in the first. A typical curve for such a property as the electrical conductivity of alloys of this group is represented in Curve III of Fig. 1. It consists of three parts, a central portion which is linear where there is a mechanical mixture of one saturated solution in another and on either side of this a portion which is similar to the initial and final parts of Curve II.

IV. The two metals *A* and *B* form a single compound *C* (Curve IV, Fig. 1) and this compound forms mechanical mixtures both with *A* and *B*. The curve representing the physical property consists of two straight lines intersecting at *C*.

V. The components form one or more intermetallic compounds. Bismuth-tellurium alloys furnish an example of this group. In this case the curves, showing the physical properties as a function of the concentration of one of the components, may assume a variety of forms according to the nature of the intermetallic compound. If the compound happens to form solid solutions with both of the components in the alloys the curve will take the form of Curve V of Fig. 1. In that case a compound *D* was formed which formed solid solutions with both *A* and *B*.

PHYSICAL QUANTITIES AND UNITS.

The specific resistance has been taken to mean the resistance in ohms or in microöhms of a wire one centimetre in cross section and one centimetre in length. The electrical conductivity is understood to be the reciprocal of the specific resistance, and it has been expressed as the reciprocal of ohms for which the term mho has been used.

The temperature coefficient of the resistance has been considered to be the rate of change of the resistance per ohm per degree Centigrade.

By the thermal conductivity is understood the quantity of heat in calories which will flow in one second through an area of one square centimetre when the temperature gradient is one degree Centigrade per centimetre.

The thermoelectric power has been measured against lead except in a few cases where it has been measured against platinum. These exceptions are clearly indicated on the figures. The thermoelectric power has been expressed as the number of microvolts for a difference of one degree Centigrade between the junctions. For the rate of variation of the thermoelectric power with the temperature the Centigrade scale has also been used.

The Thomson coefficient, which is a measure of the heat absorbed or evolved in excess of the Joulean heat by a current of electricity flowing along an unequally heated conductor, has been measured in ergs. It gives the amount of heat in ergs which is absorbed or generated in excess of Joulean heat by a current of one absolute unit flowing through a conductor in which there is a temperature gradient of one degree Centigrade per centimetre.

In the Hall constant which is a measure of the rotation of the equipotential lines under the action of a transverse magnetic field, the electric current, the transverse difference of potential and the magnetic field have been measured in absolute units and the thickness of the plate in centimetres. This constant is defined by the equation,

$$E = R\frac{Hi}{d}$$

where E = the transverse electromotive force produced by the magnetic action.

H = the intensity of the magnetic field.
i = the current in the plate.
d = the thickness of the plate.
R = the Hall constant.

For paramagnetic and diamagnetic substances the relation between the intensity of magnetization and the magnetic field which produces it may be expressed by the equation,

$$I = kH.$$

where H = the magnetic field in gausses.
k = the magnetic susceptibility per unit volume.

Sometimes the magnetic susceptibility per unit mass is used. In such a case it is called the specific magnetic susceptibility and is defined by the relation,

$$x = \frac{k}{\rho},$$

where ρ = density of the substitute.

To get a measure of the elastic properties of the alloys it has been necessary to accept the data on these properties in a variety of forms. In some cases the hardness on Brinell's scale has been used; in others the pressure necessary to cause the metal or alloy to flow, and in others the tensible strength. There is no simple way of passing from one of these kinds of data to the other. They all give some measure of the cohesive forces with which the metals and alloys are held together, and that was all that was needed in this connection.

Above the freezing point curve an attempt has been made to indicate the manner in which the metals mix to form the alloys (see Guertler, "Metallographie"). Where the alloys are mechanical mixtures for all concentrations of the constituents, this fact has been indicated by placing a plus sign between the chemical symbols of the constituents. For example $Zn + Sn$ means that tin and zinc are mechanical mixtures in all proportions. In case the metals A and B form an unbroken series of solid solutions, this has been indicated by writing $Sol\text{–}A\text{–}B$ above the freezing point curve. If one metal A is soluble to a limited extent in the other $B - a$ saturated solid solution of A in B has been denoted by I and a saturated solid solution of B in A by II. Where these saturated solid solutions then mix mechanically to form the remainder of the alloys, this fact has been indicated by $I + II$. For example in the case of the copper-silver series, the numerals above the freezing point curve indicated that copper dissolves about 3 per cent. silver, and that silver dissolves about 5 per cent. copper. These saturated solid solutions then mix mechanically to form the remainder of the alloys.

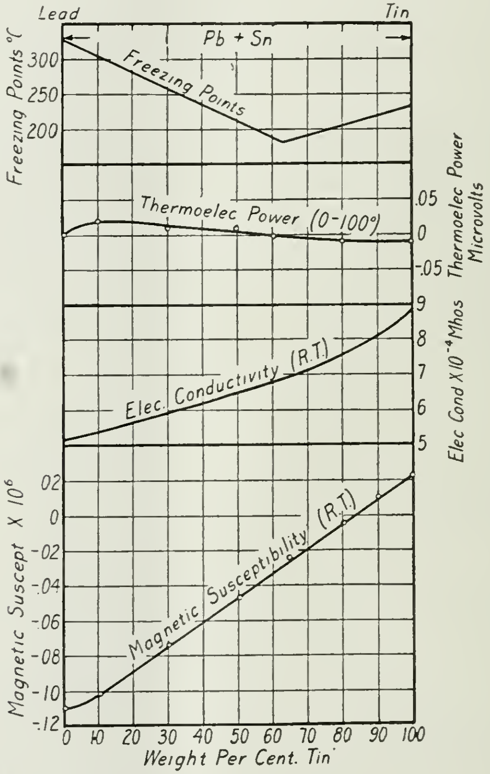
The temperature at which the observations were made or the interval of temperature over which they were made has been given on the curves as far as possible. In a number of cases the temperatures at which the observations were made were not clearly given by the observer. This is especially true for observations made in the neighborhood of room temperature. Where the observations were made near room temperature the letters (*R.T.*) have been written on the curves to indicate that fact.

METALS INSOLUBLE IN EACH OTHER.

Lead-Tin.

The freezing point curve by Degens[1] (Fig. 2) for lead-tin alloys shows a eutectic but no evidence of compounds. Tin seems to be somewhat soluble in lead and possibly lead is to a small degree soluble in tin. Between low and high concentrations of tin the alloys are heterogeneous mixtures of the crystalline phases.

FIG. 2.



The electrical conductivities as measured by Roberts[2] give nearly a straight line as is to be expected from the fact that the alloys are heterogeneous mixtures of the components. The thermoelectric powers have been determined by Rudolfi[3] and his results plotted in Fig. 2. The diamagnetic susceptibility by Honda[4] gives a curve which is nearly a straight line beginning with the value in lead and ending with the value in tin. In the alloys rich in lead (between 0 and 10 per cent. tin) where a solid solution is formed, the susceptibility decreases somewhat less rapidly than the linear relation requires. The linear relation between susceptibility and concentration follows at once from the fact that except for low and possibly high concentrations of tin the alloys are mechanical mixtures of the components.

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[1] Degens: *Zeit. anorg. Chem.*, 63, 207, 1909.

[2] Roberts: *Phil. Mag.* (5), 8, 57, 1879.

Tin-Zinc.

The two branches of the freezing point curve (Fig. 3) by Heycock and Neville[5] meet at the eutectic. The changes in curvature are all gradual and there is no evidence of compounds. The alloys consist of heterogeneous mixtures of tin and zinc.

Measurements of the electrical conductivity have been made by Matthiessen,[6] Vogt,[7] Harris and Le Chatelier.[8] More recently Schulze[9] has studied both the thermal and the electrical conductivities. His observations have been plotted in Fig. 3. The temperature coefficient is from the work of Matthiessen and Vogt. Besides the observations of Rudolfi[3] on the thermoelectric heights there are earlier observations by Rollmann and Battelli.[10] The observations of Rudolfi have been used for the curve of thermoelective forces. Except for minor variations these curves are essentially linear and typical of alloys which are formed by mechanically mixing the constituents.

Bismuth-Cadmium.

The freezing point curve (Fig. 4) by Stoffel[11] is composed of two branches meeting at the eutectic for which the temperature is 148° C. In all proportions the alloys are heterogeneous mixtures of bismuth and cadmium.

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[3] Rudolfi: *Zeit. anorg. Chem.*, 67, 65, 1910.

[4] Honda: *Sci. Rept. Tokio Univ.*, 2, 11, 1913.

[5] Heycock and Neville: *Jour. Chem. Soc.*, 71, 383, 1897.

[6] Matthiessen: *Pogg. Ann.*, 110, 207, 1860.

[7] Vogt: *Pogg. Ann.*, 122, 19, 1864.

[8] Le Chatelier: *Rev. Gen. des Sci.*, 6, 529, 1895.

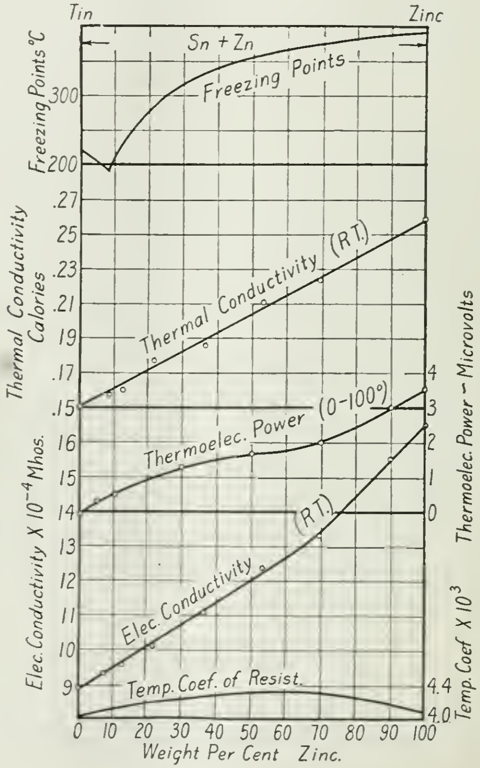
[9] Schulze: *Ann. d. Phys.*, 9, 555, 1902.

[10] Battelli: *Atti. R. Inst. Ver.* (6), 5, 1886–7.

[11] Stoffel: *Zeir. anorg. Chem.*, 53, 137, 1907.

The thermoelectric powers by Rudolfi [12] and the magnetic susceptibility by Gnesotto and Binghinnotto [13] give curves which have the form to be expected in heterogeneous mixtures. Each of these curves by their steepness where the concentration of bismuth is large suggests that cadmium is to a limited extent soluble in bismuth. There is some evidence from the study of the homogeneity of these alloys that cadmium may be soluble in bismuth up to about 1 per cent.

FIG. 3.



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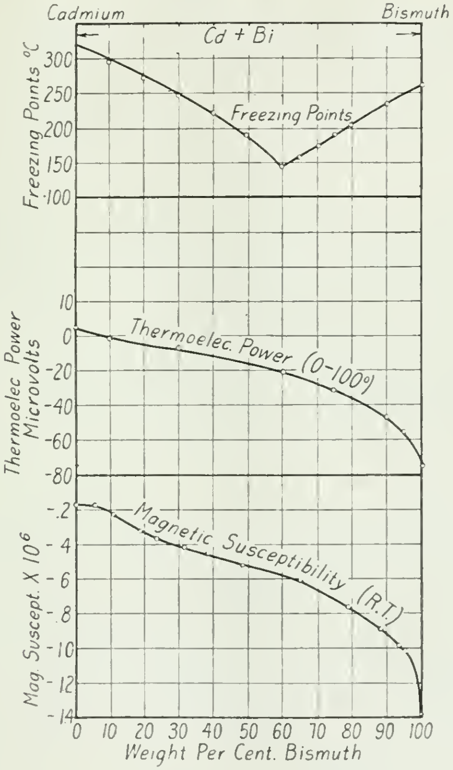
[12] Rudolfi: *Zeit. anorg. Chem.*, 67, 65, 1910.

[13] Gnesotto and Binghinnotto: *Inst. Ven.*, 69, 1382.

Cadmium-Tin.

The equilibrium diagram by Lorentz and Plumbridge [14] shows that the freezing point curve (Fig. 5) consists of two branches meeting at a eutectic when the alloy contains about 28 per cent. cadmium. These alloys are heterogeneous mixtures in all proportions.

FIG. 4.



The electrical conductivity curve by Matthiessen and Vogt [15] is a straight line. The curve giving the thermoelectric power as a function of the concentration is also a straight line. Besides these observations by Rudolfi [12] there are some earlier observations by Battelli.[16] Both of these curves are characteristic of alloys formed by mechanical mixtures of the constituents. Concerning the other thermal, electrical and magnetic properties of these alloys, no observations seem to be available.

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[14] Lorentz and Plumbridge: *Zeit. anorg. Chem.*, 83, 237, 1913.

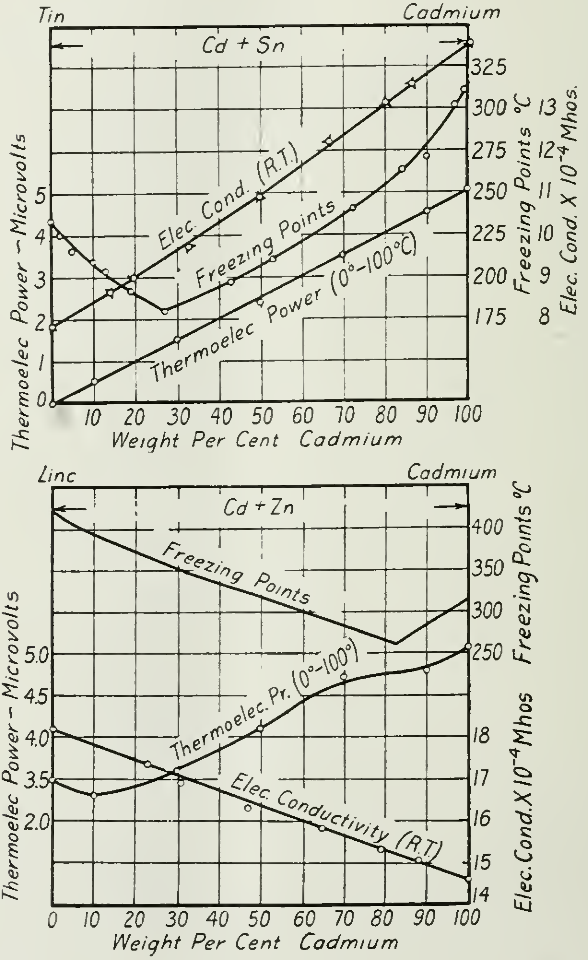
[15] Matthessien: *Pogg. Ann.*, 110, 206, 1860.

[16] Battelli: *Mem. di Torino* (2), 36, 31, 1884.

Cadmium-Zinc.

The freezing point curve for cadmium-zinc alloys by Lorentz and Plumbridge [17] (Fig. 5) consists of two branches meeting at the eutectic for which the temperature is 270° C. The alloys are mechanical mixtures of zinc and cadmium.

Fig. 5.



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[17] Lorentz and Plumbridge: *Zeit. anorg. Chem.*, 83, 236, 1913.

The electrical conductivities at room temperature have been measured by Matthiessen[18] and also by Vincentini. The curve showing the electrical conductivity as a function of the concentration of one of the constituents come out to be a straight line, as it should for this type of alloys. Battelli[19] has made a study of the thermoelectromotive forces of these alloys and a later investigation was made by Rudolfi.[20] The average values of the thermoelectric powers as determined by Rudolfi have been plotted on the curve in Fig. 5. This curve departs somewhat from a straight line and suggests by its minimum for low concentrations of cadmium that cadmium may be soluble to a limited extent in zinc.

Aluminium-Tin.

The freezing point curves of Gautier[21] and Gwyer[22] for this series of alloys differ by the fact that the freezing point curve of Gautier has a maximum corresponding to the compound AlSn and the curve by Gwyer shows no such maximum. The curve by Gwyer seems to be preferred and it has been reproduced in Fig. 6. According to Gwyer, there seems to be a eutectic which coincides nearly with the melting point of tin. Except for the possibility of slight solubility in each other these metals form alloys which are mechanical mixtures.

Besides the observations of Broniewski[23] on the electrical properties of these alloys there are available some observations by Pecheux on their thermoelectromotive force. The data given by Broniewski have been used for the curves plotted in Fig. 6. These curves in agreement with the freezing point curve lead to the conclusion that these metals do not form definite compounds and are characterized by dilute solid solution of tin in aluminium and a mixture of this solution with tin.

Aluminium-Bismuth.

Gwyer[24] found by thermal analysis that the solubility of aluminium in bismuth is about 2 per cent. and that of bismuth in aluminium is about 4 per cent. at 650° C., but inappreciable at the melting point of bismuth. The alloys may, therefore, be regarded as mechanical mixtures of aluminium and bismuth. The freezing point curve of Fig. 7 is by Gwyer.

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[18] Matthiessen: *Pogg. Ann.*, **110**, 28, 1860.

[19] Battelli: *Atti. R. Inst. Veneto* (6) **5**, 1148, 1886.

[20] Rudolfi: *Zeit. anorg. Chem.*, **67**, 65, 1910.

[21] Gautier: *C. R.*, **123**, 109, 1896.

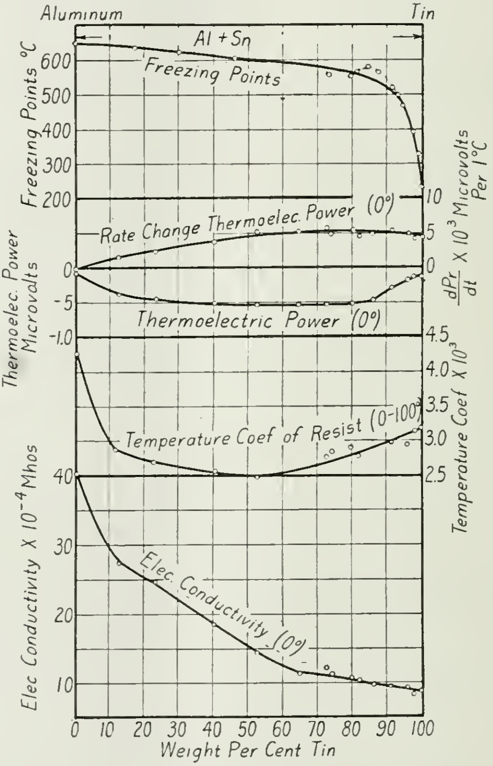
[22] Gwyer: *Zeit. anorg. Chem.*, **49**, 315, 1906.

[23] Broniewski: *Ann. de Phys. et Chem.* (8) **25**, 63, 1912.

[24] Gwyer: *Zeit. anorg. Chem.*, **49**, 316, 1906.

Some work has been done by Pecheux [25] on the thermoelectromotive forces in these alloys. They were more fully studied by Broniewski [26] from whose observations the curves in Fig. 7 have been taken. Except for a slight minimum where the concentration of aluminium is small the curve for electrical conductivities is nearly a straight line, as it should be for alloys formed by metals which are mechanically mixed. The curve for the temperature

FIG. 6.



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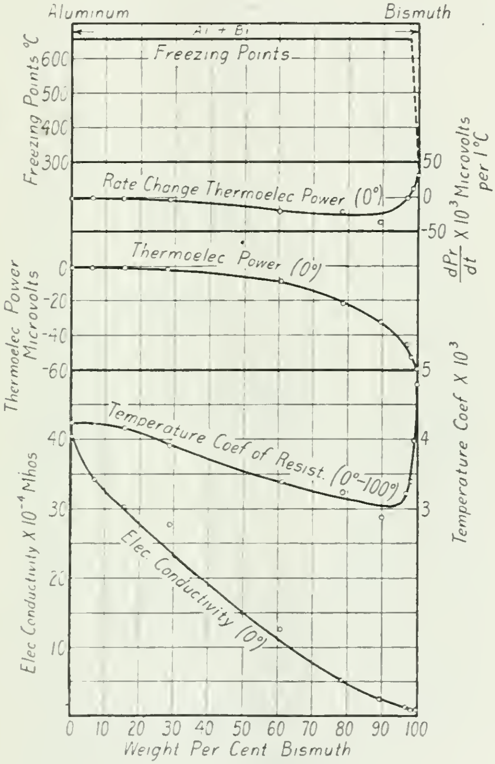
[25] Pecheux: *C. R.*, 138, 1501, 1904.

[26] Broniewski: *Ann. de Phys. et Chem.* (8), 25, 66, 1912.

coefficient of the resistance has a minimum at about 95 per cent. bismuth. This together with the slight minimum in the electrical conductivity curve indicates the formation of a solid solution of aluminium in bismuth. Since these minima are more pronounced in the unannealed than in the annealed specimens the solid solution is probably decomposed by annealing.

FIG. 7.



METALS COMPLETELY SOLUBLE IN EACH OTHER.

Indium-Lead.

Lead and indium form an isomorphous mixture in all proportions. The freezing point curve (Fig. 8) by Kurnakow and Puschin[27] is a continuous curve through the melting points of lead and indium.

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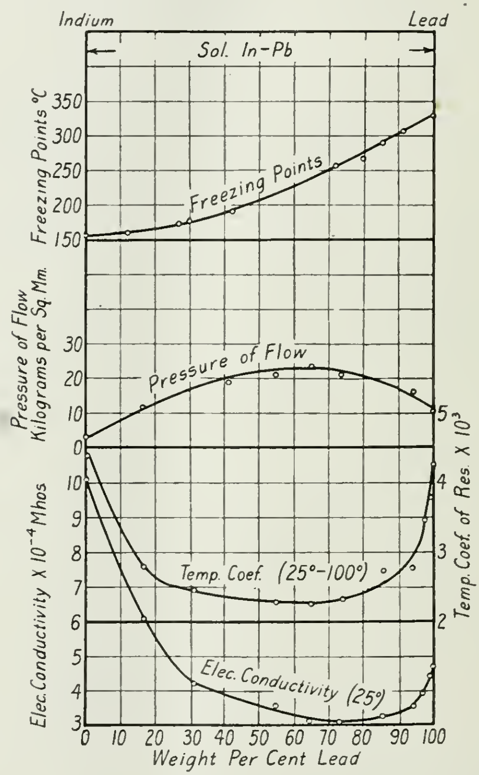
[27] Kurnakow and Puschin: *Zeit. anorg. Chem.*, 52, 430, 1907.

The electrical conductivity and the temperature coefficient have been measured by Kurnakow and Zemczuzny.[28] These authors have also determined the pressure necessary to cause these alloys to flow. All of the curves representing these physical quantities as functions of the concentration are typical of alloys in which the constituents form a continuous series of mixed crystals. The pressure necessary to produce flow has a maximum value where the electrical conductivity and the temperature coefficient of the resistance have minimum values. This indicates that the electrical properties are in part at least determined by the elastic properties.

FIG. 8.



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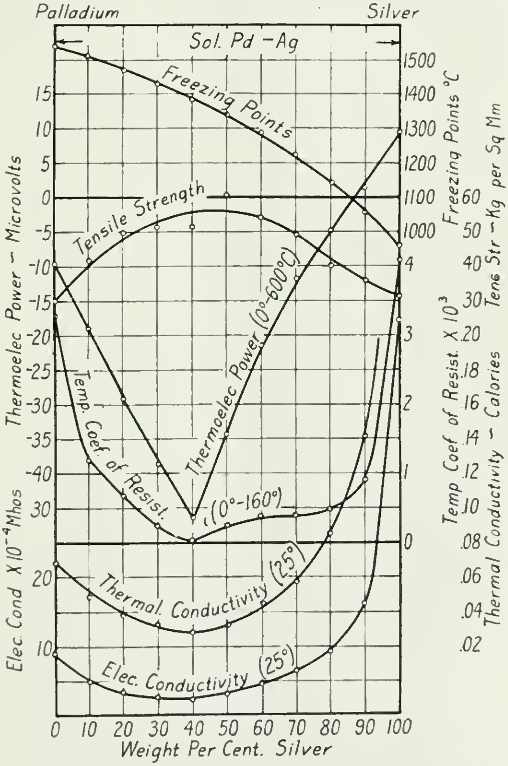
[28] Kurnakow and Zemczuzny: *Zeit. anorg. Chem.*, 64, 149, 1909.

Palladium-Silver.

The freezing point curve of these alloys (Fig. 9) is plotted from the data of Ruer.[29] The metals form solid solutions in all proportions.

The electrical and thermal conductivities have been determined by Schulze.[30] He used the same specimens which had been used by Giebel [31] for the study of the electrical conductivity, the temperature coefficient of the resistance, the thermoelectric heights and the tensile strength. All of these curves except possibly the one for thermoelectric heights are characteristic of metals which

FIG. 9.



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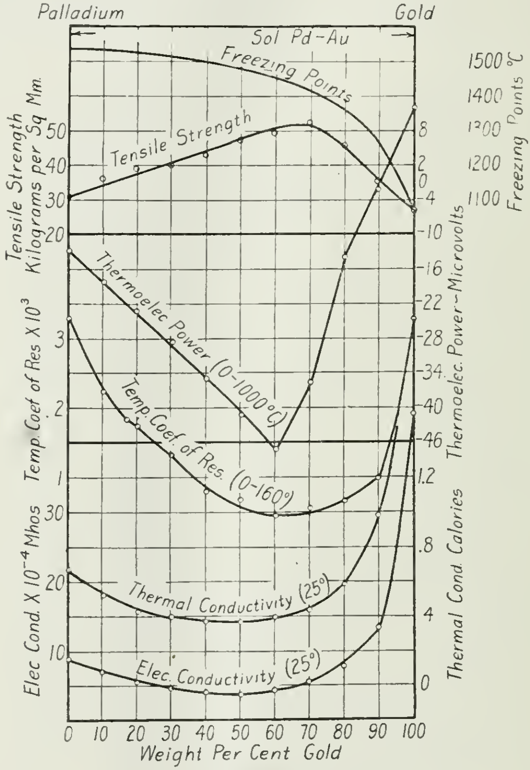
[29] Ruer: *Zeit. anorg. Chem.*, 51, 315, 1906.

[30] Schulze: *Physikal Zeitschr.*, 12, 1029, 1911.

[31] Giebel: *Zeit. anorg. Chem.*, 70, 240, 1911.

form a continuous series of mixed crystals. The elastic properties as represented by the tensible strength of these alloys have a maximum where the electrical conductivity, the thermal conductivity, the thermoelectric power and the temperature coefficient of the resistance have minimum values.

FIG. 10.



Palladium-Gold.

From the freezing point curve of Ruer [2] (Fig. 10) it is seen that the freezing point of these alloys changes gradually from the melting point of palladium to the melting point of gold. The metals form an unbroken series of mixed crystals.

The electrical and thermal conductivities have been measured by Schulze [3] and the thermoelectric powers, the temperature coefficient and the tensile strengths by Giebel [32] who also measured the electrical conductivity. All of these observations were made on the same specimens. These curves are all very similar to the corresponding curves for palladium-silver alloys and typical of alloys in which there is an unbroken series of mixed crystals. Here as in other similar cases the elastic properties are related to the electrical and thermal properties.

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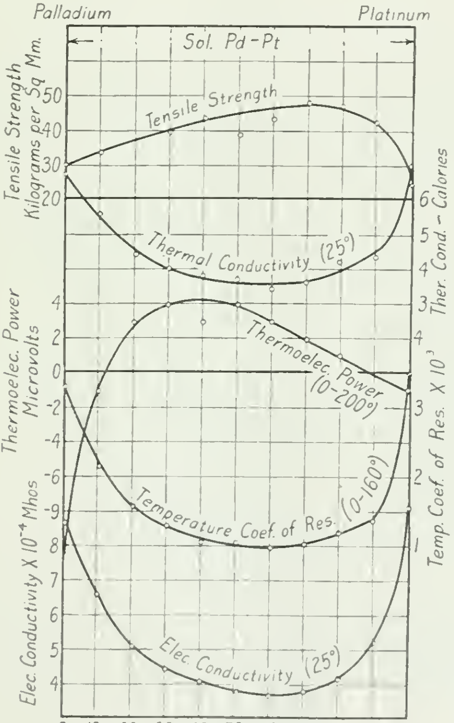
[32] Giebel: *Zeit. anorg. Chem.*, 70, 240, 1911.

Palladium-Platinum.

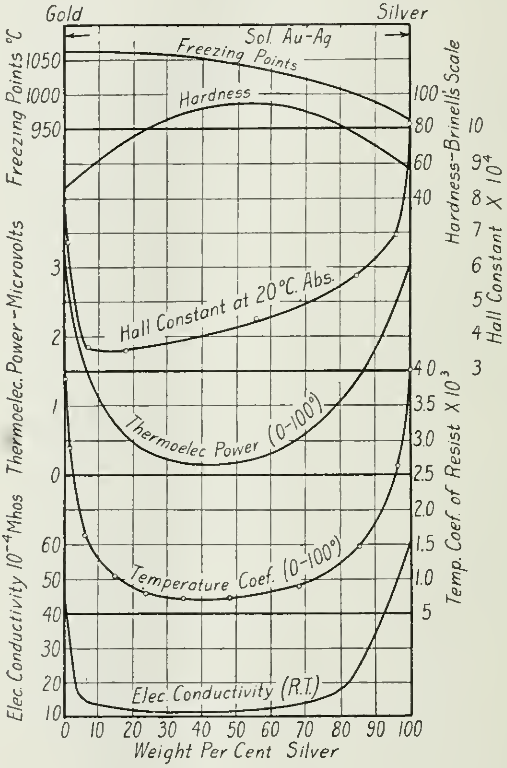
The freezing point curve for this series of alloys does not seem to have been studied. The physical properties (Fig. 11) clearly indicate that these metals mix in all proportions forming a continuous series of mixed crystals.

FIG. 11.



The thermal and electrical conductivities which have been given in Fig. 11 have been taken from the work of Schulze who made his observations on the same specimens on which Giebel[32] had made observations on the electrical conductivity, the temperature coefficient of the resistance, the thermoelectric power and the tensile strength. There is an evident parallelism between the electrical and thermal conductivities. In this series as in the palladium-silver series the curve of tensile strengths has a maximum where the electrical and thermal conductivities have minimum values.

FIG. 12.



Gold-Silver.

Roberts-Austen and Kirke Rose, confirming the work of Heycock and Neville,[33] showed that gold and silver solidify in the form of an unbroken series of mixed crystals. The freezing point changes continuously from its value in gold to its value in silver.

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[33] Heycock and Neville: *Phil. Trans. A.*, 189, A, 69, 1897.

The curve for the electrical conductivities (Fig. 12) is plotted from the observations of Matthiessen.[34] The average thermoelectric power has been calculated from the data of Rudolfi [35] and the Hall constant from the work of Beckman.[36] The hardness has been measured by Kurnakow.[37] The similarity between the four lower curves in Fig. 12 is very evident. The elastic property has a maximum value where the other properties have minimum values, indicating that the elastic forces in the alloys help to determine the Hall effect as well as the electrical conductivity, the temperature coefficient of the resistance and the thermoelectric powers.

Copper-Gold.

From the observations of Kurnakow and Zemczuzny [38] on the freezing points of copper-gold alloys (Fig. 13) it is found that the freezing point curve runs in a simple way from the melting point of copper to the melting point of gold. It has a minimum where the alloy contains about 25 per cent. copper. These metals form a continuous series of solid solutions.

The electrical conductivity of this series has been studied by Matthiessen [39] and in later times by Kurnakow, Zemczuzny and Zasedatelev [40] from whose observations have been taken both the curve for electrical conductivities and the curve for temperature coefficient of resistance. The hardness which has been measured by Kurnakow and Zemczuzny [41] shows the maximum which is characteristic of a series of solid solutions. The average thermoelectric power as found by Rudolfi [1] has been used for the thermoelectric height curve. Except for the two peaks in the curve for the electrical conductivity and two corresponding peaks in the curve for the temperature coefficient of the resistance, this set of curves is very similar to the set found for alloys of platinum and palladium in which there was a continuous series of solid solutions.

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[34] Matthiessen: *Pogg. Ann.*, 110, 190, 1861.

[35] Rudolfi: *Zeit. anorg. Chem.*, 67, 65, 1910.

[36] Beckman: *Com. fr. Phys. Lab. Univ. of Leiden*, 130, 27, 1912.

[37] Kurnakow: *Zeit. anorg. Chem.*, 60, 1, 1908.

[38] Kurnakow and Zemczuzny: *Zeit. anorg. Chem.*, 54, 163, 1907.

[39] Matthiessen: *Pogg. Ann.*, 110, 217, 1860.

[40] Kurnakow, Zemczuzny and Zasedatelev: *J. Inst. of Metals*, 15. 305, 1916.

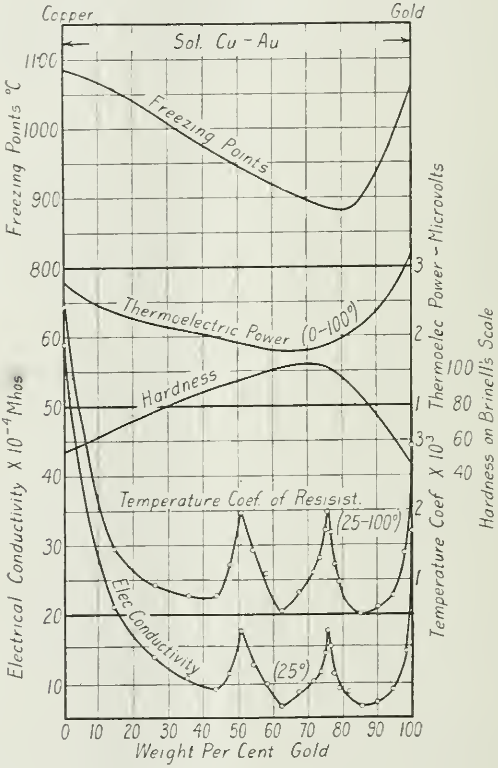
[41] Kurnakow and Zemczuzny: *Zeit. anorg. Chem.*, 60, 1, 1908.

Potassium-Rubidium.

These metals form isomorphous mixtures in all proportions. The freezing point curve has not been located and it seems to be unknown.

The observations on the electrical conductivity and on the rate of change of the resistance with the temperature (Fig. 14) are due to Kurnakow and Nikitinsky.[42] They also determined the pressures required to cause the alloys to flow. The electrical conductivity as a function of the concentration of one of the components is represented by a continuous curve with a very slight minimum. This is characteristic of isomorphous mixtures. With rising temperature the minimum is displaced toward rubidium—

FIG. 13.



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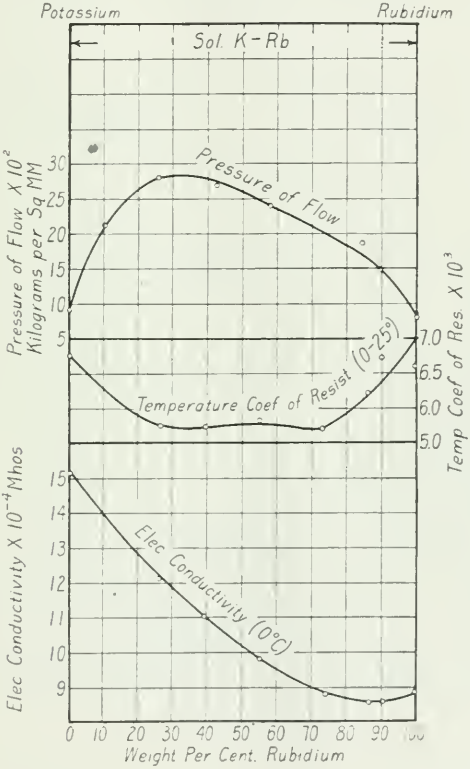
[42] Kurnakow and Nikitinsky: *Zeit. anorg. Chem.*, 88, 151, 1914.

the metal with the least conductivity. The curve for the rate of change of the resistance with the temperature shows the minimum characteristic of this type of alloys. The only difference between these curves and those obtained under similar conditions for most isomorphous mixtures is the smaller decrease in the electrical conductivity which is produced by adding rubidium to potassium. In most cases in which solid solutions are formed this decrease is much larger than it is in this case. The elastic properties as measured by the pressure necessary to cause the alloys to flow show the characteristic maximum found in isomorphous mixtures.

FIG. 14.

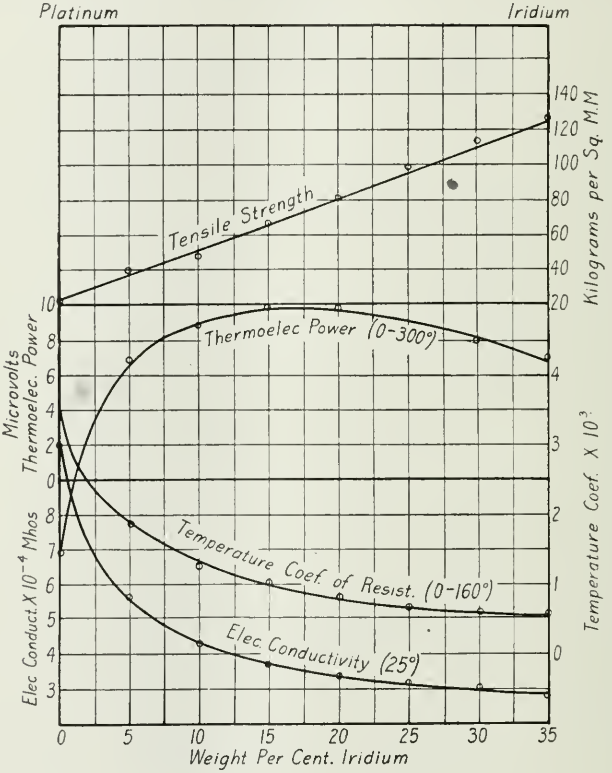


Platinum-Iridium.

The equilibrium diagram for this series has not been located. These metals probably form solid solutions.

The electrical conductivity, the temperature coefficient, the thermoelectric power, and the tensile strength have been studied by Giebel[43] for this series of alloys for concentrations of iridium between 0 and 35 per cent. These curves are reproduced in Fig. 15. The tensile strength is a linear function of the concentration of iridium over this interval. The thermoelectric power passes through a maximum between 15 and 20 per cent. iridium. The addition of iridium to platinum lowers both the electrical conductivity and the temperature coefficient in the way to be expected on the assumption that the metals form solid solutions.

FIG. 15.



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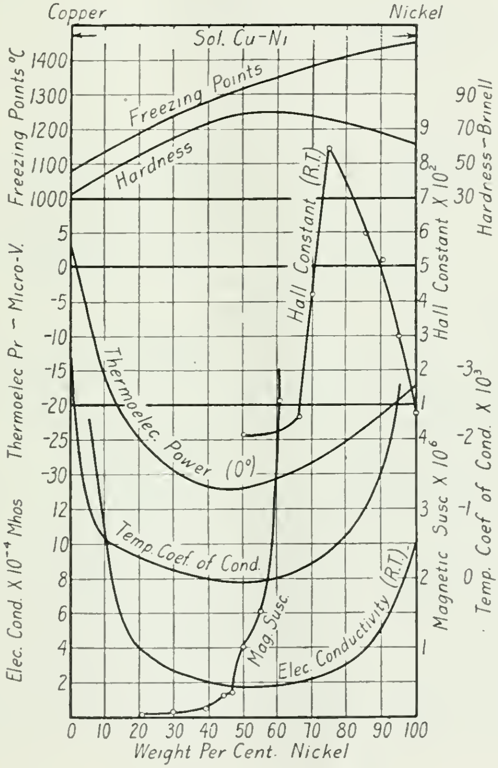
[43] Giebel: *Zeit. anorg. Chem.*, 70, 247, 1911.

Copper-Nickel.

The freezing point curve for this series of alloys (Fig. 16) is the one worked out by Guertler and Tammann.[44] These metals form an unbroken series of solid solutions.

The electrical conductivity and the temperature coefficient have been measured by Feussner.[45] The curves thus obtained are characteristic of a continuous series of solid solutions. The thermoelectric power by Englisch [46] and the hardness by Kurnakow and Papke [47] also give the type of curve to be expected in alloys which

FIG. 16.



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[44] Guertler and Tammann: *Zeit. anorg. Chem.*, 53, 281, 1907.

[45] Feussner: *Verhand. d. physik Gesel. zu Berlin*, 10, 109, 1891.

[46] Englisch: *Phys. Consts. Soc. Fran. de Phys.*, p. 654, 1893.

[47] Kurnakow and Papke: *Zeit. anorg. Chem.*, 87, 274, 1914.

are solid solutions. The curve of hardness shows a maximum where the curve for the electrical conductivity has a minimum, thus showing that the elastic properties in a measure determine the electrical conductivity and its variation with the temperature. The curve for the magnetic susceptibilities has been taken from the observations of Gans and Fouseca.[48] The Hall constant is by the author.[49] Neither the Hall constants nor the magnetic susceptibilities seem to depend on the concentration in the way to be expected for solid solutions.

Iron-Nickel.

The freezing point curve (Fig. 17) by Guertler and Tammann [50] indicates that iron and nickel form a continuous series of solid solutions. On the freezing point curve as sometimes given there seems to be a change in curvature at the concentration corresponding to the compound Ni_2Fe, from which this compound is sometimes inferred.

The specific heat, the electrical conductivity, the temperature coefficient, the thermal conductivity and the thermoelectric heights have been measured by Ingersoll and others.[51] The flux densities given in Fig. 17 are from the observations of Yensen [52] and were measured for an external magnetic field of 400 gausses. The specific heat is a maximum at the concentration of the possible compound Ni_2Fe. The electrical conductivity, the temperature coefficient and the thermoelectric power have minimum values where the intermetallic compound might be formed. Of all these curves only the one for the thermal conductivities has the general form to be expected in a series of alloys which are an unbroken series of solid solutions. The complexity of the curves in this case is doubtless due to the fact that both nickel and iron are polymorphic.

Magnesium-Cadmium.

The freezing point curve by Grube [53] has a point of inflection where the concentration corresponds to the compound MgCd.

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[48] Gans and Fouseca: *Ann. d. Phys.*, 61, 742, 1920.

[49] Smith: *Phys. Rev.*, *N. S.*, 17, 24, 1921.

[50] Guertler and Tammann: *Zeit. anorg. Chem.*, 45, 205. 1905.

[51] Ingersoll: *Phys. Rev.*, *N. S.*, 16, 85, 1920.

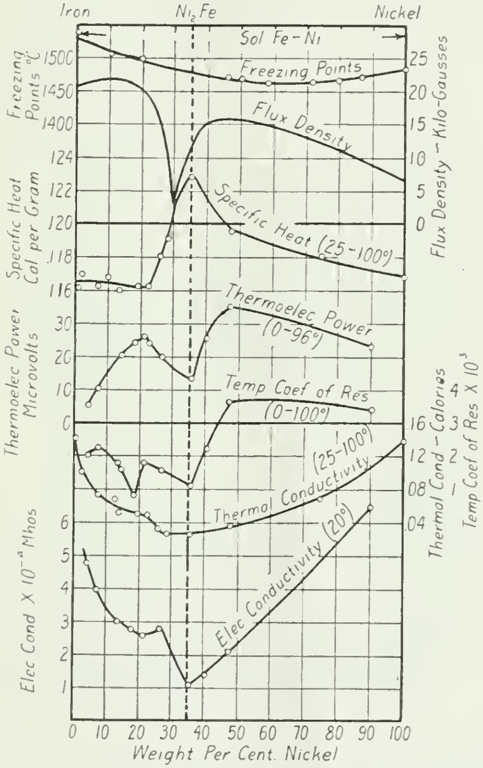
[52] Yensen: *Jour. A. I. E. E.*, 396, 1920.

[53] Grube: *Zeit. anorg. Chem.*, 49, 72, 1906.

This change in curvature is not very evident in the curve as plotted in Fig. 18. According to this equilibrium diagram the alloys are solid solutions of two crystalline phases. About the equilibrium in this series there is still considerable doubt.

The electrical conductivity, the temperature coefficient, and hardness have been measured by Ourazow.[54] On both of these curves the compound MgCd is clearly marked by a cusp. This compound is also indicated on the curve for hardness. On either side of the compound there is a second cusp in the curve for electrical conductivity and for the temperature coefficient. If these cusps were absent the curves would have the normal course

FIG. 17.



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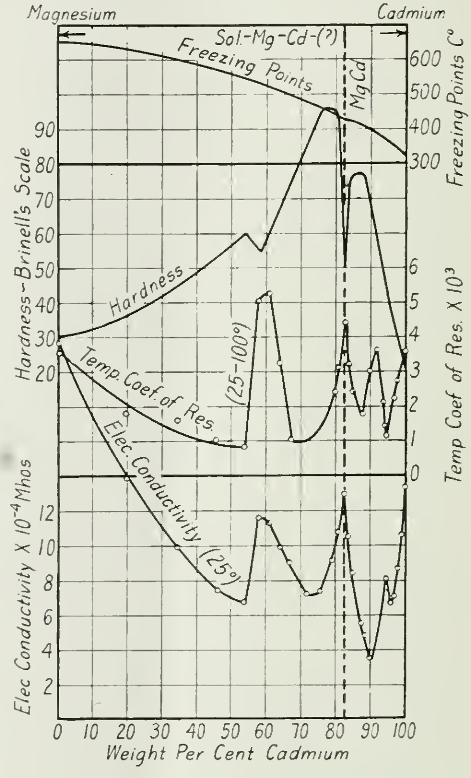
[54] Ourazow: *Zeit. anorg. Chem.*, 73, 31, 1912.

to be expected on the assumption that alloys to the right of the compound are solid solutions of cadmium and the compound MgCd and those to the left of the compound solid solutions of magnesium with the compound MgCd. The thermal analysis is

FIG. 18.



not in agreement with the indications given by the thermal and electrical properties in this case.

METALS WITH LIMITED SOLUBILITY IN EACH OTHER.

Aluminium-Zinc.

The freezing point curve of aluminium-zinc alloys from Gautier[55] (Fig. 19) is made up of two branches which meet at the temperature of fusion of the eutectic. There are no compounds and Shepherd[56] concludes from a micrographic study that aluminium-zinc alloys are formed of two solid solutions and a mechanical mixture of these solid solutions.

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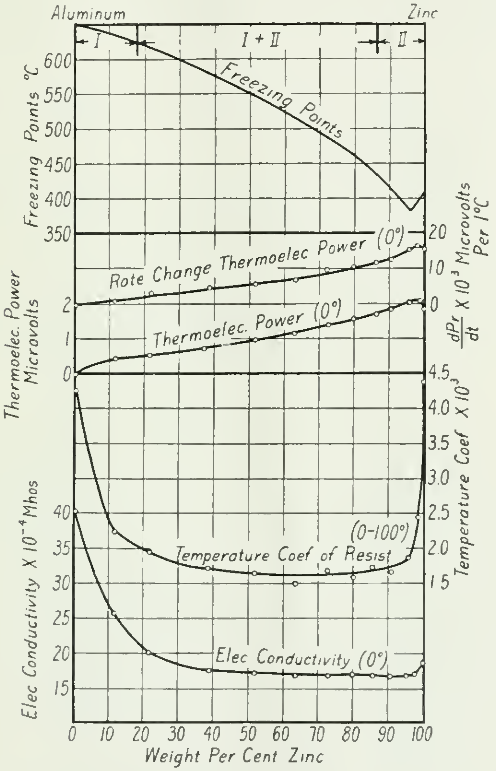
[55] Gautier: *Bull. Soc. Encour.* (5), 1, 1293, 1896.

Besides the observations of Broniewski[57] on the electrical properties of these alloys there are earlier observations by Battelli[58] and Pecheux.[59] The curves in Fig. 19 have been plotted from the observations of Broniewski. The curves all belong to that group of alloys which are formed from metals which are soluble in each other to a limited extent and in which the solid solutions

FIG. 19.



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[56] Shepherd: *Jour. Phys. Chem.*, 9, 504, 1905.

[57] Broniewski: *Ann. de Chem. et Phys.* (8), 25, 1, 1912.

[58] Battelli: *Atti. R. Inst. Veneto.* (6), 5, 1148, 1886-7.

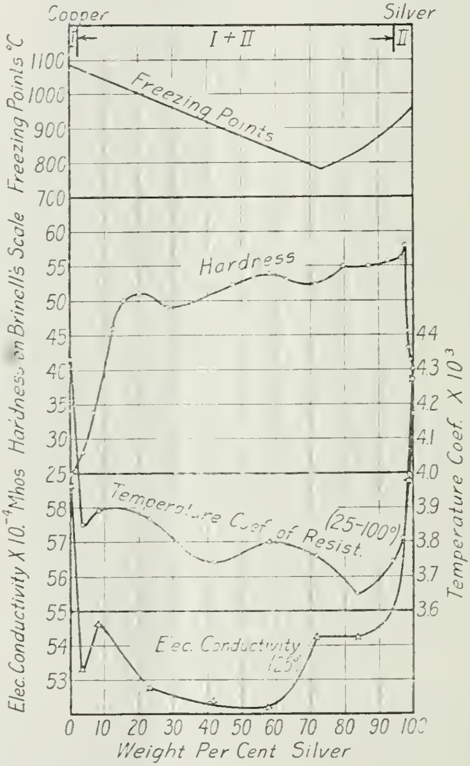
[59] Pecheux: *C. R.*, 138, 1103, 1904.

thus formed mix with each other mechanically. The central portions of the curves are nearly linear as should be the case for mechanical mixtures.

Copper-Silver.

A thermal analysis of copper-silver alloys has been made by Heycock and Neville.[60] Their results have been confirmed by the work of Friedrick and Leroux and Lepowiski,[61] from whose work the freezing point curve (Fig. 20) is taken. Copper forms a solid solution with silver and silver with copper until the concentration of the copper in one case and silver in the other is about

FIG. 20.



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[60] Heycock and Neville: *Phil. Trans. A.*, 189, 25, 1897.

[61] Lepowiski: *Zeit. anorg. Chem.*, 59, 289, 1908.

5 per cent. The remainder of the alloys are heterogeneous mixtures of these saturated solid solutions.

The hardness, the electrical conductivity and the temperature coefficient of this series are due to Kurnakow, Puschin and Semkowsky.[62] Where a solid solution is formed between copper and silver there is a marked lowering of both the electrical conductivity and the temperature coefficient. Hence for high as well as low concentrations of silver these curves are very steep. The central portions of these curves, although somewhat irregular, approach the form to be expected in a region where there is a mechanical mixture of two crystalline phases. There is a marked increase in hardness over the intervals in which solid solutions are formed. The remainder of the hardness curve is such as is to be found where the alloys are mechanical mixtures of two crystalline phases. Except for some irregularities this set of curves clearly belongs to metals which form solid solutions with each other to a limited extent and then these solid solutions mix mechanically to form the remainder of the alloys.

Bismuth-Lead.

The freezing point curve by Kapp and Charpy[63] (Fig. 21) has a eutectic at 56.5 per cent. bismuth. Herold[64] finds that for suitable concentrations lead and bismuth form mixed crystals, but the region over which these solid solutions extend is not clearly defined. The solutions, however, are rather dilute. The remainder of the alloys are mechanical mixtures of two crystalline phases.

The hardness curve indicates by the increase in hardness for low and high concentrations of bismuth the formation of solid solutions at the beginning and end of this series. The remainder of the curve of hardness over the region where the alloys are heterogeneous mixtures is a straight line. The thermal and electrical conductivities by Schulze[65] have minima for alloys containing 4 or 5 per cent. of lead. This as well as the initial drops in these curves for alloys containing from 0 to 15 per cent. bismuth is further evidence for the formation of solid solutions at either end of this series. The thermoelectromotive force which has been studied by Battelli[66] shows a rapid decrease for the addition of small quantities of lead to bismuth. The characteristics of this curve in this region are similar to the characteristics of the other curves over this same region.

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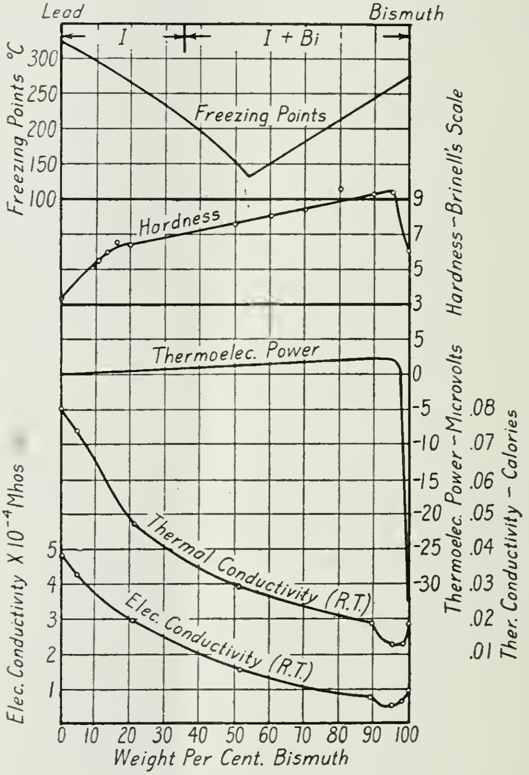
[62] Kurnakow, Puschin and Semkowsky: *J. d. russ. phys. Chem.*, 42, 733, 1910.

[63] Barlow: *Zeit. anorg. Chem.*, 70, 183, 1911.

[64] Herold: *Zeit. anorg. Chem.*, 112, 131, 1920.

[65] Schulze: *Ann. d. Phys.*, 9, 564, 1902.

FIG. 21.



Antimony-Tin.

The freezing point curve by Williams[67] (Fig. 22) consists of three parts. Later study of equilibrium in this system has been made by Konstantinow and Smirnow, LeGris and Loebe. Between 90 and 100 per cent. antimony there is a solid solution of tin in antimony and between 0 and 10 per cent. tin there is a solid solution of antimony in tin. When the metals are present in equal concentrations a new crystalline phase is formed, probably the intermetallic compound SnSb. Those alloys which are not solid solutions for large and small concentrations of antimony are mechanical mixtures of two crystalline phases.

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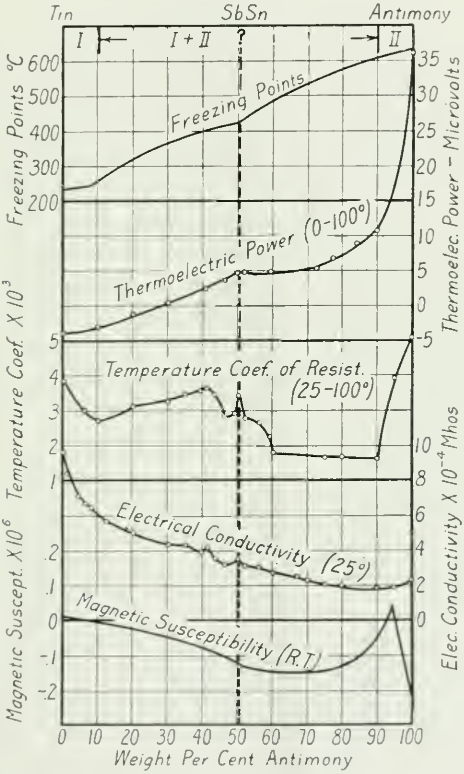
[66] Battelli: *Atti. R. Inst. Veneto* (6), 5, 1148, 1886.

[67] Williams: *Zeit. anorg. Chem.*, 55, 12, 1907.

The electrical conductivity and the temperature coefficient are

FIG. 22.



by Konstantinow and Smirnow[68] and the magnetic susceptibility by Leroux.[69] The initial decrease in the electrical conductivity, the temperature coefficient of the resistance and the thermoelectric power[70] caused by the addition of tin to antimony give evidence of the formation of a solid solution over this region. On the other side of the diagram where the concentration of antimony is less than 10 per cent. the curve for the electrical conductivity and for the temperature coefficient show again the presence of solid solutions. The curve for the magnetic susceptibilities indicates the compound by a change in its curvature at that concentration. The evidence for the structure of this series is not conclusive.

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[68] "Annual Tables of Constants and Numerical Data," Vol. 2, p. 345.

[69] Leroux: *C. R.*, 156, 1764, 1913.

[70] Hutchins: *Jour. Am., Amer. Jour. Sci.*, 48, p. 226, 1894.

Lead-Thallium.

The freezing point curve for lead-thallium alloys has a very flat maximum between 30 and 40 per cent. lead. This was regarded by Lewkonja[71] as evidence of the compound $PbTh_2$. Kurnakow and Puschin[72] found that this maximum is displaced by the addition of tin to the alloys. If the maximum were due to a true compound, this displacement should not occur. Hence the existence of the compound is in doubt. Thallium forms a solid solution with lead until the concentration of thallium is about 75 per cent., and lead dissolves in thallium until the concentration of lead is about 4 or 5 per cent. Rejecting the evidence for the existence of the compound, the alloys between 5 and 25 per cent. lead are heterogeneous mixtures of a saturated solution of lead in thallium with a saturated solution of thallium in lead.

The electrical conductivity and the mean temperature coefficient as determined by Kurnakow and Schemtschuschny[73] have been plotted in Fig. 23. There is a minimum in both curves. This minimum is in the region over which lead and thallium form an isomorphous mixture. The addition of lead to thallium causes a decrease in the electrical conductivity and also in the temperature coefficient of the resistance. This decrease is followed by an increase which extends to the concentration at which the solution is saturated with lead. Between 5 and 25 per cent. lead there is a straight line portion in the curve. This is over the region where the alloys are mechanical mixtures of two crystalline phases. The pressure required to produce flow in these alloys is taken from these same observers. The curve thus obtained shows the characteristic relation between the electrical and the elastic properties.

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[71] Lewkonja: *Zeit. anorg. Chem.*, 52, 452, 1907.

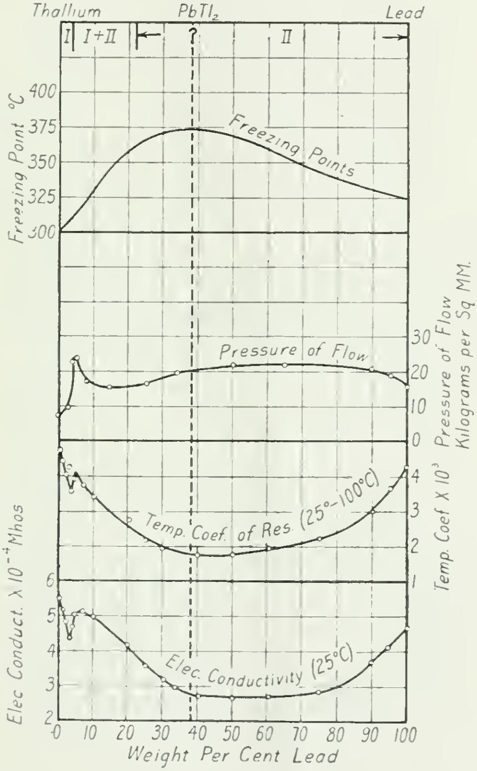
[72] Kurnakow and Puschin: *Zeit. anorg. Chem.*, 52, 430, 1907.

[73] Kurnakow and Schemtschuschny: *Zeit. anorg. Chem.*, 64, 156, 1909.

Lead-Antimony.

The freezing point curve (Fig. 24) by Gontermann[74] shows a eutectic in the neighborhood of 13 per cent. antimony. Gontermann finds some evidence of an intermediate crystalline constituent. Aside from this possible exception these alloys may be considered mechanical mixtures.

FIG. 23.



The curve for electrical conductivities is by Matthiessen;[75] for the thermoelectric powers by Rudolfi;[76] for magnetic susceptibilities by Leroux,[77] and for the electromotive force of dissolution by Pouchine. The curve for the thermoelectric powers is a straight line until the alloy contains 90 per cent. antimony. At that concentration the thermoelectric power rises rapidly to its value in pure antimony. At about this same concentration the curve for the magnetic susceptibilities and for the electromotive forces of solution show peculiarities. There seems to be nothing in the structure of the alloys to offer an explanation of this sudden change in the direction of the curves at this concentration.

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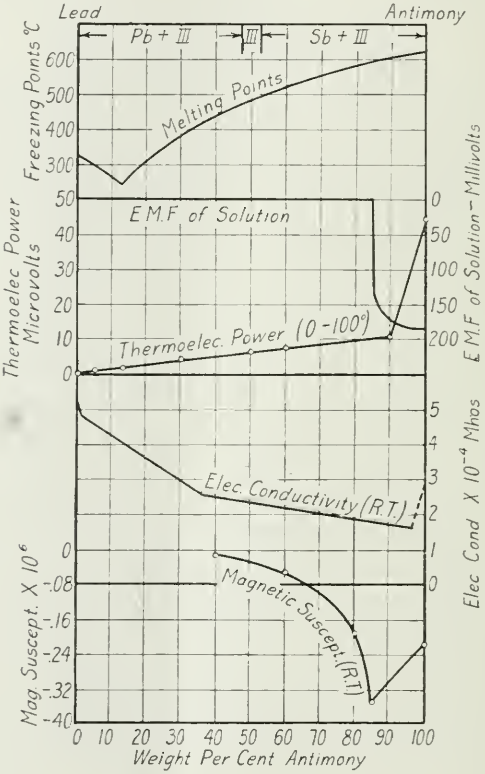
[74] Gontermann: *Zeit. anorg. Chem.*, 55, 419, 1907.

[75] Matthiessen: *Pogg. Ann.*, 110, 28, 1860.

[76] Rudolfi: *Zeit. anorg. Chem.*, 67, 65, 1910.

[77] Leroux: *C. R.*, 156, 1764, 1913.

FIG. 24.



Copper-Cobalt.

The equilibrium diagram by Sahmen [78] from which the freezing point curve (Fig. 25) is taken shows that copper and cobalt form mixed crystals with each other over a limited region. Copper is soluble in cobalt up to 8 per cent. and cobalt in copper up to 5 per cent. cobalt. The remainder of the alloys are a heterogeneous mixture of a saturated solution of cobalt in copper and a saturated solution of copper in cobalt.

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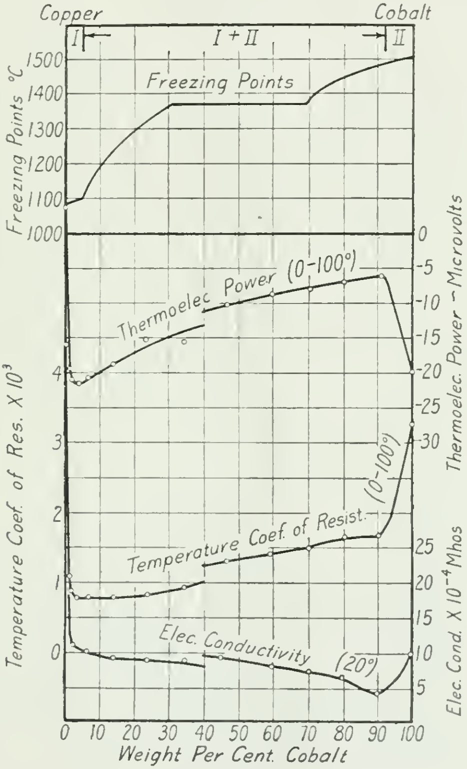
[78] Sahmen: *Zeit. anorg. Chem.*, 57, 1908.

The curve for electrical conductivity, temperature coefficient and thermoelectric height are from the observations of Reichardt.[79]

FIG. 25.



Each of these curves shows a discontinuity since alloys with less than 60 per cent. copper were so brittle that they could not be forged into wires and were investigated in the form of castings. Aside from these discontinuities the curve for electrical conductivity and the curve for temperature coefficient of resistance are typical of alloys in which the constituents are soluble in each other to a limited extent. The central portion of the curves are essentially linear, as they should be for alloys formed by the mixture of two crystalline phases. The curve for thermoelectric powers is peculiar in view of the fact that the addition of cobalt to copper causes a rapid decrease in the thermoelectric height and the addition of copper to cobalt causes a somewhat less rapid increase in the thermoelectric height of cobalt.

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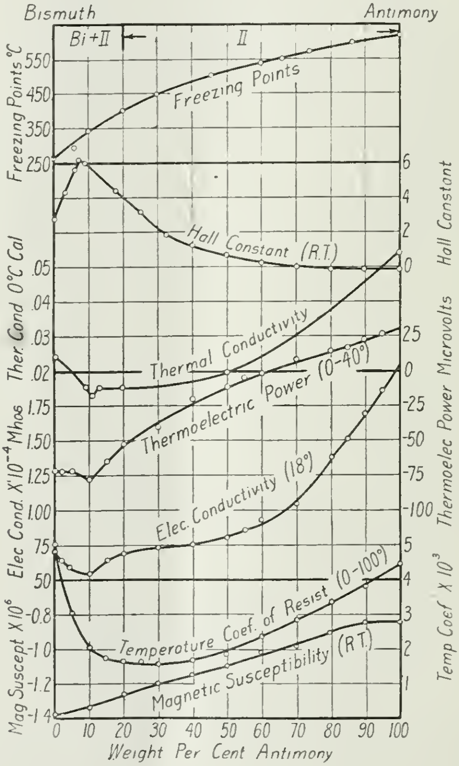
[79] Reichardt: *Ann. d. Phys.*, 6, 842, 1901.

FIG. 26.



Bismuth-Antimony.

The freezing point curve (Fig. 26) by Huttner and Tammann[80] shows that the freezing points of these alloys decrease gradually from the melting point of bismuth. Concerning the structure of the alloys there seems to be some doubt. Between 18 and 100 per cent. antimony they may be considered solid solutions of bismuth and antimony and between 0 and 18 per cent. antimony they are mixtures of bismuth and a saturated solution of antimony and bismuth.

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[80] Huttner and Tammann: *Zeit. anorg. Chem.*, **44**, 131, 1905.

The thermoelectromotive forces of this series have been studied by Seebeck, Rollmann, Matthiessen, Becquerel, Sundell, Battelli, Hutchins and more recently by Haken [81] from whose data the curve of Fig. 26 is taken. The electrical conductivity is known from the work of Matthiessen, Calvert and Johnason and Haken.[81] The magnetic susceptibility by Honda and Sone [82] is a linear function of the concentration until the alloy contains about 90 per cent. antimony where the proportionality fails. The curve for the thermal conductivities by Gehlhoff and Neumaier [83] is very similar to the curve for electrical conductivities by Haken. This shows that Wiedmann and Franz's law holds approximately for these alloys. The curve for the temperature coefficient is characteristic of alloys which are solid solutions. The Hall constant [84] is evidently closely related to the thermoelectric power in agreement with the suggestion of Beattie that there is a proportionality between these two quantities.

Bismuth-Tin.

The freezing point curve (Fig. 27) from data of Stoffel [85] and Lepkowski [86] consists of two branches meeting at a eutectic. Except for small and possibly large concentrations of tin where the metals may be soluble in each other to a limited extent, these alloys are mechanical mixtures of bismuth and tin.

The thermoelectric powers of these alloys have been studied by Hutchins [87] and also by Caswell.[88] The curve for the Thomson effect as determined by Laws and also by Caswell is very similar to the curve for the thermoelectric heights. Each curve shows a pronounced maximum when a small quantity of tin is present in the alloy. The curves for electrical and thermal conductivities by Schulze [89] have minima where the preceding curves have maxima.

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[81] Haken: *Ann. d. Phys.*, 32, 291, 1910.

[82] Honda and Sone: *Sci. Repts. Univ. Tokio*, 2, 5, 1913.

[83] Gehlhoff and Neumaier: *Verh. d. Deutsch. Phys. Ges.*, p. 876, 1913.

[84] Smith: *Phys. Rev.*, 32, 178, 1911.

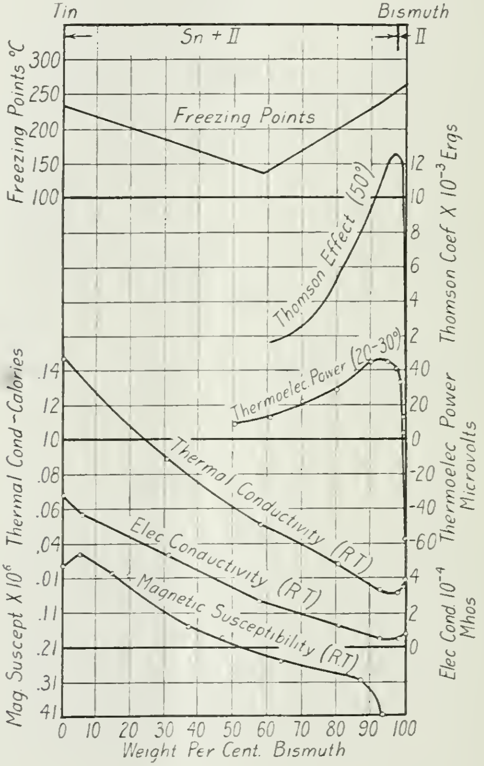
[85] Stoffel: *Zeit. anorg. Chem.*, 53, 148, 1907.

[86] Lepkowski: *Zeit. anorg. Chem.*, 59, 287, 1908.

[87] Hutchins: *Amer. Jour. Sci.*, 48, 226, 1894.

[88] Caswell: *Phys. Rev., N. S.*, 12, 226, 1918.

FIG. 27.



These maxima and minima occur in the region in which the alloys seem to be dilute solid solutions of tin in bismuth. The remainder of the curve for thermal conductivity as well as the curve for electrical conductivity is roughly linear, the form to be expected in alloys which are heterogeneous mixtures. The curve for magnetic susceptibilities has been contributed by Gnesotto and Binghinnotto.[90] The addition of tin to bismuth rapidly decreases the diamagnetic susceptibility in the interval where the preceding curves showed either maxima or minima, that is, in the interval of possible solid solutions. The remainder of the curve suggests mechanical mixtures except for the irregularities where the alloys are nearly all tin.

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[89] Schulze: *Ann. d. Phys.*, 9, 566, 1902.

Copper-Zinc.

The freezing point curve (Fig. 28) is by Shepherd and others.[91] The structure of this series is complex. Copper dissolves zinc until the concentration of zinc is about 35 per cent. and zinc dissolves copper until the concentration of copper is 2 or 3 per cent. The intermediate alloys may be considered heterogeneous mixtures of two crystalline phases.

The electrical conductivity, the temperature coefficient, the thermoelectric powers and the rate of change of thermoelectric power have been measured by Norsa.[92] Between 0 and about 35 per cent. zinc these curves have the form characteristic of alloys which are solid solutions. The remainder of the curves seem too complicated to admit of analysis in terms of the structure of the alloys. The thermal conductivity is known from the work of Calvert and Johnson.[93] The course of this curve is somewhat irregular. The curve for the magnetic susceptibility has been plotted from data by Weber.[94] Between 0 and 35 per cent. zinc it is a straight line.

Copper-Tin.

The structure of this series of alloys is complex. The freezing point curve (Fig. 29) by Heycock and Neville[95] gives evidence of one compound Cu_3Sn. Tin is soluble in copper until there is about 13 per cent. copper present. The remainder of the alloys may be considered as mechanical mixtures of two crystalline phases.

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[90] Gnesotto and Binghinnotto: *Inst. Ven.*, 69, 1382.

[91] Guertler: *Metallographie*, i, 459.

[92] Norsa: *C. R.*, 155, 348, 1912.

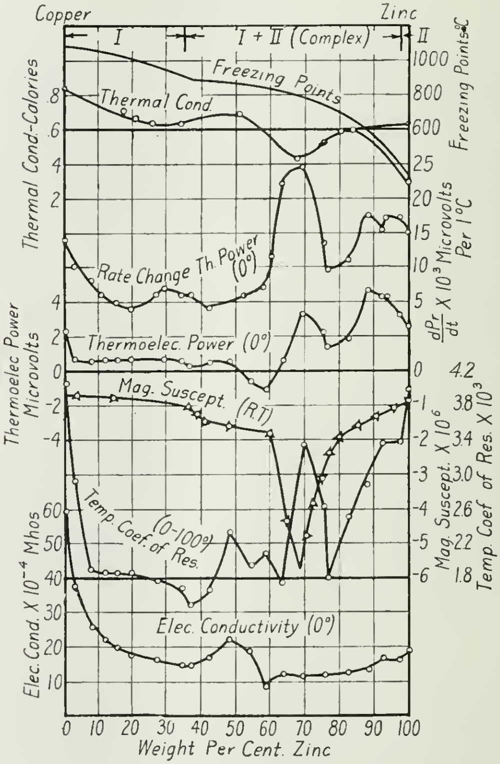
[93] Calvert and Johnson: *Phil. Mag.*, 18, 354, 1850.

[94] Weber: *Ann. d. Phys.*, 62, 666, 1920.

[95] Heycock and Neville: *Phil. Trans. A.*, 202, 1, 1904.

The curve for hardness by Kurnakow and Zemczuzny [96] consists of two straight lines and a curve of gentle slope. The two straight lines intersect where the concentration of tin is about 11 per cent. and thus mark the concentration at which tin ceases to be soluble in copper. The intersection of the second straight line with the third portion of the curve marks the concentration for the compound Cu_3Sn. The curves for the electrical conductivity, the temperature coefficient, the thermoelectric power and its variation

FIG. 28.



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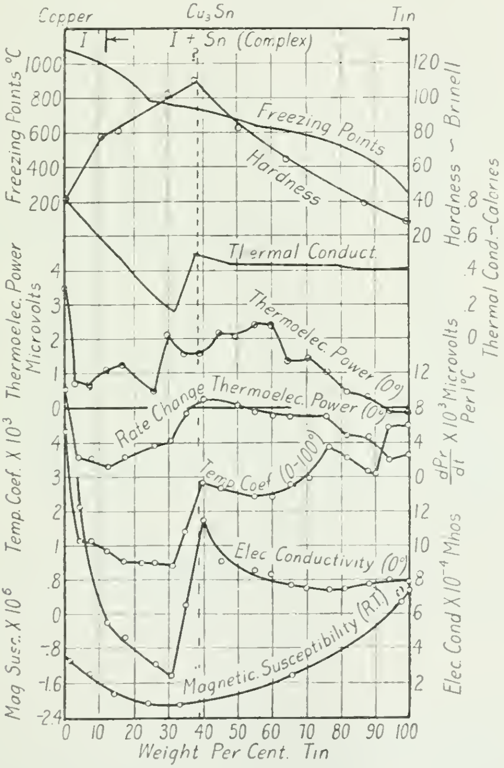
[96] Kurnakow and Zemczuzny: *Zeit. anorg. Chem.*, 60, 9, 1908.

with the temperature have been plotted from the observations of Leroux.[97] Between 0 and 35 per cent. tin the curves for electrical conductivities and that for the temperature coefficients are typical of alloys which are solid solutions. Where the solution becomes saturated the direction of the curves suddenly changes. The position of the intermetallic compound is marked on these curves by cusps. The curve for thermal conductivities is very similar to the curve for electrical conductivities. Wiedemann and Franz's law must, therefore, hold approximately for these alloys. The curve for the magnetic susceptibilities by Clifford [98] does not show

FIG. 29.



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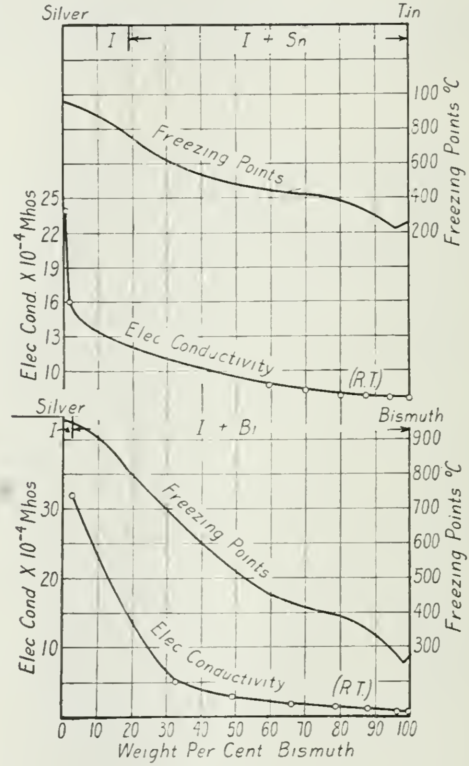
[97] Leroux: *C. R.*, 155, 35, 1912.

[98] Clifford: *Phys. Rev.*, 26, 424, 1908.

the compound, but this is probably due to the fact that the points on the curve near the compound are too far apart.

FIG. 30.



Silver-Tin.

The freezing point curve for silver-tin alloys (Fig. 30) by Petrenko[99] is in complete agreement with the curve obtained by Heycock and Neville. Silver is only slightly soluble in tin and tin is soluble in silver until the concentration of tin is about 18 per cent. The other alloys are, therefore, heterogeneous mixtures of two crystalline phases.

The electrical conductivity curve by Matthiessen[100] shows an initial rapid drop for alloys rich in silver. This is characteristic of alloys which are solid solutions. The remainder of the curve has the general shape of curves for a mechanical mixture of two crystalline phases.

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[99] Petrenko: *Zeit. anorg. Chem.*, 50, 138, 1906; also 53, 200, 1907.

[100] Matthiessen: *Pogg. Ann.*, 110, 215, 1860.

Silver-Bismuth.

Petrenko[101] has also given the freezing point curve for silver-bismuth alloys (Fig. 30). It shows a eutectic for alloys containing 2.5 per cent. silver. Under suitable conditions bismuth is somewhat soluble in silver. Most of the alloys are heterogeneous mixtures of a saturated solid solution of bismuth in silver and of bismuth.

The form of the electrical conductivity curve by Matthiessen[102] suggests the formation of solid solutions of bismuth in silver followed by a region in which the alloys are heterogeneous mixtures. The electrical conductivity curve for this series is very similar to the curve for silver-tin alloys. Some observations on thermoelectromotive forces have been made by Battelli.[103]

Lead-Cadmium.

The freezing point curve, according to Stoffel[104] (Fig. 31), consists of two branches meeting at a eutectic for which the temperature is 249° C. Lead and cadmium form a solid solution until the concentration of cadmium is about 5 per cent., at which concentration the solution is saturated. The remainder of the alloys are a mechanical mixture of this saturated solution and cadmium.

The electrical conductivity by Matthiessen[102] is nearly a linear function of the concentration. There are no points on the curve in the region between 95 and 100 per cent. lead in which the solid solutions are now known to be formed.

Lead-Silver.

The freezing point curve by Petrenko[101] (Fig. 31) shows a eutectic for which the temperature is 303.9° C. Lead is soluble in silver until the alloy contains about 5 per cent. lead. The remainder of the alloys are heterogeneous mixtures of this solid

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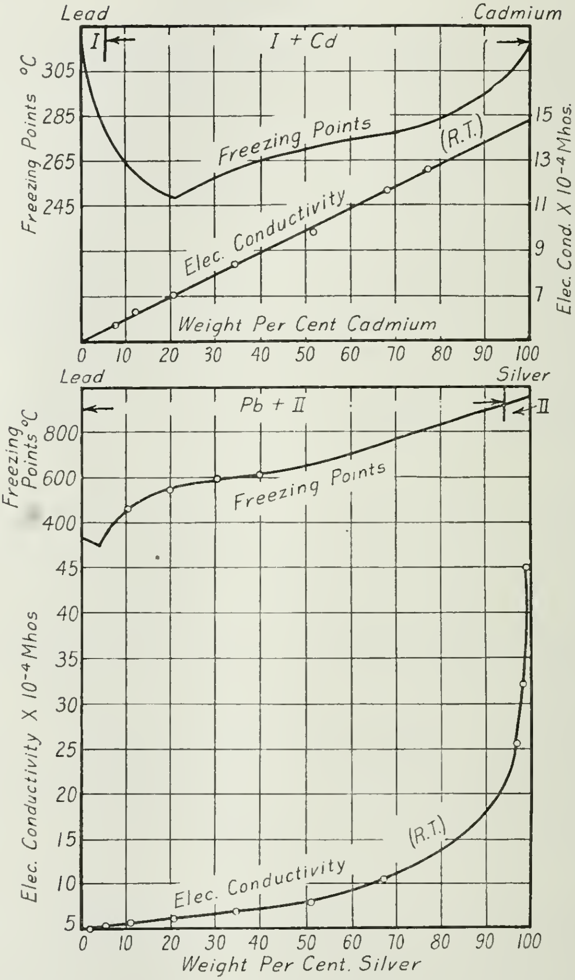
[101] Petrenko: *Zeit. anorg. Chem.*, 50, 138, 1906; also 53, 200, 1907.

[102] Matthiessen: *Pogg. Ann.*, 110, 208, 1860.

[103] Battelli: *Atti. R. Inst. Ven.* (6), 5, 1148, 1886.

[104] Stoffel: *Zeit. anorg. Chem.*, 53, 152, 1907.

FIG. 31.



solution and lead. The structure of these alloys is very similar to the structure of the lead-cadmium alloys.

The curve for electrical conductivities by Matthiessen[105] is very steep between 100 and 97 per cent. silver. This is the region in which a solid solution of lead in silver is formed. For alloys containing less than 50 per cent. silver the curve becomes nearly a straight line, which is characteristic of mechanical mixtures of two crystalline phases.

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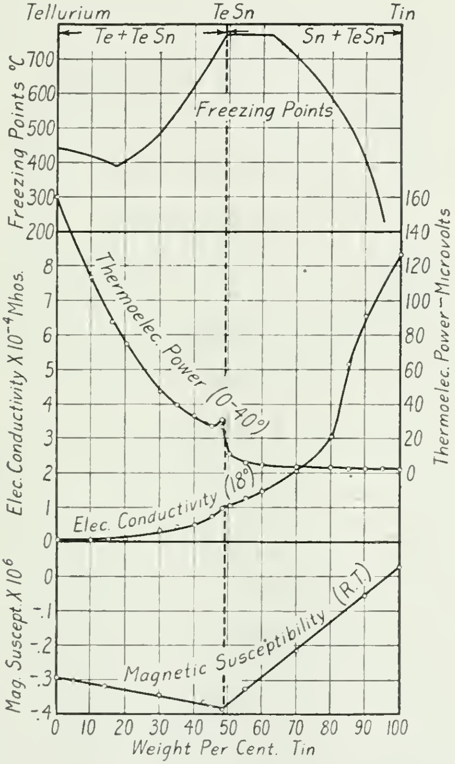
[105] Matthiessen: *Pogg. Ann.*, 110. 212, 1860.

FIG. 32.



METALS FORMING COMPOUNDS WITH EACH OTHER.

Tellurium-Tin.

The freezing point curve (Fig. 32) by Fay[106] gives the intermetallic compound TeSn, with a eutectic on either side. The results of Kobayashi[107] are in agreement with those of Fay. Alloys containing less than 48 per cent. tin are mechanical mixtures of tellurium; the compound TeSn and those containing more than 48 per cent. tin are mechanical mixtures of tin and the compound TeSn.

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[106] Fay: *Jour. Amer. Chem. Soc.*, 29, 1265, 1907.

[107] Kobayashi: *Zeit. anorg. Chem.*, 69, 1, 1911.

The thermoelectric powers and the electrical conductivities in Fig. 32 are by Haken [108] and the magnetic susceptibilities from the work of Honda and Sone.[109] The compound is indicated on each of the curves. The curve for magnetic susceptibilities consists of two straight lines which intersect at the concentration giving the compound TeSn. One of these straight lines corresponds to mixtures of tellurium and TeSn and the other to mixtures of tin and TeSn. Between 55 and 100 per cent. tin the thermoelectric power curve is a straight line of small slope.

Bismuth-Tellurium.

The freezing point curve of this system (Fig. 33) by Monkemeyer [110] indicates the compound Bi_2Te_3 with a eutectic on either side. Alloys to the left of the compound are mixtures of Bi and Bi_2Te_3 and those to the right are mixtures of Te and the compound Bi_2Te_3.

The thermoelectric power and the electrical conductivity are taken from the observations of Haken,[111] the Hall constant at room temperature from Trabacci [112] and the magnetic susceptibility from the work of Honda and Sone.[113] The presence of the compound is clearly marked on each of the curves. The curve for the Hall constant is very similar to the curve for the thermoelectric powers. A proportionality between the Hall constants and the thermoelectric powers has been recognized by Beattie and these curves are in agreement with the suggestion. Beside the work of Honda and Sone on the diamagnetic susceptibility of these alloys there is some earlier work by Mendenhall and Lent [114] who failed to find in their curve an indication of the compound. Honda and Sone point out that this was probably due to the fact that Mendenhall and Lent did not take the points in the neighborhood of the compound close enough together. Between 1 and 41 per cent. tellurium and between 60 and 100 per cent. tellurium the susceptibility curve is nearly a straight line which is characteristic of mechanical mixtures.

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[108] Haken: *Ann. d. Phys.*, 32, 291, 1910.

[109] Honda and Sone: *Sci. Repts. Imp. Univ. Tokio*, 2, 10, 1913.

[110] Monkemeyer: *Zeit. anorg. Chem.*, 46, 415, 1905.

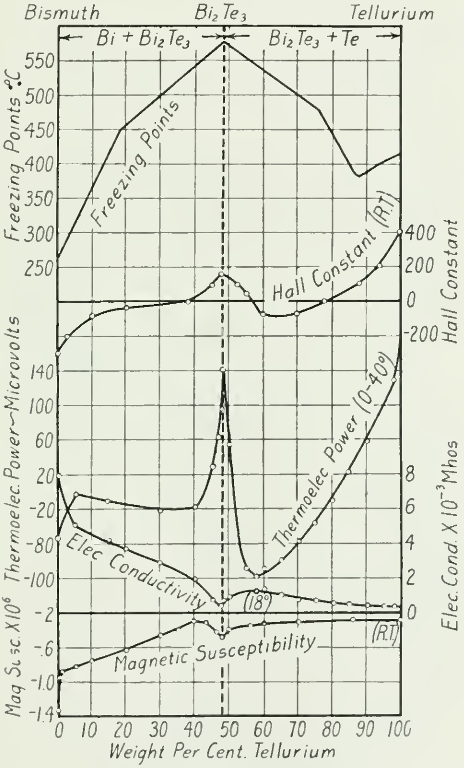
[111] Haken: *Ann. d. Phys.*, 32, 291, 1910.

[112] Trabacci: *Nuovo Cim.*, 9, 95, 1915.

[113] Honda and Sone: *Sci. Repts. Imp. Univ. Tokio*, 2, 12, 1913.

[114] Mendenhall and Lent: *Phys. Rev.*, 32, 406, 1911.

FIG. 33.



Bismuth-Magnesium.

The freezing point curve (Fig. 34) by Grube[115] gives one compound Bi_2Mg_3 and one eutectic for this combination of metals. Another eutectic is probably formed between bismuth and the compound Bi_2Mg_3, but the temperature of this eutectic nearly coincides with the melting point of bismuth. Alloys to the left of the compound consist of heterogeneous mixtures of bismuth and the compound Bi_2Mg_3 and those to the right of mixtures of magnesium with the compound Bi_2Mg_3.

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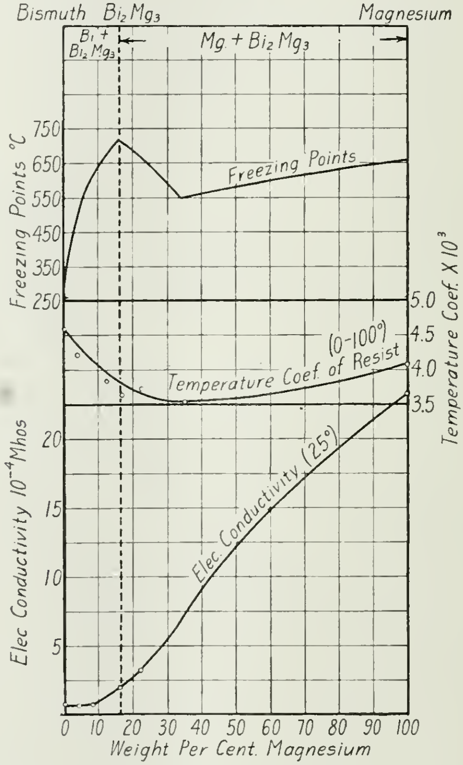
[115] Grube: *Zeit. anorg. Chem.*, 49, 85, 1906.

The electrical conductivity and the average temperature coefficient have been determined by Stepanow.[116] Neither of these curves show the presence of the compound Bi_2Mg_3. This may be due to the fact that the points in the neighborhood of this compound were not taken sufficiently close together. The course of the curves on either side of the compound somewhat resembles straight lines, indicating that the alloys on the two sides of the compound are mechanical mixtures.

FIG. 34.



Magnesium-Tin.

The equilibrium diagram for magnesium and tin shows one intermetallic compound, Mg_2Sn. There is a eutectic on either side of this compound. The freezing point curve of Fig. 35 is by Grube.[117] Magnesium does not seem to be soluble in tin, but Grube finds that magnesium dissolves about 6 per cent. of tin. With the exception of alloys in this region, the alloys of this series are mechanical mixtures of two crystalline phases.

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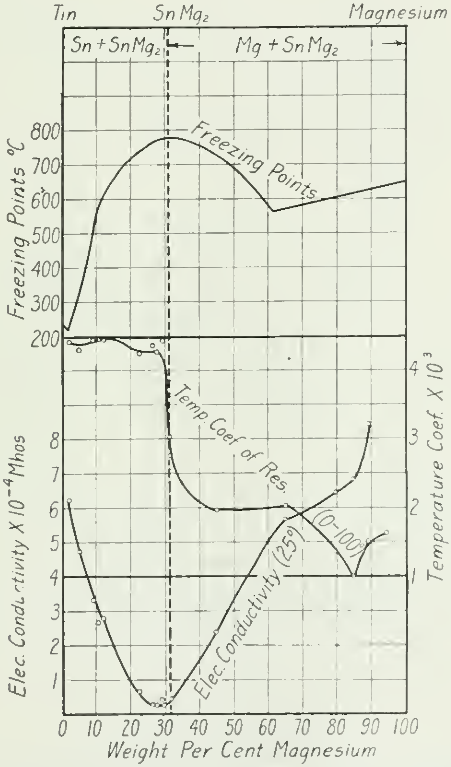
[116] Stepanow: *Zeit. anorg. Chem.*, 78, 1, 1912.

FIG. 35.



The electrical conductivities and the temperature coefficients of the resistance are known from the work of Stepanow.[116] The addition of tin to magnesium causes a rapid drop in the electrical conductivity and the temperature coefficient. The steepness of these curves for large concentrations of magnesium confirms the existence of solid solutions of magnesium and tin where the concentration of tin is small. The compound is marked by a rapid drop in the temperature coefficient and a minimum in the electrical conductivity. Between 0 and about 28 per cent. and between 40 and 65 per cent. magnesium the temperature coefficient is nearly constant. This constancy could be inferred from the fact that the alloys over these regions are mechanical mixtures, in the former case of tin and the compound $SnMg_2$ and in the latter case of the compound $SnMg_2$ and a saturated solid solution of magnesium and tin.

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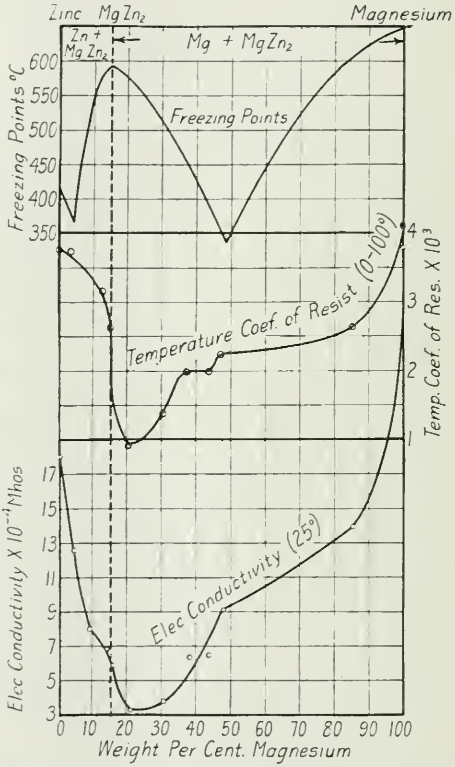
[117] Grube: *Zeit. anorg. Chem.*, 46, 1905.

FIG. 36.



Magnesium-Zinc.

These two metals, according to Grube,[118] from whose work the freezing point curve of Fig. 36 is taken, form one intermetallic compound with the formula $MgZn_2$. On either side of this compound is a eutectic and the alloys to the left of the compound may be regarded as heterogeneous mixtures of zinc and the compound $MgZn_2$ and those to the right of the compound, mixtures of magnesium and the compound $MgZn_2$. It seems possible that zinc may form dilute solid solutions with magnesium.

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[118] Grube: *Zeit. anorg. Chem.*, 49, 80, 1906.

The electrical conductivities and the temperature coefficients of the resistance are from the observations of Stepanow.[119] These curves are similar in form to the corresponding curves for magnesium-tin alloys. In the region where the concentration of zinc is small and dilute solid solutions of zinc in magnesium may be formed both of these curves are steep, especially the curve of electrical conductivities. The position of the compound is marked by a rapid decrease in both the temperature coefficient and the electrical conductivity.

Bismuth-Thallium.

The freezing point curve by Chickashige[120] (Fig. 37) indicates a compound at the concentration corresponding to Bi_5Tl_3 with a eutectic on either side of it. The freezing point curve seems to have three maxima, but only one of them corresponds to a simple atomic ratio, and that is the one giving the compound Bi_5Tl_3. A micrographic examination also locates a compound in this neighborhood.

The electrical conductivity curve for these alloys are by Whitford[121] and the magnetic susceptibilities by Mendenhall and Lent.[122] Both the electrical conductivity and the magnetic susceptibility were measured at room temperature. The presence of the intermetallic compound is clearly indicated on both of these curves.

Aluminium-Magnesium.

Grube[123] whose freezing point curve is reproduced in Fig. 38 finds one maximum which corresponds to either Al_2Mg_3 or Al_3Mg_4. These compounds lie so close together that it is difficult to choose between them, but Grube concludes that Al_3Mg_4 is the more probable. To the right of this compound the alloys may be considered mechanical mixtures of Mg and the compound Al_3Mg_4 and for lower concentrations of magnesium they are mechanical mixtures of aluminium and the compound Al_3Mg_4.

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[119] Stepanow: *Zeit. anorg. Chem.*, 78, 25, 1912.

[120] Chickashige: *Zeit. anorg. Chem.*, 51, 328, 1906.

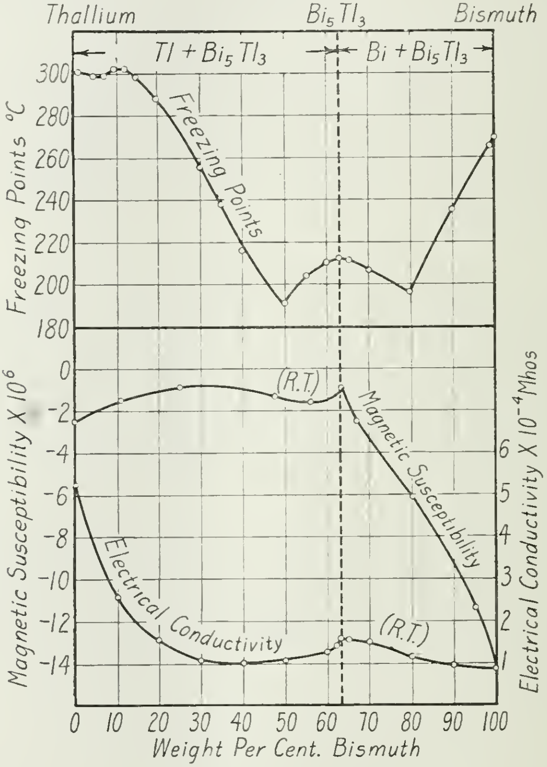
[121] Whitford: *Phys. Rev.*, 35, 144, 1912.

[122] Mendenhall and Lent: *Phys. Rev.*, 32, 415, 1911.

[123] Grube: *Zeit. anorg. Chem.*, 45, 225, 1905.

Besides the observations of Broniewski[124] from which the curves in Fig. 38 have been plotted there are observations by Pecheux[125] on the thermoelectromotive forces of some members of this series. According to the interpretation of Broniewski, two compounds are indicated by his curves: *viz.*, AlMg and Al_2Mg_3. Of these compounds Al_2Mg_3 is much more clearly marked than the compound AlMg.

FIG. 37.



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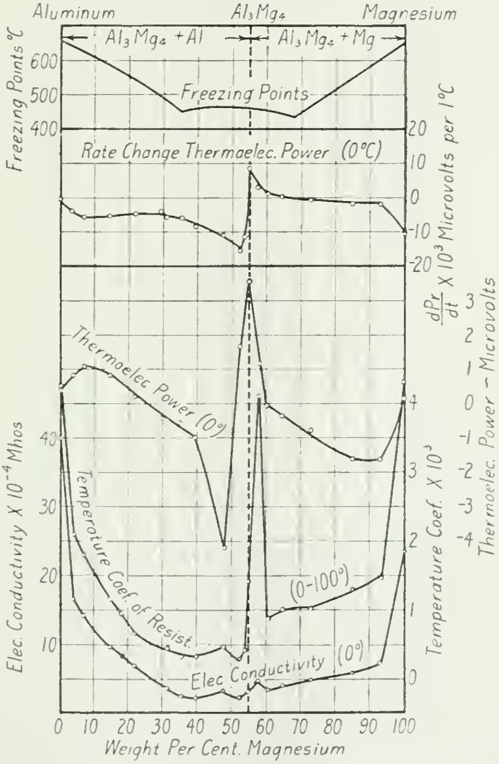
[124] Broniewski: *Ann. de Phys. et Chem.* (8), 25, 76, 1912.

[125] Pecheux: *C. R.*, 139, 1202, 1904.

Copper-Arsenic.

The freezing point curve of Fig. 39 is by Friedrich.[126] From the equilibrium diagram and from a metallographic study of these alloys it is found that copper and arsenic form two compounds Cu_3As and Cu_3As_2 and that arsenic dissolves in copper up to 4 per cent.

FIG. 38.



Some observations on the electrical conductivity of copper-arsenic alloys have been made by Matthiessen and Holtzmann[127] and later by Hampe.[128] Friedrich also gives some data on the electrical resistance of these alloys for low concentrations of arsenic. The specific resistance of these alloys and the temperature coefficient (Fig. 39) are the values given by Puschin and Dischler.[129] The electrical resistance of copper is much increased by the addition of small quantities of arsenic, and the temperature coefficient is decreased. This occurs in the region where a solid solution of arsenic in copper is formed and is characteristic of the formation of such solutions. When the concentration of arsenic is about 28.5 per cent. the specific resistance has its maximum value. This maximum value comes at the concentration for the compound Cu_3As. The temperature coefficient decreases from its value in pure copper, passes through a minimum for 6 per cent. arsenic and then increases until the concentration corresponding to the compound Cu_3As is reached. Beyond this concentration the temperature coefficient remains nearly constant.

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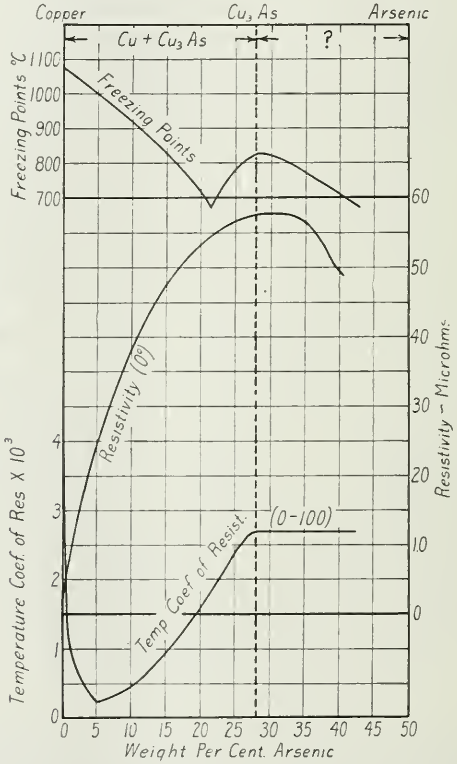
[126] Friedrich: *Metallurgie,* 5, 529, 1908.

[127] Matthiessen and Holtzmann: *Pogg. Ann.,* 110, 229, 1860.

[128] Hampe: *Chemiker, Ztg.,* 726, 1892.

FIG. 39.



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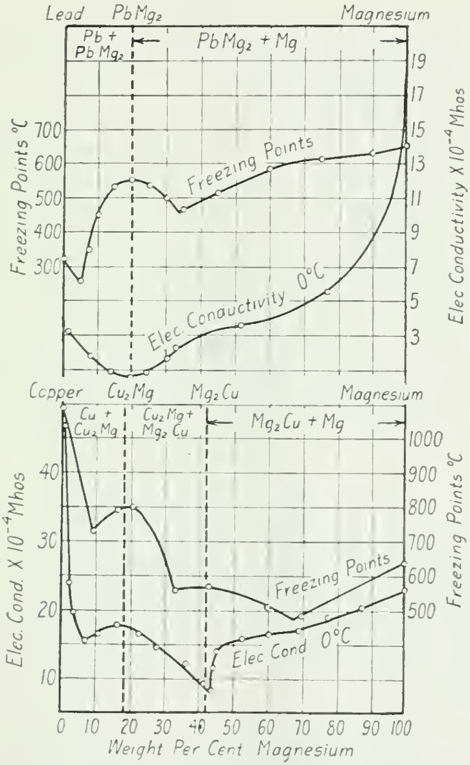
[129] Puschin and Dischler: *Zeit. anorg. Chem.*, 80, 65, 1913.

Lead-Magnesium.

The freezing point curve[130] (Fig. 40) shows a maximum corresponding to the compound Mg_2Pb with a eutectic on either side. For concentrations of magnesium less than that corresponding to the compound the alloys are heterogeneous mixtures of lead and Mg_2Pb and for greater concentrations they are mixtures of the compound Mg_2Pb and magnesium.

FIG. 40.



The curve for electrical conductivities by Stepanow[131] (Fig. 40) shows that the electrical conductivity decreases rapidly with the addition of lead to magnesium until the alloy contains about 5 per cent. lead. At the concentration corresponding to the compound Mg_2Pb the electrical conductivity passes through a minimum.

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[130] Grube: *Zeit. anorg. Chem.*, 44, 117, 1905.

[131] Stepanow: *Zeit. anorg. Chem.*, 78, 12, 1912.

Magnesium-Copper.

The freezing point curve (Fig. 40) as determined by Urasow[132] and also Sahem[133] shows two maxima corresponding to the intermetallic compounds Cu_2Mg and $CuMg_2$. There are, therefore, in the equilibrium four types of crystalline substances to be considered: Cu, Cu_2Mg, $CuMg_2$ and Mg. None of these substances seems able to form solid solutions with any of the others. The alloys then become divided into three groups, mixtures of Cu with Cu_2Mg; mixtures of Cu_2Mg with $CuMg_2$ and mixtures of Mg and $CuMg_2$.

The electrical conductivity has been measured by Stepanow.[134] He also gives data from which the temperature coefficient can be calculated. These data are, however, very irregular and have not been plotted. Between 100 and 45 per cent. magnesium the curve is nearly a straight line, as it should be for a mechanical mixture of Mg and $CuMg_2$. Between 43 and 20 per cent. magnesium it is again a straight line corresponding to the mixture of $CuMg_2$ and Cu_2Mg. The early part of the curve for small concentrations of magnesium is steep and suggests a solid solution rather than a mechanical mixture.

Antimony-Tellurium.

The freezing point curve of this series by Fay and Ashley[135] (Fig. 41) gives a maximum corresponding to the compound Sb_2Te_3. There is also a eutectic for which the temperature is 421° C. The compound forms a continuous series of mixed crystals with antimony but does not mix in the same way with pure tellurium.

The electrical conductivity and the thermoelectric power for this series have been measured by Haken.[136] The addition of tellurium to antimony causes a rapid decrease in both the electrical conductivity and the thermoelectric power. Both of these quantities pass through a minimum and rise rapidly to their value for the compound Sb_2Te_3. The course of these curves between 0 and 60 per cent. tellurium is typical of alloys formed of two crystalline phases, forming solid solutions. The compound is clearly marked on both curves. The magnetic susceptibility of these alloys has been studied by Honda.[137] Between 100 and 62 per cent. tellurium the curve is nearly a straight line. Over this region the susceptibility varies little. The compound is marked by a sudden change in the direction of the curve at the concentration of the compound.

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[132] Urasow: *Jour. russ. Chem. Ges.*, 39, 1566, 1909.

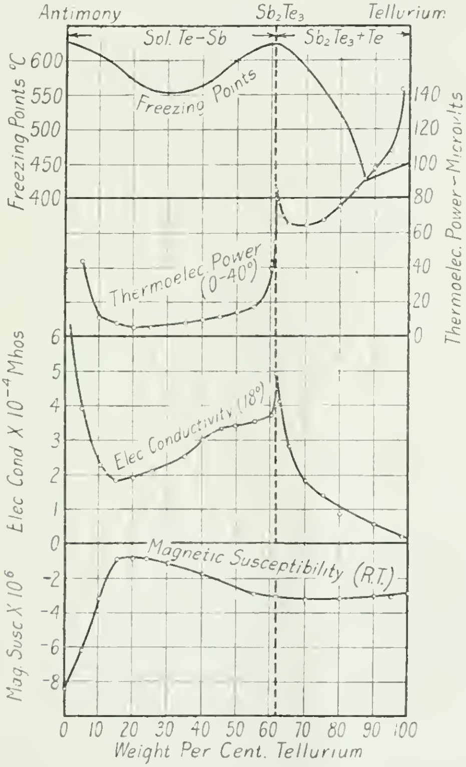
[133] Sahem: *Zeit. anorg. Chem.*, 57, 3, 1908.

[134] Stepanow: *Zeit. anorg. Chem.*, 78, 20, 1912.

[135] Fay and Ashley: *Amer. Chem. Jour.*, 27, 1902.

[136] Haken: *Ann. d. Phys.*, 32, 291, 1910.

FIG. 41.



Antimony-Aluminium.

The freezing point curve (Fig. 42) by Gautier[138] has two maxima. One corresponds to the compound AlSb, but the existence of a second compound is questioned.

The diamagnetic susceptibility of these alloys has been contributed by Honda.[139] The curve showing the susceptibility as a function of the concentration is made up of two straight lines which intersect where the alloy contains 18.4 per cent. aluminium, *i.e.*, at the concentration for the compound AlSb. The character of the susceptibility curve indicates that alloys containing less than 18.4 per cent. aluminium are mechanical mixtures of antimony and the compound AlSb, and those containing more than 18.4 per cent. aluminium are mixtures of aluminium and the compound AlSb.

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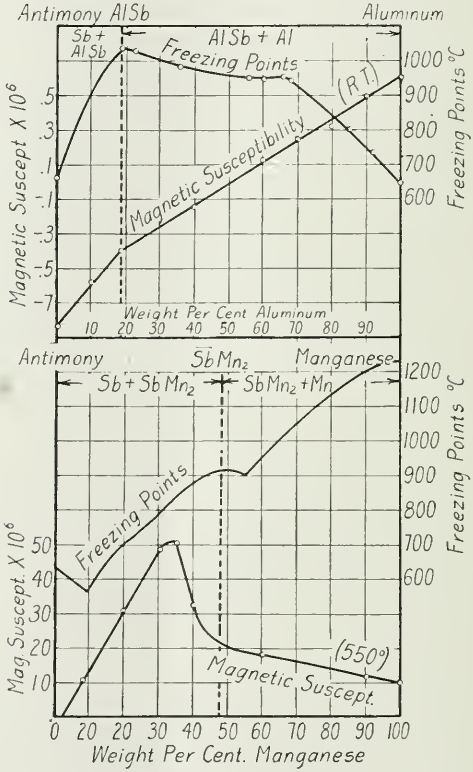
[137] Honda: *Sci. Rept. Imp. Univ. Tokio,* **2**, 9, 1913.

[138] Gautier: " Contribution a l'etude des alleages," **112**, 1901.

FIG. 42.



Antimony-Manganese.

The compound $SbMn_2$ is indicated on the freezing point curve (Fig. 42) by Williams.[140] Between 100 and about 52 per cent.

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[139] Honda: *Sci. Rept. Imp. Univ. Tokio*, 2, 9, 1913.

[140] Williams: *Zeit. anorg. Chem.*, 55, 1, 1907.

manganese the alloys are heterogeneous mixtures of Mn and the compound $SbMn_2$; between 0 and about 31 per cent. manganese they are a mixture of antimony and a secondary crystalline phase.

The thermal and electrical properties of these alloys have not been studied. The magnetic susceptibility at 550° C. has been determined by Honda.[139] His observations were made with a field of 10.9 kilogausses. The susceptibility for this temperature is nearly independent of the temperature. From 0 to 31.2 per cent. manganese the susceptibility is nearly a linear function of the concentration. In this region the alloys are mechanical mixtures of Sb and $SbMn_2$. Between 31.2 and 40.7 per cent. manganese the character of the susceptibility curve changes because a new crystalline phase appears. Between 40.7 and 47.8 per cent. Mn there is another straight line portion, followed by another linear portion from 50.5 to 100 per cent. Mn, the region over which the alloys are heterogeneous mixtures of manganese and a second crystalline phase.

Antimony-Zinc.

The freezing point curve for this series as determined by Monkmeyer [141] (Fig. 43) has two maxima, one corresponding to the compound SbZn and the other to the compound Sb_3Zn_2. The alloys divide themselves into three groups—heterogeneous mixtures of Zn and Sb_3Zn_2, mixtures of Sb_3Zn_2 and SbZn and mixtures of SbZn and Sb. The structure of the series of alloys is very similar to the structure of the antimony-cadmium series.

The curve for the magnetic susceptibilities by Honda [142] consists essentially of two straight lines intersecting at the concentration for the compound SbZn. Over the region where there is supposed to be a mixture of Sb_3Zn_2 and SbZn the curve departs somewhat from a straight line. The linear relation between the susceptibility and the concentration confirms the existence of the mechanical mixtures indicated by thermal analysis. The curves for the specific resistance, the thermoelectric power and the Hall constant [143] show clearly the compound SbZn, but they give no evidence of the compound Sb_3Zn_2.

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[141] Monkmeyer: *Zeit. anorg. Chem.*, 43, 182, 1905.

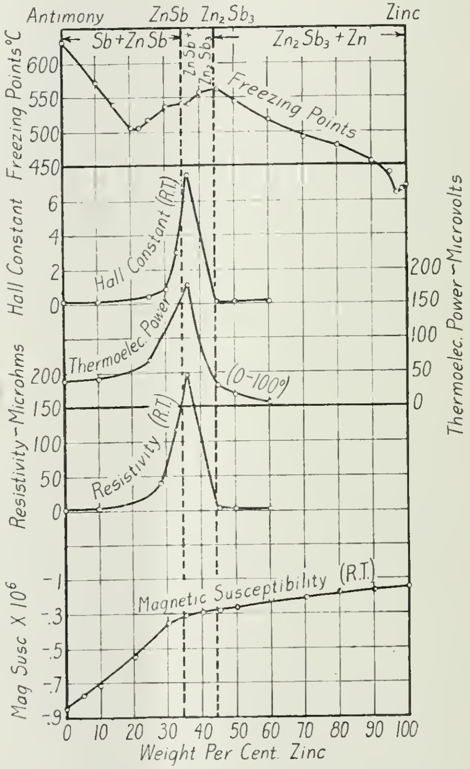
[142] Honda: *Sci. Rept. Imp. Univ. Tokio*, 2, 6, 1913.

[143] Smith: *Phys. Rev.*, 32, 178, 1911.

Antimony-Cadmium.

The freezing point curve (Fig. 44) by Treitschke[144] gives a compound SbCd which is stable and another Sb_2Cd_3 which is probably unstable. Between 0 and about 40 per cent. antimony the alloys are mixtures of Cd and Sb_2Cd_3; between 40 and 52 per cent. mixtures of SbCd and Sb_2Cd_3 and between 52 and 100 per cent. mixtures of SbCd and Sb.

FIG. 43.



The curves for the specific resistance, temperature coefficient and thermal conductivity are from the observations of Eucken and Gehlhoff.[145]

The thermoelectric power of this series has been frequently studied. The curve of thermoelectric powers in Fig. 44 is from

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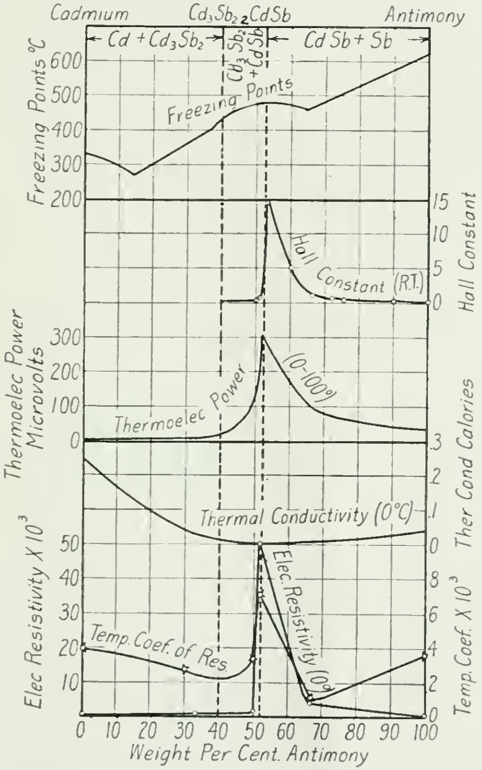
[144] Treitschke: *Zeit. anorg. Chem.*, 50, 217, 1906.

[145] Eucken and Gehlhoff: *Verh. d. deut. Phys. Ges.*, 169, 1912.

Haken.[146] The Hall constant is by the author.[147] The position of the compound is very evident on all of these curves except the curve for the thermal conductivities. On either side of this compound and at some distance from the concentration at which it appears neither the resistance, nor the thermoelectric power, nor the Hall constant changes rapidly with a change in the composition of the alloys.

FIG. 44.



Magnesium-Silver.

The freezing point curve[148] (Fig. 45) gives one maximum at the concentration for the compound MgAg. There is probably a second compound Mg_3Ag. Between 0 and 8.5 per cent. magnesium there is a solid solution of magnesium in silver followed by a mechanical mixture of this solid solution and the compound MgAg. There then follows a region in which there is a mixture of Mg_3Ag and MgAg. Between 0 and 60 per cent. silver the alloys are mixtures of Mg and Mg_3Ag.

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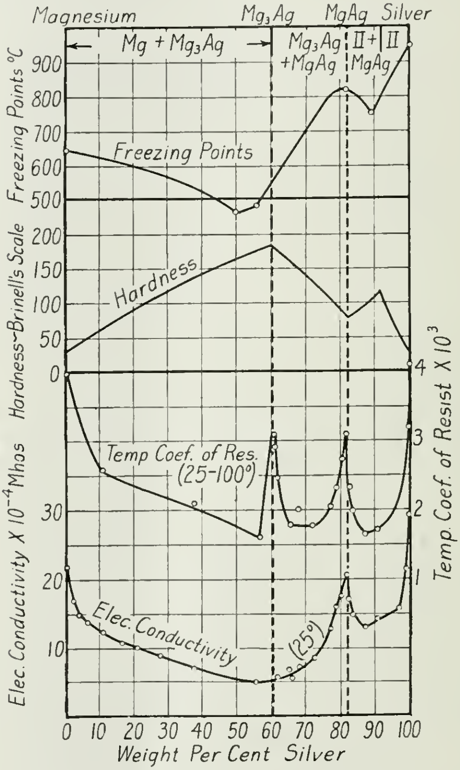
[146] Haken: *Ann. d. Phys.*, 32, 291, 1910.

[147] Smith: *Phys. Rev.*, 32, 178, 1911.

[148] Zemczuzny: *Zeit. anorg. Chem.*, 49, 403, 1906.

FIG. 45.



The curve for the hardness[149] of these alloys shows these regions quite clearly. It is made up of four straight lines. One of these lines extends from 0 to 60 per cent. silver, over the region in which the alloys are mechanical mixtures of Mg and Mg_3Ag; the next from 60 to 82 per cent. silver where there is a mixture of MgAg and Mg_3Ag; the third one from 82 to 91 per cent. silver where there is a mixture of MgAg and a saturated solid solution of magnesium in silver; the last from 91 to 100 per cent. silver, the region of the solid solutions of magnesium in silver. The curves for the electrical conductivity and the temperature coefficient are by Smirnow and Kurnakow.[150] The positions of both compounds are marked on these curves. The initial rapid decrease in the electrical conductivity and the temperature coefficient for fairly small concentrations of magnesium in silver is in agreement with the conception that solid solutions are formed over this interval. The nearly linear course of these curves between 10 and 58 per cent. silver follows from the fact that the alloys are mechanical mixtures of two crystalline phases over this interval.

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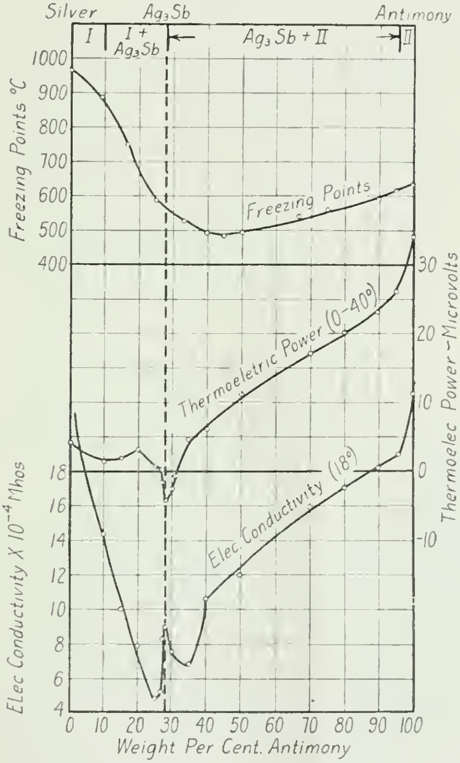
[149] Desch: "Intermetallic Compounds," p. 15.

FIG. 46.



Silver-Antimony.

Petrenko[151] gives the freezing point curve (Fig. 46). By a sudden change in curvature it suggests the compound Ag_3Sb. A solid solution of silver in antimony is formed until the concentration of the silver becomes about 4 per cent. and a solid solution of antimony in silver until the concentration of antimony is about 10 per cent. Between about 10 and 28 per cent. antimony there is a mechanical mixture of Ag_3Sb and a saturated solid solution of antimony in silver and between 28 and 96 per cent. a mixture of Ag_3Sb and a saturated solution of silver in antimony.

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[150] Smirnow and Kurnakow: *Zeit. anorg. Chem.*, 72, 31, 1911.

[151] Petrenko: *Zeit. anorg. Chem.*, 50, 1906.

The curve for the thermoelectric heights as well as that for the electrical conductivities has been worked out by Haken,[152] and these curves are reproduced in Fig. 46. The compound is clearly indicated and the region in which silver is soluble in antimony is marked by the steepness of both curves in this region. In like manner the interval in which antimony is soluble in silver is indicated by the characteristic drop in the electrical conductivity. The decrease in the thermoelectric power for small concentrations of antimony is less marked.

Aluminium-Copper.

By thermal analysis Gwyer[153] finds evidence of three compounds: Al_2Cu, AlCu and $AlCu_3$ (Fig. 47). The compound Al_2Cu is decomposed before fusion. Between 0 and 9 per cent. copper there are solid solutions of aluminium in copper, followed by a region of mixed crystals of Cu_3Al and CuAl; a second region of mixtures of CuAl and $CuAl_2$, and a third region of mixtures of $CuAl_2$ and a saturated solution of copper in aluminium.

A number of observations have been made on the electrical properties of these alloys. Among the earlier observations in this region are those made by LeChatelier,[154] Dewar and Flemming,[155] Battelli,[156] Steinmann,[157] Pouchine[158] and Pecheux.[159] The curves in Fig. 47 have been plotted from the data given by Broniewski.[160] On the curve for electrical conductivity and also on the one for the temperature coefficient the three compounds are indicated by a change in the slope of the curves at the concentrations corresponding to the compounds. On the curve for the thermoelectric power and also on the curve for the variations of the thermoelectric power with the temperature the compounds Al_2Cu and $AlCu_3$ are marked by minima in the curves. Over the region between 91 and 100 per cent. copper where solid solutions of aluminium in copper are formed the electrical conductivity and the temperature coefficient show the decreases characteristic of such solutions.

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[152] Haken: *Ann. d. Phys.*, 32, 291, 1910.

[153] Gwyer: *Zeit. anorg. Chem.*, 57, 113, 1908.

[154] LeChatelier: *C. R.*, 101, 454, 1890.

[155] Dewar and Flemming: *Phil. Mag.* (5), 36, 271, 1893.

[156] Battelli: *Atti. R. Inst. Veneti.* (6), 5, 1148, 1886–7.

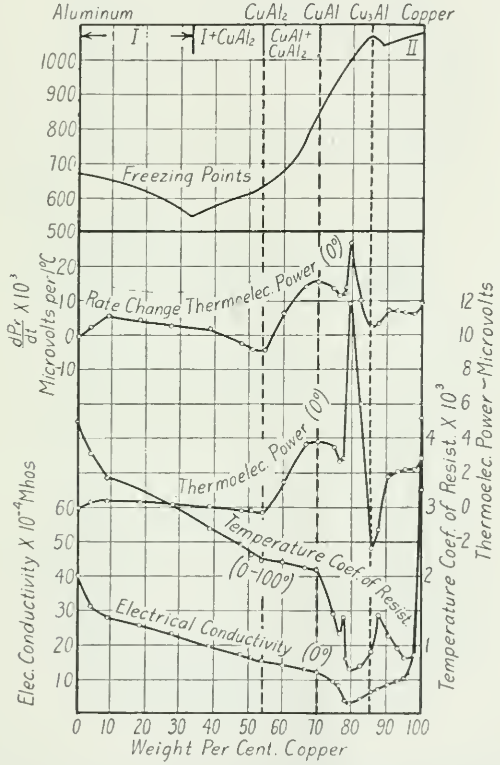
[157] Steinmann: *C. R.*, 130, 1300, 1900.

[158] Pouchine: *Jour. Soc. phys. Chem. russ.*, 39, 528, 1907.

[159] Pecheux: *C. R.*, 148, 1041, 1909.

[160] Broniewski: *Ann. de Phys. et Chem.* (8), 25, 91, 1912.

FIG. 47.



Aluminium-Silver.

The freezing point curve (Fig. 48) by Petrenko[161] gives two compounds, $AlAg_2$ and $AlAg_3$. These compounds are indicated by a change in the curvature of the freezing point curve at the concentration at which the compound occurs. This change in curvature is not very evident from the curve as plotted in Fig. 48. Between 0 and 87.5 per cent. silver the alloys are mechanical mixtures of aluminium and the compound $AlAg_2$; between 87.5 and 91.5 per cent. silver mixed crystals of $AlAg_2$ and $AlAg_3$; between 91.5 and 94 per cent. silver mixtures of $AlAg_3$ and saturated solid solutions of aluminium in silver, and between 94 and 100 per cent. silver solid solutions of aluminium in silver.

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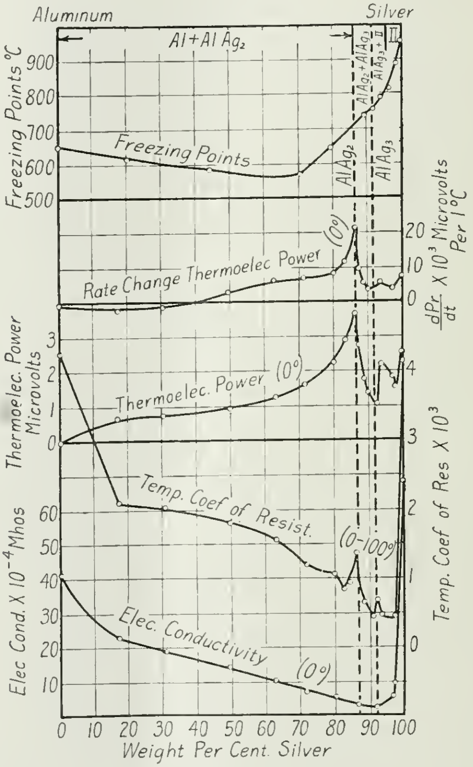
[161] Petrenko: *Zeit. anorg. Chem.*, 46, 49, 1905.

FIG. 48.



The curves for the electrical conductivity, temperature coefficient, thermoelectric power and variations of the thermoelectric power with the temperature have been taken from the observations of Broniewski.[162] The compound $AlAg_2$ is marked clearly by a peak in each of the curves except the curve for the electrical conductivity where the peak is small. On the other hand, a minimum occurs in each of the curves except the curve for electrical conductivities where the concentration corresponds to the compound $AlAg_3$. Except for the two peaks in the curve for the temperature coefficient that curve as well as the curve for electrical conductivities has the form which is found in binary alloys of metals which form limited solid solutions with each other and then these solid solutions mix mechanically to form the remainder of the alloys of the series.

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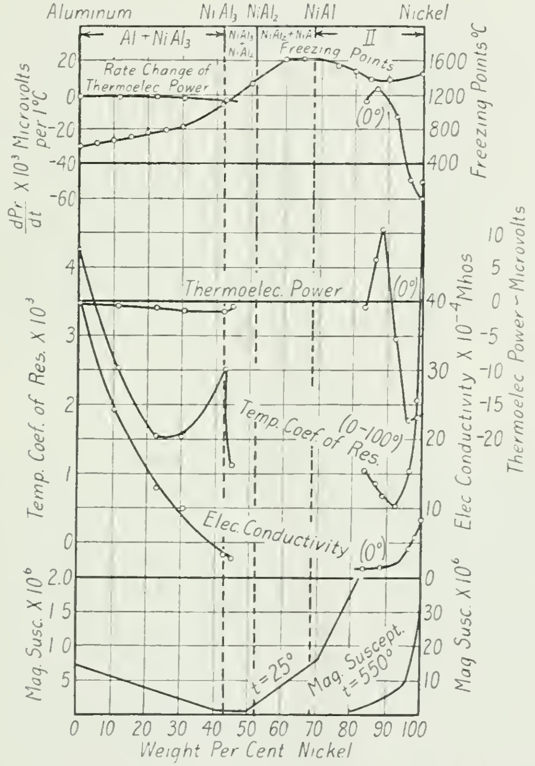
[162] Broniewski: *Ann. de Phys. et Chem.* (8), 25, 83, 1912.

FIG. 49.



Aluminium-Nickel.

The freezing point curve (Fig. 49) by Gwyer [163] gives a maximum near the concentration corresponding to the compound NiAl. Between 0 and 42 per cent. nickel the alloys are mixtures of Al and $NiAl_3$; between 42 and 52 per cent. nickel mixtures of $NiAl_3$ and $NiAl_2$; between 52 and 68 per cent. nickel mixtures of $NiAl_2$ and NiAl, and between 68 and 100 per cent. nickel an unsaturated solution of aluminium in nickel.

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[163] Gwyer: *Zeit. anorg. Chem.*, 57, 133, 1908.

The electrical conductivity, the temperature coefficient, the thermoelectric power and its variation with the temperature have been plotted from the observations of Broniewski.[164] In the interval between 45 and 84 per cent. nickel there are no observations on account of the brittleness of the alloys in this region. The compound $NiAl_3$ is indicated by a peak on the temperature coefficient curve. The magnetic susceptibilities by Honda [165] have been measured at 25° C. for alloys containing less than 80 per cent. nickel. For alloys containing more than 80 per cent. nickel the susceptibilities were determined at 550° C. A magnetic field of from 5 to 12 kilogausses was used. Until the alloys contain 80 per cent. nickel the susceptibility curve consists of four straight lines intersecting at the concentrations at which a new crystalline phase appears.

Nickel-Tin.

The composition of this series of alloys is very complex. The freezing point curve by Gautier [166] has been reproduced in Fig. 50. Five different kinds of crystals are present in the solidified alloys.

No observations have been found on the thermal and electrical properties of these alloys. The magnetic properties have been studied by Honda.[165] On the alloys containing less than 60 per cent. nickel the observations were made at 25° C. On the remainder of the alloys the observations were made at 550° C.—a temperature above the transformation point. The external magnetic field was from 5 to 12 kilogausses. The ferro-magnetic properties disappear when the concentrations of the constituents correspond to the compound Ni_3Sn. So long as the alloys are composed of the same two kinds of crystals in varying concentrations the susceptibility is a linear function of the concentration. Where one type of crystal disappears and is replaced by another the slope of the curve suddenly changes.

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[164] Broniewski: *Ann. de Chem. et Phys.* (8), **25**, 108, 1912.

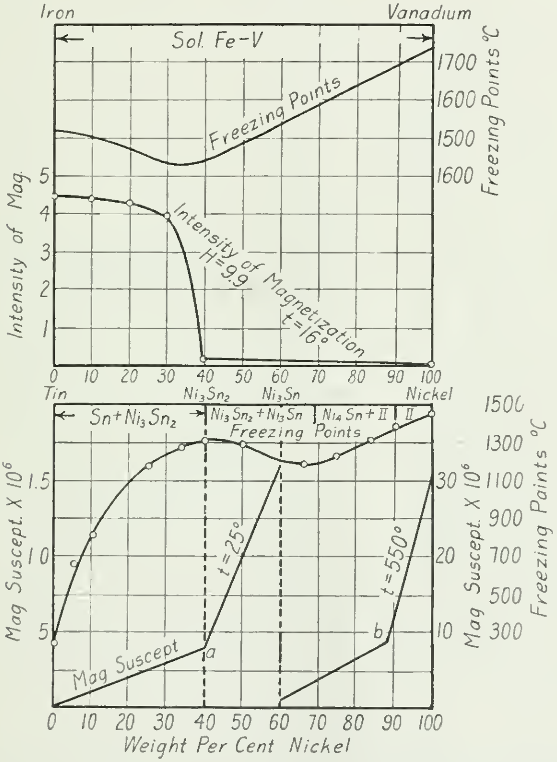
[165] Honda: *Ann. d. Phys.*, **32**, 1015, 1910.

[166] Gautier: *C. R.*, **122**, 109.

Iron-Vanadium.

The composition of iron and vanadium alloys has been studied by Vogel and Tammann,[167] and it has been found that they solidify in the form of an unbroken series of mixed crystals. The freezing point curve (Fig. 50) has a minimum in the neighborhood of 35 per cent. vanadium.

FIG. 50.



The electrical and thermal properties of this series of alloys do not seem to have been studied. There are observations by Honda[168] on the intensity of magnetization. These observations were made at 16° C. and with a magnetic field of 9.9 gausses. The intensity of magnetization of iron decreases slowly with an increasing concentration of vanadium until that concentration is reached at which the freezing point curve has its minimum. Here there is an extraordinarily rapid decrease in the intensity of magnetization so that an alloy containing more than 40 per cent. of vanadium is very feebly magnetic.

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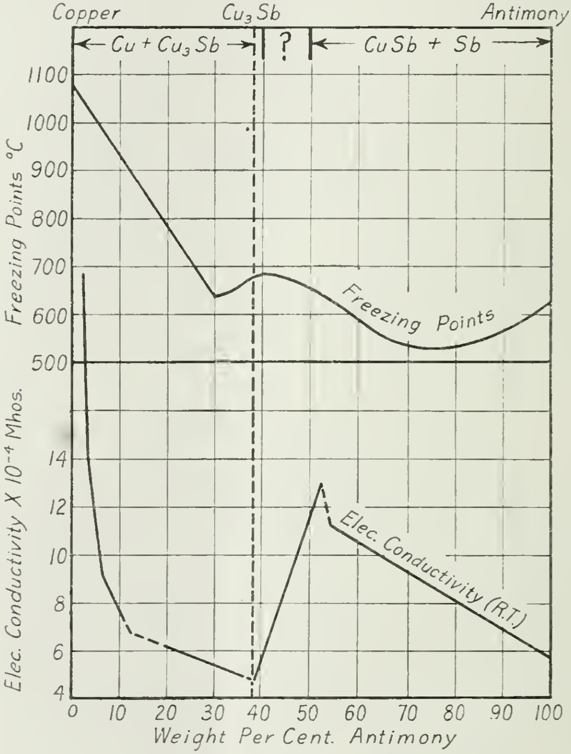
[167] Vogel and Tammann: *Zeit. anorg. Chem.*, 58, 79, 1908.

[168] Honda: *Ann. d. Phys.*, 32, 1910, 1912.

FIG. 51.



Copper-Antimony.

Baikow[169] has given a complete equilibrium diagram from which the freezing point curve (Fig. 51) is taken. The compound Cu_3Sb is formed. Alloys to the right or left of this compound are heterogeneous mixtures of two crystalline phases. Copper, according to Stead, dissolves about 0.3 per cent. antimony.

For small concentrations of antimony the electrical conductivity curve by Matthiessen[170] is very steep. This is in the region where antimony and copper form dilute solid solutions. The electrical conductivity curve shows a sudden change in direction where the concentrations correspond to the compound Cu_3Sb. Between 55 and 100 per cent. antimony the conductivity curve is a straight line. Over this interval the alloys are mechanical mixtures. A third region of mechanical mixtures is indicated by the straight line representing the electrical conductivity between 38.4 and 52 per cent. antimony.

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[169] Baikow: *Jour. reiss. Phys. Chem. Ges.*, 36, 111, 1904.

[170] Matthiessen: *Pogg. Ann.*, 110, 190, 1860.

CHANGE OF THERMOELECTROMOTIVE FORCE WITH TEMPERATURE.

The rate of variation of the thermoelectromotive force with the temperature for a large number of aluminium alloys has already been discussed. The observations on these alloys were made by Broniewski. In his work the observations were extended over a limited range of temperature. The most important work in this connection is that of Giebel,[171] in which the observations were extended over a large range of temperature. He studied the following series of alloys: Palladium-gold, palladium-platinum, palladium-silver, platinum-silver, platinum-rhodium, and platinum-iridium. The thermoelectromotive forces were measured against platinum in this case and the temperature of the cold junction was kept at 0° C. Observations were made at intervals of 100° C. between 0° and 900° C. The observed thermoelectromotive forces have been plotted against the temperatures in Figs. 52, 53 and 54. From Figs. 52 and 54 it is seen that the higher the temperature the more rapid is the rate of change of the thermoelectromotive force with the temperature for palladium-silver and for platinum-silver alloys. In the palladium-platinum series (Fig. 53) there is nearly a linear relation between the thermoelectromotive force and the temperature after the alloy contains about 40 per cent. platinum. When the concentration of platinum is increased above 40 per cent. this linear relation is more nearly realized.

In Fig. 55 the thermoelectromotive force in platinum-rhodium alloy at a particular temperature has been plotted against the concentration of rhodium in the alloy. These curves show that the thermoelectromotive force increases rapidly with the increase in the concentration of rhodium until the alloy contains about 5 per cent rhodium. Between 5 and 10 per cent. rhodium the increase is much less rapid, especially at low temperatures. Increasing the concentration of rhodium beyond 10 per cent. changes only slightly the thermoelectromotive force until the temperature of the hot junction exceeds 1000° C. At very high temperatures the thermoelectromotive force continues to increase with increasing concentration of rhodium, but this rate of increase decreases with increasing concentration of rhodium.

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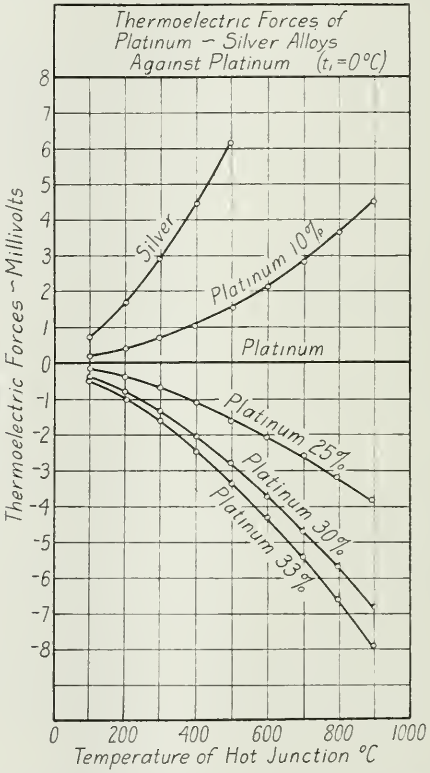
[171] Giebel: *Zeit. anorg. Chem.*, **69**, 38, 1910, and **70**, 240, 1911.

FIG. 52.



The relation between the thermoelectromotive force and the difference in temperature between the junctions may be expressed by an equation of the form

$$E = a t + b t^2 + c t^3$$

where one junction is kept at 0° C. and the other at t° C. For a large number of metals and alloys it has been found that only the second power of the temperature need be considered. The equation then becomes, to a very good approximation,

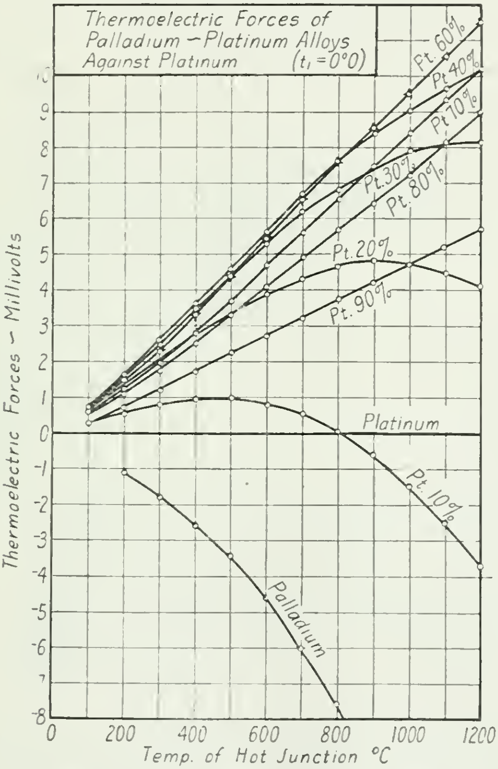
$$E = a\,t + b\,t^2.$$

In such a case the rate of variation of the thermoelectromotive force with the temperature becomes

$$\frac{d\,E}{d\,t} = P = a + 2\,b\,t.$$

This equation states that the thermoelectric power is a linear function of the difference of temperature between the junctions. In some of the platinum-palladium alloys it has already been seen to be independent of the temperature. The rate of variation of the thermoelectric power with the temperature is

FIG. 53.



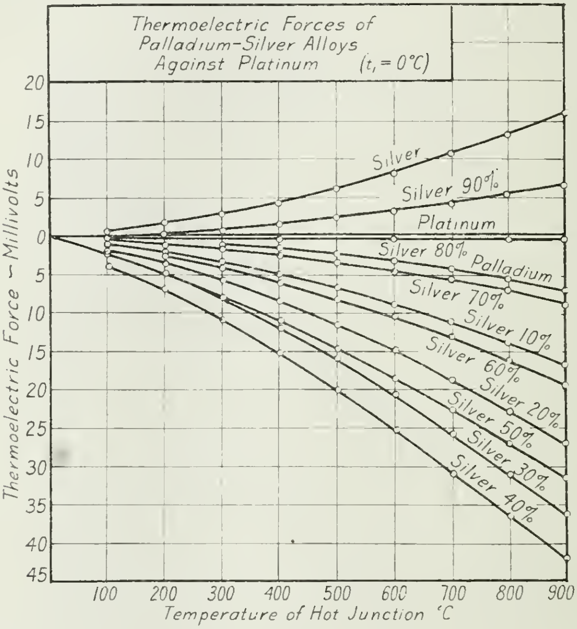
$$\frac{d\,P}{d\,t} = 2\,b.$$

In so far as the approximation introduced above is correct the variation of the thermoelectric power with the temperature is the same for all temperatures.

THEORIES OF RESISTANCE AND THERMOELECTROMOTIVE FORCES.

Lord Rayleigh[172] and Liebenow,[173] independent of each other, came to the conclusion that the increase in specific resistance which alloys show in excess of the resistance calculated from the resistance of their components may be attributed to the thermoelectromotive forces which arise between the junctions of the metals forming the alloys. According to these theories, the electrical current passing from one layer of metal to another layer of the other metal in the alloy develops or absorbs heat at the surface of contact between the components of the alloy, on account of the Peltier effect. These temperature differences cause thermoelectromotive forces to be set up in the alloy in such a way that they are equivalent to a large number of small cells connected in series, so that they oppose the flow of the current through the substance. They have, therefore, the effect of an added resistance. Since the difference in temperature between the contacts is proportional to the current flowing in the conductor, and since the thermoelectro-

Fig. 54.



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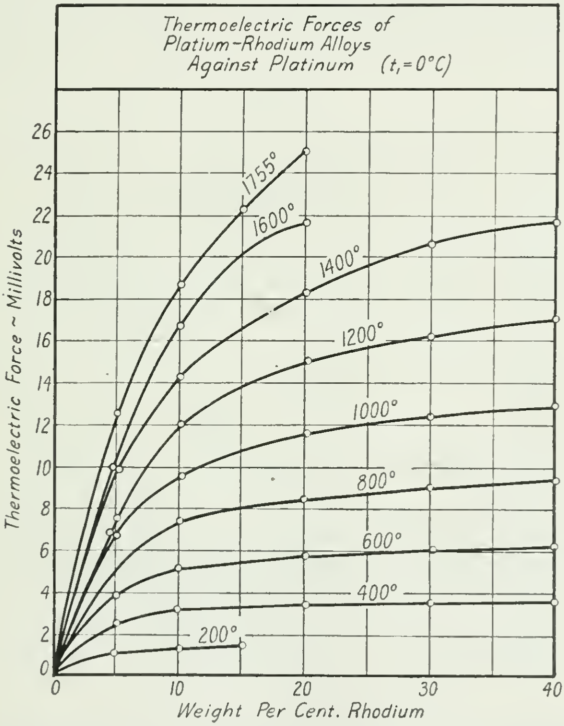
[172] Lord Rayleigh: *Nature*, 54, 154, 1896.

[173] Liebenow: *Zeit. Electrochem.*, 4, 201 and 217.

motive forces are proportional to the difference in temperature, this back electromotive force is proportional to the current. The opposition to the flow of current through the alloy from this cause will, therefore, behave like a resistance and it will be impossible to distinguish between the ordinary resistance and that which arises from these thermoelectromotive forces. It is of importance

FIG. 55.

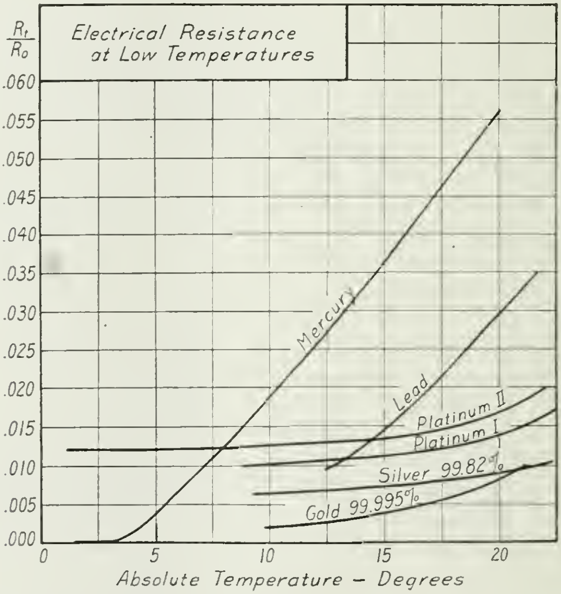


to note that where the constituents form compounds in which the resistance is high, it is necessary on the basis of this theory to assume a thermoelectromotive force between the molecules. Such an assumption is not very probable.

Some evidence for the correctness of this theory is found in the consideration of the way in which the electrical resistance of metals and alloys behaves at extremely low temperatures. When the temperature of a pure metal is decreased, the electrical resist- temperature, until the temperature is 3 or 4 degrees above the ance is found to be very accurately proportional to the absolute

absolute zero where the superconducting state appears and the electrical resistance almost entirely disappears. If, on the other hand, the temperature of an impure metal or alloy is decreased indefinitely, the resistance does not disappear at the absolute zero, but approaches a constant value. This suggests that there is in the resistance of an alloy an added constant term which does not disappear at the absolute zero. The theory of Lord Rayleigh leads at once to the existence of such a term. The added resistance arising from the causes considered in this theory does not seem large enough to account for the very high resistance of some alloys.

FIG. 56.



By reference to Fig. 56 it will be seen that the curve showing the relation between the electrical resistance of mercury and its temperature is nearly a straight line except near the origin. The curve for pure lead is also observed to be a straight line, but the corresponding curves for metals containing small admixtures of other metals do not seem to pass through the origin when prolonged backward. In such cases the electrical resistance seems to approach a constant value which persists to the absolute zero. Such a constant value is to be expected on the basis of the theory proposed by Lord Rayleigh and may, therefore, be taken as partial evidence of the correctness of that theory. From the above point of view it is clear that it is impossible to prepare a good conductor by mixing two or more metals with each other. There will always be present this added resistance which makes the alloy at least not a better conductor than the constituents of which it is composed. Ordinarily the alloy is found to have a lower conductivity than would be calculated from its constituents by the additive law.

This theory of Lord Rayleigh may be stated more fully as follows:

Let R_o = the resistance of the alloy at 0° C.
γ = the temperature coefficient of resistance of the alloy.
R = the resistance of the alloy at 0° C. calculated from its constituents by the additive law.
α = the temperature coefficient of the alloy calculated by the additive law.
R_c = the contact resistance occurring at the surface of two metals forming the alloy.
β = the temperature coefficient of the constant resistance.

Then,

$$R_o\,(l+\gamma\,t) = R\,(l+\alpha\,t) + R_c\,(l+\beta\,t).$$

The resistance R_c is, according to Lord Rayleigh and Liebenow, caused by thermoelectromotive forces.

Where it is possible to assume that the temperature coefficients γ, α and β are independent of the temperature, it is possible to write,

$$R_o = R + R_c, \text{ and } 0^\circ \text{ C.},$$

and therefore,

$$R_o\,\gamma = R\,\alpha + R_c\,\beta,$$

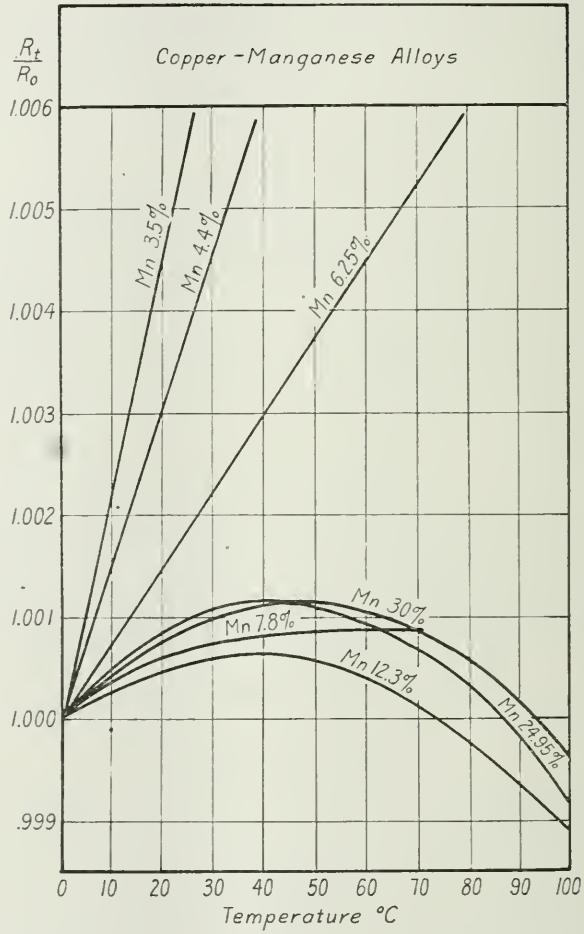
at any temperature.

Whence,

$$\gamma = \frac{R\,\alpha + R_c\,\beta}{R_o}\,.$$

From this equation it is seen that γ may be negative when β is negative and $R_c\,\beta > R\alpha$. The temperature coefficient of alloys has been found negative in some cases. A well-known example is the case of copper-manganese alloys. Curves showing the ratio of the resistance at any temperature to the resistance at zero for some copper-manganese alloys have been reproduced in Fig. 57. They are taken from the work of Guertler.[174] For an alloy containing 12.3 per cent. of manganese the temperature coefficient is positive above and negative below 40° C.

FIG. 57.



If it is possible to assume that $\beta = o$, that is, that the contact resistance is independent of the temperature,

$$R_o\,(l + \gamma\,t) = R\,(l + \alpha\,t) + R_c,$$

---

[174] Guertler: *Jahr. der Rad. und Elekt.*, 5, 17, 1908.

and since

$$R_0=R+R_c,$$
$$R_0\gamma=R\alpha$$
$$\frac{R_0}{R}=\frac{\alpha}{\gamma}.$$

This equation is equivalent to the empirical rule stated by Matthiessen and Vogt; namely, that the observed temperature coefficient of the resistance divided by that calculated from the additive law is equal to the observed electrical conductivity divided by the electrical conductivity calculated from the additive law. In so far as this rule may be accepted the curve for the electrical conductivity of a series of alloys ought to be the same as the curve for the temperature coefficient of that same series of alloys. In the preceding pages it has been seen that in very many cases there is a parallelism between the curve for the electrical conductivity and that for the temperature coefficient of the resistance. This parallelism offers satisfactory proof of the correctness of this rule in many cases.

The theory of Lord Rayleigh also leads to the second rule stated by Matthiessen and Vogt. Since

$$R_0(+l\gamma t)=R(l+\alpha t)+R_c,$$

and

$$R_0=R+R_c,$$
$$R_c=R_0-R=$$

difference between observed and calculated resistance at 0° C., and

$$R_c=R_0\,(l+\gamma t)-R\,(l+\alpha t)=$$

the difference between the observed and calculated resistance at t° C. Since each of these differences is equal to the contact resistance which has been assumed independent of the temperature, these differences ought to be the same for all temperatures. In other words, the difference between the observed and the calculated resistance is the same whatever the temperature. The large number of cases in which this rule has been found to be verified gives evidence that the temperature coefficient of the contact resistance is either zero or very small.

The increase in the electrical resistance of solid solutions over the resistance of the pure metals of which they are composed does not find an easy explanation on the basis of the electron theory of metallic conduction. One assumption which has been made to explain this increase is that the number of free electrons in the alloy is much less than in pure metals. This assumption when considered in connection with Wiedemann and Franz's law does not lead to satisfactory results. The departures from this law, which states that the ratio of the thermal to the electrical conductivity is a constant for any particular temperature, are greater for alloys than for pure metals. These departures are always of such a nature that the electrical conductivity has been decreased more than the thermal conductivity by the formation of the alloy. This, with other considerations, has lead Schenck [175] to suggest that the increase in the electrical resistance of the alloy over the value calculated by the additive law could be accounted for by assuming that it arises from an increase in the frictional resistance which the electrons encounter in their motion through the alloy. This increase in frictional resistance to the motion of the electrons may be thought of as produced in a way very analogous to the way in which on the basis of the kinetic theory the addition of one gas to another causes an increase in the viscosity. Schenck thinks that the slowly diffusing molecules of the added metal hand on their energy and thus participate in the thermal conductivity but not in the process of electrical conduction. For this reason the ratio of the thermal to the electrical conductivity in mixed crystals is greater than for pure metals, and this quotient increases with increasing concentration of the added metal. The ratio of these conductivities in alloys as in pure metals is approximately proportional to the absolute temperature.

According to the electron theory, the different concentration of the electrons in the two metals is the source of a diffusion current which is the cause of the thermoelectromotive force which arises when the junctions of two metals or alloys are at different temperatures.

Let $e =$ the thermoelectromotive force per degree difference in temperature.
$R =$ the gas constant in ergs.
$F = 965{,}450$ coulombs.
$N_a =$ number of electrons per cubic centimetre in A.
$N_b =$ number of electrons per cubic centimetre in B.

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[175] Schenck: *Ann. d. Phys.*, 32, 261, 1910.

Then,

$$e = \frac{R}{F} \operatorname{Log}_e \frac{Na}{Nb}.$$

Many alloys give such large values of the thermoelectromotive force against one of the pure metals of which they are composed that in the application of the above equation it is necessary to assume very large changes in the number of free electrons to be produced by adding one metal to the other. In order to avoid these improbable assumptions Schenck has introduced the assumption already referred to—that it is the friction of the free electrons rather than their number that is changed by alloying one metal with another. By using this assumption that there is an increase in the frictional resistance of the electrons in the alloys over the resistance which they experience in pure metals, Schenck has been able to derive a relation between the thermal and electrical conductivity of the alloy, the thermal and electrical conductivity of the pure metallic solvent, and the thermoelectric power of the alloy against the pure solvent.

Let k = thermal conductivity of the pure solvent.

σ = electrical conductivity of the pure solvent.

k' = the thermal conductivity of the alloy.

σ' = the electrical conductivity of the alloy.

π = the thermo-electromotive force of the solid solution against the pure solvent for 1° C. temperature difference between the junctions.

R = the gas constant.

e = the specific electrical charge = 96,540 coulombs.

Then for dilute solutions Schenck shows that

$$\pi = \frac{R}{2e} \operatorname{Log}_e \left(\frac{k'}{\sigma'} \div \frac{k}{\sigma} \right).$$

Some observations have been made by Bernoulli[176] to test the validity of this equation. The following table shows the kind of agreement which exists between the observed and calculated values.

---

[176] Bernoulli: *Ann. d. Phys.*, 33, 690, 1910.

Solvent.	Element in solution.	π Observed.	π Calculated.
Silver	2.73 per cent. Th.	2.8	2.6
	4.76 per cent. Th.	10.3	8.4
	4.00 per cent. Sn.	7.6	8.9
Cadmium.........	5.14 per cent. Hg.	2.8	2.2
	10.0 per cent. Hg.	2.6	3.0
Copper...........	5.00 per cent. Sn.	3.4	3.6
	3.11 per cent. Zn.	2.9	3.6
	5.00 per cent. Zn.	1.4	6.6
	3.94 per cent. Ni.	13.3	3.9
	17.30 per cent. Ni.	27.3	11.2

From this table it is seen that for dilute solutions fair agreement exists between the observed and the calculated values. The more concentrated the solution the less satisfactory is the agreement.

RESISTANCE AND HARDNESS.

A theory of electrical conductivity proposed by March[177] offers a possible explanation of the relation between the elastic properties and the electrical conductivity of alloys. According to this theory, the number of free electrons in a metal or alloy, and therefore the electrical conductivity, is in part, at least, determined by the characteristic frequency of vibration of the atoms about their positions of equilibrium. Whatever changes this characteristic frequency would change at the same time the elastic properties and the thermal and electrical properties. On the basis of this theory the number of free electrons decrease with increasing frequency of the characteristic vibration of the atoms. Whatever, therefore, decreases the frequency of the characteristic vibrations of the atoms increases the electrical conductivity of the substance. Hence, at the absolute zero where the characteristic frequency of vibration of the atoms is very small or else disappears, the electrical conductivity will be very large. This is, of course, exactly what is observed in metals at very low temperatures where they pass into the superconducting state.

The formation of a solid solution produces a change in the elastic properties which causes the characteristic frequency of vibration of the atoms to increase. This arises out of the fact that in an alloy in which the components *A* and *B* form mixed crystals, a molecule of *A* and a molecule of *B* act on each other with greater force than that with which a molecule of *A* acts on a molecule of *A* or a molecule of *B* on a molecule of *B*. There is on this account an increase in the cohesive forces and consequently an increase in the characteristic frequency of vibration of the atoms. With this increase in frequency is associated a decrease in the number of free electrons and a corresponding increase in the electrical resistance. From this point of view it is possible to see that where such elastic properties as hardness or tensile strength have maximum values, the electrical resistance will have a maximum value and the electrical conductivity a minimum value. Many illustrations of this have been seen in the preceding curves.

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[177] March: *Ann. d. Phys.*, 49, 710, 1916.

If the two components *A* and *B* of the alloy are insoluble in each other, the force which the molecule *A* exerts on another molecule *A* or the force which a molecule *B* exerts on another molecule *B* exceeds the force which a molecule *A* exerts on a molecule *B* or that which a molecule *B* exerts on a molecule *A*. Consequently each of these classes of molecules will form a group of crystals and the alloy will be a conglomerate formed of groups of crystals of the two constituents. In such an alloy the characteristics of each constituent are retained and the physical properties are additive. The components will, therefore, retain their characteristic vibrations and their elastic properties as well as the number of free electrons, and the electrical conductivity will change in conformity to the additive law. The hardness which increases with the frequency of the characteristic vibration of the atoms should in such cases be a linear function of the concentration of one of the constituents in the alloy. In such alloys the electrical conductivity will also be a linear function of the concentration of one of the constituents. Many cases of this kind have been noted in the preceding pages. This theory seems to offer a possible explanation of some of the interesting relations noted among the physical properties of alloys. Although the theory may not be satisfactory in many particulars, it is without doubt very suggestive.

Grateful acknowledgment is made to the Engineering Experiment Station of the Ohio State University for generous financial assistance in aid of this work.

PHYSICAL LABORATORY,
OHIO STATE UNIVERSITY.

THE OHIO STATE UNIVERSITY
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VOLUME 27 DECEMBER 30, 1922 NUMBER 7

THE DETERMINATION
OF
GUMS IN SUGAR PRODUCTS

BY
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AND
JAMES R. WITHROW

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COLUMBUS, OHIO

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INTRODUCTION

The manufacture and refining of sugar is an industry in Ohio. It is generally classified as a chemical industry. This industry in Ohio is confined to sugar from sugar beets. The output from this source in Ohio has steadily increased in volume during the last fifteen years. What its future will be in the economics of the State we do not know.

In common with industries of its kind, the sugar industry has numerous problems in one locality which are closely related to the problems in the same industry as it exists in other parts of the world. Hence though unlike each other in many respects, some of the fundamental problems in the beet sugar industry are exactly the same as the similar problems in the cane sugar industry. In some respects each of these industries is in turn, in advance of the other in its practice. Both of them, however, have difficulty at times in obtaining what the chemical examination of the raw products indicates as an adequate yield of commercial granulated sugar. Various explanations have been given for this fact. One of them has been that the presence of gum-like bodies is responsible for the failure of the expected amount of sugar to actually materialize at the end of manufacture. A strike-pan or evaporator full of syrup may be boiled and boiled at times without granulated sugar crystallizing. This partakes of the nature of a calamity to the sugar producer. His product appears to slip through his fingers.

Since it is felt by factory operators that gums are a factor in causing this difficulty, any method for determining gums therefore, would enable the refiner to have a better measure or grasp of the situation, a thing for which he always keenly seeks. Unfortunately the methods hitherto suggested for gum determination require many hours for their accomplishment, and are, therefore, so unsatisfactory as to be almost never used.

This work in which the Engineering Experiment Station of The Ohio State University has to some extent coöperated with financial help from the industry, has succeeded in modifying the methods previously proposed in such a way as to make them both accurate and low in time consumption. This gives, therefore, to both the beet sugar and the cane sugar men a new tool for the handling or study of the gum problem and its influence on sugar production and refinery operation—a tool which has not hitherto been available.

This work has already aroused interest, and favorable comment, in the United States, Porto Rico and Cuba among sugar men.

Because of the shortness of the beet sugar season in Ohio and the necessity for our work to continue throughout the year, we have used cane sugar products as a rule in this particular study. This, however, need not interfer with the value of the method developed when applied to beet sugar products. No cane sugar is manufactured or refined in Ohio.

Beet gums are at times of a highly sticky character and difficult to handle in a determination, but we have found the method developed in our work satisfactory on those beet sugar products examined by us.

Note.—We are also indebted in this work for coöperation by John C. Hebden, Consulting Chemical Fngineer, New York City, and for financial assistance from Mr. Arthur H. Lamborn of Lamborn and Company, New York City.

The Determination of Gums in Sugar Products[1]

By H. T. Ruff and James R. Withrow

Laboratory of Industrial Chemistry, Ohio State University,
Columbus, Ohio

Sugar boilers and others responsible for the production of refined from raw sugar recognize that different yields and rates of crystallization may be obtained from two raw sugars of similar sucrose, invert sugar and ash content, that have had the same treatment. It is generally conceded that this difference is due to the gums. The gums are presumed to be the strongest molasses formers among the organic nonsugars. This situation is also encountered in the manufacture of raw sugar from the cane.

The problem with which this work is concerned is the selection of a method for determining these gums that could be used for control work or to classify sugar products further than the usual sugar and ash determinations. Our work indicates:

1—Of the various methods proposed for gum determination, the precipitation by ethyl alcohol with hydrochloric acid acidification was found to be the most suitable for a rapid method adapted to control work.

2—The concentration of hydrochloric acid can be varied within rather wide limits without materially affecting the results, but the concentration of the alcohol must be kept within close limits.

3—With suitable precautions, denatured or methyl alcohol can be substituted for the ethyl alcohol.

4—The technic of the method can be simplified, so that its use is not prohibitive for control work.

THE METHODS for determining gum in sugar products may be classified into four groups, according to the precipitant used:

1—Hydrochloric acid-ethyl alcohol methods.[2]

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[1] Presented before the Division of Sugar Chemistry at the 63rd Meeting of the American Chemical Society, Birmingham, Ala., April 3 to 7, 1922.

Reprinted from The Journal of Industrial and Engineering Chemistry, December, 1922.

[2] L. B. Langguth, Steuerwald, and T. Van der Linden, *Arch. Suikerind.*, **22** (1914), 1033 [*C. A.*, **8** (1914), 3511]; J. J. Hazewinkel, *Ibid.*, **18** (1910), 746 [*C. A.*, **5** (1911), 1850]; *Ibid.*, **19** (1911), 313 [*C. A.*, **5** (1911), 244]; Weichmann, "Sugar Analysis," 3rd ed., p. 134, John Wiley & Sons, Inc., N. Y.; Noel Deerr, "Cane Sugar," **1911**, p. 482.

2—Calcium chloride-ethyl alcohol methods.[3]

3—Ferric chloride followed by reprecipitation with alcohol.[4]

4—Lead acetate in alcohol methods.[5]

In addition to these there are methods involving dialysis, but they could not be considered as control methods.

We checked representative methods from each group, using black strap molasses for the tests. Both ethyl and methyl alcohol were used. The following average results were obtained:

	Methyl Alcohol Per cent	Ethyl Alcohol Per cent
HCl-Alcohol (Weichmann)	2.27	2.53
$CaCl_2$-Alcohol (Bellier)	7.38	5.74
Fe_2Cl_6-(Auguet)	0.68	1.00
$Pb(C_2H_3O_2)_2$-Alcohol (Chauvin)	15.9	19.1

In the literature the HCl-alcohol method is generally conceded to give the most reliable results. Some of the methods are proposed only on a basis of their results on solutions of gums in water or pure sugar solution. This may help to account for the large variation that we have found.

For our purpose the $CaCl_2$-alcohol method is objectionable because the precipitation requires 24 hrs. The filtration and washing require more time than the HCl-alcohol method. This is the only case in which the methyl alcohol gives higher results than the ethyl. Possibly $CaCl_2.(CH_3OH)_x$ is formed.

Auguet's Fe_2Cl_6 method from a manipulative viewpoint is the most unsatisfactory of the four. It has these disadvantages: two precipitations and filtrations are involved. The first filtration requires longer than an entire HCl-alcohol gum determination. The gums from the Fe_2Cl_6 precipitation dissolve with difficulty in hot water, and this requires a comparatively large amount of water, which requires as much as three times the alcohol used in an HCl-alcohol precipitation. Compared with the other methods, this method gives abnormally low results.

The $Pb(C_2H_3O_2)_2$-alcohol method has the very obvious disadvantage that it precipitates many other organic nonsugars in addition to the alcohol-insoluble gums, when applied to a product like black strap. This method involves a lead correction and washing with alcohol and ether. It is slower than the HCl-alcohol method.

[3] Bellier, *Ann. fals.*, **3** (1910), 528 [*C. A.*, **5** (1911), 928].

[4] A. Auguet, *Ibid.*, **2** (1909), 136 [*Chem. Zentr.*, **11** (1909), 1011].

[5] A. C. Chauvin, *Mon. sci.*, [5] **1** (1911), 1, 317 [*C. A.*, **5** (1911), 3353]; *Ann. fals.*, **5** (1912), 27 [*C. A.*, **6** (1911), 1074]. X. Rocques and G. Sellier, *Ann. chem. anal.*, **16** (1911), 218 [*C. A.*, **5** (1911), 3635].

REQUIREMENTS OF A METHOD FOR GUM DETERMINATION FOR CONTROL WORK

1—For control work a gum determination should be easily made and should be as rapid as possible.

2—The precipitate obtained in the case of a product which has been defecated probably does not represent entirely the true cane gum present in the cane. This precipitate is thus a more or less indirect measure of the gum or substances that retard crystallization, and only comparative results are obtained. In view of this inability to obtain absolute results, almost any method that will give consistent comparative results and is adapted to rapid and convenient manipulation is justifiable.

3—It is our opinion that no method should be used that throws out large amounts of organic nonsugars other than gums. Since the pure cane gum cannot be isolated by any simple procedure, the alcohol concentration should be such that dextrin will be precipitated. Dextrin is also a gum and retards crystallization.

The most suitable method for control work has been shown to be the HCl-alcohol method. This was studied for the purpose of obtaining such alcohol and acid concentrations that slight changes in the concentration would have little effect on the gum precipitate.

Ethyl alcohol is frequently hard to procure, and for this reason substitutes like methyl and denatured alcohol are desirable. Throughout this work we have compared these with ethyl alcohol.

EFFECT OF SODIUM BENZOATE AS PRESERVATIVE

Five g. of sodium benzoate were added to the stock sirup,[6] using 10-cc. sirup samples. Analysis before and after addition of the preservative showed the following average results:

Before sodium benzoate	After sodium benzoate
0.0296 g. gums	0.0302 g. gums

There is a slight increase in the gum content after adding the sodium benzoate, but this may be an analytical variation. The sirup can be thus kept 4 wks. at ordinary temperatures before any evidences of fermentation appear. Even when bubbles appear there is no perceptible change in the gum content.

EFFECT OF HCl CONCENTRATION ON THE GUM PRECIPITATE

The alcohol concentration was kept constant in these tests while the acid concentration was varied. Ten cc. stock sirup were taken and varying amounts of acid added, but water was added with the acid to make the total added volume 2.5 cc. Then 50 cc. of alcohol (94.5 per cent by volume for the ethyl and 92.7 per cent by weight for the methyl alcohol) were added, and after 30 min. the precipitate was filtered in alundum crucibles, dried 1 hr. at 100° C., weighed, ashed, and reweighed. The gum column in the table represents the net gum content after deducting the ash.

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[6] Used in all following determinations except as expressly noted otherwise. 1000 g. 96° raw sugar (gums, 0.14 per cent) and 200 g. black strap molasses (gums 2.53 per cent), with water to 2000 cc. This makes a sirup of 47 to 48 per cent solids.

	Ethyl Alcohol			Methyl Alcohol	
Expt. No.	HCl G.	Gums G.	Ash in Gums Pptd. G.	HCl G.	Gums G.
1	0.00	0.1022	0.0472	0.00	0.0987
2	0.02	0.0794	0.0349	0.02	0.0809
3	0.05	0.0364	0.0220	0.05	0.0386
4	0.10	0.0292	0.0213	0.10	0.0283
5	0.15	0.0288	0.0213	0.20	0.0285
6	0.20	0.0288	0.0183	0.30	0.0271
7	0.30	0.0294	0.0105	0.40	0.0264
8	0.40	0.0291	0.0017	0.60	0.0262
9	0.60	0.0279	0.0015	1.00	0.0261
10	1.00	0.0345	0.0014		

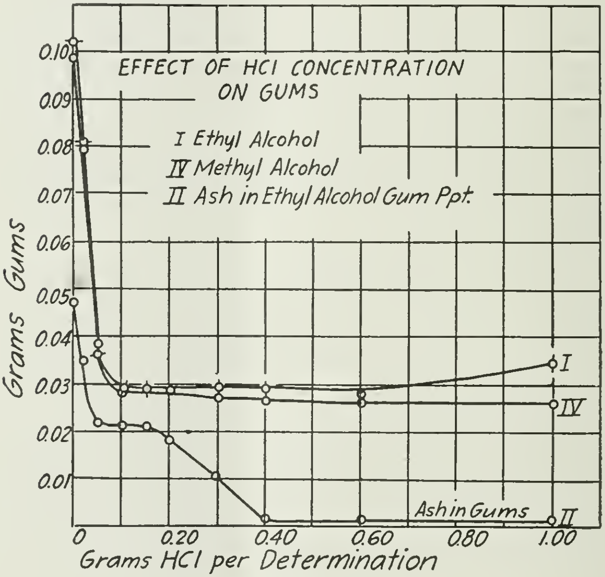

Fig. 1

These results are shown in Curves I, II, and IV of Fig. 1. The first portion of the gum curve has a very sharp slope, which breaks off sharply. Calcium salts of the various organic acids can be precipitated by alcohol of this concentration, while the free acids cannot. As soon as sufficient HCl has been added to liberate all these acids, the gum precipitate becomes practically constant. After this point is reached the acid concentration may be varied over quite a range without materially affecting the amount of gums obtained. The concentration recommended by Noel Deerr and by Weichmann falls on the flat portion of this curve. It corresponds to a concentration of about 0.27 g. HCl, as plotted on this curve.

The combustible matter present in the gum precipitate, and not the ash, determines the amount of gum. It is not necessary to determine this ash to obtain the gums, as is necessary where the filtration is on filter paper. The ash was determined here only as a matter of information.[7]

EFFECT OF VARYING ALCOHOL CONCENTRATION WITH ACIDITY CONSTANT

The acidity used in these experiments is 1 cc. of concentrated HCl per determination for 100 cc. of alcohol. This concentration permits a considerable variation in the acid measurement without affecting the results. This is a convenient concentration and is the same as is recommended by Noel Deerr and by Weichmann. Ten cc. of sirup were used and 0.5 cc. of concentrated HCl added, and then the desired quantity of alcohol in a volume of 50 cc. Filtration was in alundum crucibles for the ethyl alcohol experiments, and in an asbestos mat Gooch for the methyl alcohol experiments. The precipitate was dried 1 hr. at 100° C., weighed, ashed, and reweighed.

In the ethyl alcohol runs, Expts. 9 and 10 filtered so slowly that the washing was imperfect. In general, the

Expt. No.	G. Alcohol per 100 Cc. Mixture	Gums	Ash in Gum Pptd.
	ETHYL ALCOHOL		
1	65.4	0.0307	0.0204
2	63.7	0.0290	0.0187
3	60.5	0.0289	0.0156
4	57.3	0.0242	0.0169
5	51.0	0.0190	0.0036
6	44.6	0.0159	0.0022
7	38.2	0.0154	0.0020
8	31.8	0.0154	0.0031
9	19.1	0.0175	0.0023
10	6.4	0.0172	0.0019
	METHYL ALCOHOL		
1	62.2	0.0303	......
2	59.2	0.0262	......
3	56.1	0.0208	......
4	49.6	0.0104	......
5	31.1	0.0089	......
6	12.4	0.0087	......

---

[7] The fact that loss on ignition without actual determination of ash is all that is necessary to determine gums is made possible by our use of alundum crucible in place of filter paper commonly recommended. For still more rapid, less precise, work even the loss on ignition might be omitted, if a series of gum determinations were being made on a product with uniform ash character, by taking advantage of the fact, shown on the curves, that above 0.4 g. of HCl per determination, the ash is low and constant. In such a case the ash constancy should be checked occasionally by determining loss on ignition.

methyl alcohol precipitates settle faster and filter faster than those of the ethyl alcohol.

The data are represented by Curves III and V of Fig. 2.

The higher alcohol concentrations are on a rather steep part of the curves. This abrupt drop in the gums is apparently a dextrin loss. If the alcohol concentrations were carried much higher, there would be danger of throwing out much organic nonsugar and possibly sugar. It is evident that the alcohol concentration should be kept within close limits, and as high as the higher concentrations used in these tests.

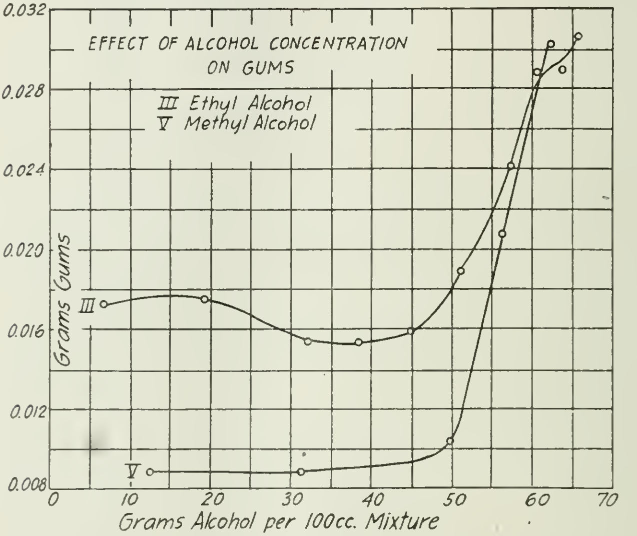

Fig. 2

Comparison of Ethyl, Methyl, and Denatured Alcohols

Tests on 94° Molasses-House Sugar—This sugar was made into a 50 per cent sirup and filtered by suction through asbestos. 20 cc. of sirup, 1 cc. of HCl, and 100 cc. of alcohol were used for each determination.

Average Percentage of Gums

A	B	C
0.252	0.252	0.247

A—Pure ethyl alcohol, 94 per cent by volume.
B—90 per cent ethyl alcohol and 10 per cent methyl alcohol, 92 per cent by weight.
C—Methyl alcohol (commercial), 92 per cent by weight.

These results indicate that for this sugar these alcohols give results that are practically identical.

Another sample of sugar was checked against completely denatured alcohol and the following average results obtained:

Ethyl Alcohol (94 per cent)	Denatured Alcohol
0.266 per cent gums	0.281 per cent

This alcohol was denatured by the following formula (parts by volume): 100, ethyl alcohol; 2, methyl alcohol; 0.25, pyridin; and 0.50, benzine. This denatured alcohol showed 94 per cent by volume.

The results show a slight increase in the case of the denatured alcohol, but the agreement for most purposes is quite satisfactory.

Tests on Stock Sirup, Using Alcohols Described Above—Ten cc. of sirup, 0.5 cc. of concentrated HCl, and 50 cc. of alcohol were used. Series 2 was run with twice the acid concentration used above. The following average results were obtained:

	Grams of Gums		
Series	Ethyl Alcohol	Denatured Alcohol	Methyl Alcohol
1	0.0304	0.0301	0.0259
2	0.0301	0.0280	0.0267

The methyl alcohol gives results that are quite low on a product like black strap. The denatured alcohol is more sensitive to change in acid concentration than either the ethyl or methyl alcohol.

The Technic of Gum Determinations

Filtration of Gums—All the methods found in the literature, with the exception of that recommended by Noel Deerr and by Weichmann, are foreign methods, and none of the foreign methods suggest the use of a Gooch crucible. Since the ash in the gums need not be known, only two weighings are necessary when the Gooch crucible is used. This saves two weighings over tared filters, and has the further advantages of greater accuracy and ease of manipulation.

Asbestos mat Gooch crucibles are to be preferred to alundum crucibles chiefly because of the better filtration obtained with the former. Alundum crucibles are also rather difficult to wash properly. Since there is no filtration between weighings, the asbestos mat need not be washed free from fine fibers, thus making mat preparation very simple. There is a loss of only 0.0010 g. between drying and ignition, owing to asbestos and crucible losses for a mat weighing 0.20 to 0.25 g. dry. The corresponding loss for an alundum crucible is at least 0.0020 g.

Effect of Time of Standing after Precipitation on Rate of Filtration and Weight of Gums Obtained—These results were obtained from 10 cc. of stock sirup, 0.5 cc. of concentrated HCl, and 50 cc. of 94 per cent alcohol.

Expt. No.	Time of Standing before Filtration	Time for Filtering and Washing	Gums Obtained Grams
1	0 min.	4 hrs.	0.0282
2	15 min.	8 min.	0.0281
3	30 min.	18 min.	0.0279
4	1 hr.	14 min.	0.0283
5	2.5 hrs.	14 min.	0.0284
6	16 hrs.	15 min.	0.0288

For filtering through asbestos mat crucibles, 15 min. standing is all that is necessary. No serious error is introduced by different times of standing.

TEMPERATURE OF DRYING AND TIME REQUIRED TO REACH CONSTANT WEIGHT—Drying at 90° C. in 30-min. periods comes to constant weight in 90 min. The precipitate goes from 0.0290 to 0.0280 g. from first to last period.

Drying at 100° C.; constant weight in 60 min.; 0.0282 to 0.0280 g.
Drying at 105° C.; constant weight in 30 min.
Drying at 110° C.; constant weight in 30 min.

Using the same gum precipitate, the drying was carried constant at 90° C., then constant at 95° C., etc., with the following results:

90° C. constant at 0.0282 g.; 90 min. required
95° C. constant at 0.0281 g.; 90 min. required
100° C. constant at 0.0277 g.; 60 min. required
105° C. constant at 0.0271 g.; 30 min. required
110° C. constant at 0.0269 g.; 30 min. required

These results show that even a 5° change in drying temperature will cause but little error. The time required to reach constant weight will also vary somewhat with the weight of the precipitate. For this reason it is desirable to dry at least 1 hr. at a temperature of 100° to 105° C.

RECOMMENDED METHOD FOR GUM DETERMINATION

SAMPLE—If the sample is a sirup or juice, dilute to or concentrate to about 50 per cent solids. For work in which only comparative results are desired on products of nearly the same density, this is not necessary but it is desirable. The sample should be free from suspended matter. We find filtering through asbestos or centrifuging fairly satisfactory. Care must be taken that no clarification that has a tendency to remove gums is used. If the gum content is low, use a 20-cc. sample; if high, use a 10-cc. sample. If the density of the sample is not accurately known, it should be weighed to the nearest 0.1 g.

A sugar can be dissolved in an equal weight of water and should usually be filtered. Sodium benzoate satisfactorily keeps sirups for gum analysis.

ACIDITY—Use 0.5 cc. of concentrated HCl for the 10-cc. sample and 1.0 cc. for the 20-cc. sample. Add the acid to the sirup; otherwise, in the case of very low-grade products an unfilterable precipitate may be obtained.

Alcohol—Use alcohol at about 93 to 96 per cent by volume. In case methyl or denatured alcohol is used, a large supply should be kept so that the results will be comparable. This old supply should not be entirely used before checking against the new supply on the products usually analyzed. With ethyl alcohol it is only necessary to see that the density is the same. For some products methyl or denatured alcohol will give results nearly the same as the ethyl alcohol, but on other products the results may be lower. Denatured alcohol especially should be watched, as the nature of the substance used for denaturing may be a factor in its behavior as a gum precipitant. Alcohol denatured with methyl alcohol alone is preferable for gum precipitation. All alcohol should be filtered before use.

Precipitation—The precipitation is preferably conducted in 120-cc. Erlenmeyer flasks or in beakers. The alcohol should be added from a rather slow delivery pipet, and the mixture should be well agitated during the precipitation. Use 50 cc. of alcohol for the 10-cc. sample and 100 cc. for the 20-cc. sample. To avoid slow filtration allow the precipitate to stand 15 min. before filtering.

Filtration—Use an asbestos mat Gooch crucible. The mat should contain at least 0.20 g. of dry asbestos for a precipitate weighing from 0.2 to 0.5 g. It is unnecessary to wash the mat free from fine fibers or to weigh the crucible before filtering the gums. The flask containing the precipitate should be drained before rinsing, and the rinsings of wash alcohol should not be added until the last drop of the original liquor has passed through the mat. The filtration and washing of precipitates weighing about 0.01 g. ordinarily need not require more than 5 min., and precipitates of about 0.05 g. can be handled in 15 min. Alundum crucibles if used need not be weighed before filtration of the gums.

Wash Alcohol—The requirement for the wash alcohol is that it shall be of higher alcohol concentration and of less acidity than the alcohol in the mixture of sirup, alcohol, and acid. This avoids any solvent effect on the precipitate. It is advisable to use the same strength alcohol for washing as was used in precipitating the gums. Occasionally a little acid in the wash alcohol will speed up the filtration.

Drying—Dry at 100° to 105° C. for 1 hr. For very large precipitates it is well to see whether this is sufficient to give constant weight.

Weighing—After drying and ignition the crucible should be cooled in a desiccator and weighed. These weighings should be on a balance accurate to 0.0001 g.

Ignition—The ignition of the organic matter may be accomplished in a muffle or by a Bunsen flame, and must be continued until all the carbon is consumed. This usually requires 10 to 15 min. The difference between the weight after drying and after ignition represents the dried gums. For very accurate work or where the precipitate weighed is very small, a correction must be applied for the loss in weight of the dried crucible and mat during ignition. For a 14- to 18-g. Gooch crucible with a 0.20 to 0.25 g. asbestos mat, this correction amounts to 0.0010 g. This correction may be determined once for all for any crucible and mat. The mats nearly constant in weight are prepared by using the same volume of a thoroughly shaken stock suspension of asbestos and water.

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By

D. J. DEMOREST

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A Preliminary Survey Made to Determine Suitability and Economic Value of Various Ohio Coals for Gas Making Purposes.

By
D. J. DEMOREST
Professor of Metallurgy

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GASIFICATION OF OHIO COALS

A Preliminary Survey Made to Determine Suitability and Economic Value of Various Ohio Coals for Gas-Making Purposes

By D. J. Demorest
Professor of Metallurgy, Ohio State University

The natural gas production of Ohio has dropped so far below the demands of that state that out of the total consumption of 145,000,000,000 cu. ft. in 1920 Ohio produced only 60,000,000,000 cu. ft., the difference being made up by gas piped in from West Virginia. The new wells drilled in Ohio all come in with lower pressure year after year. It is obvious that the natural gas supply will grow less with each year and will soon be entirely inadequate for the State's needs even with gas from West Virginia.

These facts were so evident several years ago that it became a matter of immediate importance to institute a survey to determine what resources the state possesses for the making of manufactured gas to supplement and ultimately completely to supplant the natural gas. The writer presented the case to the Engineering Experiment Station of the Ohio State University and the Legislature was asked to appropriate a sum sufficient to erect an experimental plant whereby the gas- and byproduct-making values of the coals of Ohio might be determined. The Legislature made such a grant to the Engineering Experiment Station, out of which $10,000 was allocated for this coal-testing work to cover the cost of erecting the plant. The gas and coal interests of Ohio were then asked to appropriate funds for operating the plant for one year. The Southern Ohio Coal Operators' Association, through W. D. McKinney, its secretary, made a grant of $200 per month for one year and the Gas and Oil Men's Association, through W. H. Thompson, its secretary, granted $100 a month for one year.

THE PURPOSE OF THE SURVEY

It must be distinctly borne in mind that the researches reported here constitute only a preliminary survey. It is hoped and expected that this preliminary survey will permit the classification of Ohio coals so that we shall know which ones to select as the most economical for gas making; then, later, to concentrate more detailed work upon those coals in order to discover exactly the best conditions for their gasification, and to establish with a fair degree of accuracy their economic values to the gas industry.

Each coal yields its best value upon gasification under its own peculiar conditions. For instance, some coals must be gasified at comparatively high, others at comparatively low temperatures. The amounts of gas made, the total number of heat units in the gas per cubit foot, the amount and nature of light oils recoverable, the amount and nature of tar produced, the amount of ammonia recoverable, the impurities in the gas (such as hydrogen sulphide and naphthalene) and the quality of the coke all vary with the rate of gasification and the temperature. The amounts and qualities of these products which can be recovered vary also with the fineness to which the coal is crushed before charging in the retort and with the amount of moisture in the coal. It is also true that many coals which are not economically gasified alone may be very valuable for gasification purposes when mixed with a characteristic amount of some other coal.

In this preliminary survey it was possible only to carry out the carbonization experiments at the average temperature of 1,000 deg. C. This will probably give the best results for some coals but is too low a temperature for some other coals and possibly too high for still others. Furthermore, it was not possible in this preliminary survey to discover the optimum fineness for crushing nor to attempt to find the results obtainable by mixing coals or regulating the moisture content. In the further work which is to be done on certain selected coals, the economies resulting from steaming the coals and from the use of sized coal will be ascertained.

In this work it was realized that the most highly refined methods for determining the benzol yield and the distribution of sulphur in the gas were not justified, because of the fact that in the preliminary work the time and money were not available for ascertaining the retort conditions which would give the best yields for each coal. Hence only comparative results were sought.

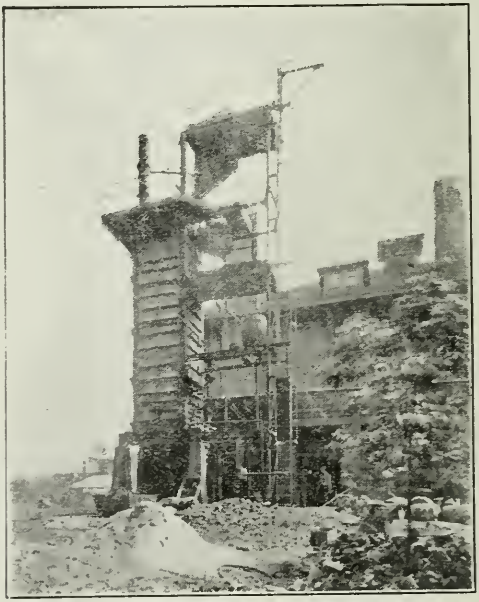

FIG. 1—FRONT VIEW OF RETORT STRUCTURE

It is believed, however, that the results obtained from this preliminary survey are reasonably accurate and afford a basis for the selection of those of our coals upon which further detailed work should be done. Furthermore, they give a rough evaluation of all the coals for gas making.

The Nature of Coal

Coal is made up of the partially decomposed remains of vegetation, chiefly of the Carboniferous Age, mixed with inorganic minerals. These inorganic minerals remain—some changed, some unchanged—when coal is burned and make up the "ash."

Chief of these materials are:

Kaolinite (the chief mineral of fire clay) $Al_2O_3 2SiO_2 2H_2O$.

Quartz (SiO_2).

Pyrite (FeS_2).

In addition to these, there are smaller amounts of all the innumerable minerals which may be found in ordinary soil or clay.

The remainder or true coal is made up of partially decomposed vegetal remains, a very complex mixture of many organic compounds of complex structure concerning which very much remains to be learned. Woody matter which constituted the original vegetation is composed largely of the following classes:

Ligno-celluloses—essentially $nC_{12}H_{18}O_9$.

Resins, gums, waxes.

Nitrogenous bodies, i. e. proteid bodies containing nitrogen, carbon, hydrogen, oxygen and sulfur.

During the period when the debris of the ancient vegetation was accumulating on the swamp bottoms on which the vegetation was thriving, the decomposing action of bacteria and atmospheric agencies (decay) brought about very considerable changes in the chemical composition of the organic compounds of the original vegetation; at the same time floods brought dirty water into the swamps which deposited clayey material in the accumulating organic matter and thus made the "ash" of the coal. As these deposits were submerged and ultimately covered with hundreds and even thousands of feet of rock, usually sedimentary, the temperature was raised somewhat, possibly to 300° C. The slowly rising temperature and high pressure continuing through long geologic ages brought about further chemical changes in the already partially decomposed vegetal matter, and finally produced the semi-carbonized and highly complex substance which we call coal.

Of these three types of organic matter, the most unstable are the cellulosic and proteid types which have undergone such profound alteration in the process of coal formation that in many coals the original characteristics are completely masked. The resins are less effected.

The results of these organic changes are extremely complex and not very well known, but in general we may say that the products are classifiable as below indicated:

Humic Bodies. These have resulted from the decomposition of the ligno-celluloses and have the composition of approximately:

Carbon	64%
Hydrogen	4%
Oxygen	32%

Resinic Bodies. As mentioned above, these are not so greatly different from the resins of the original vegetation and have approximately the composition:

Carbon	77%
Hydrogen	10%
Oxygen	13%

Nitrogenous Constituents. In the original vegetation these compounds were largely proteids, but during the millions of years of change through which the coal has passed, the proteids have suffered alteration, the resulting compounds being very inadequately known. The ammonia which is obtained from coal when it is retorted comes largely from the breakdown of these proteid remains. Many coals are believed to have a considerable portion of their sulfur, as "organic sulfur," which was probably in the proteids.

Hydrocarbons. Free solid hydrocarbons, such as "paraffin," are found to a limited extent in coals and seem to come from the decomposition of resins. Some researches, however, throw considerable doubt as to the existence of free hydrocarbons in the coal.

Free Carbon. The ultimate result of the complete decomposition of coal-making compounds is free carbon. In those coals in which it is nearly complete, such as anthracite, the amount of free carbon is very large.

CLASSIFICATION OF COALS

Coals may be classified in many ways, but for the present purpose it will be most convenient to use the graph given in Professional Paper 100-A by M. R. Campbell of the United States Geological Survey. See Fig. A.

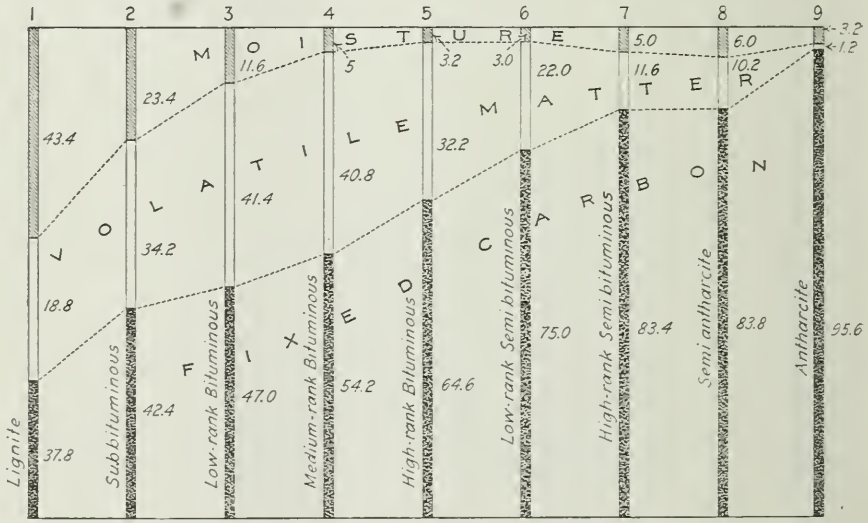

FIG. A

The best gas coals are generally regarded as those containing about 35% volatile matter with low ash and sulfur and would fall in this graph between the "high rank bitumenous" and "medium rank bitumenous." However, it is well known that the determination of volatile matter in coal is far from an accurate means of estimating its gas making value. The only satisfactory method is an actual retorting test. Even a retort test is not the final answer to the question, "will any given coal be satisfactory for manufactured gas-making," because what is really desired is an answer to the question, "will it be profitable to make manufactured gas and recover the byproducts from a given coal." It is obvious that there will be places where it will be more profitable to retort a poor but cheap coal than a rich but expensive coal. For instance, it might be found profitable to select for gas making at some Ohio city one of our poorer coals rather than one of our richer coals or one of the richer coals from another state. One must be careful, therefore, in declaring that any given coal is or is not a proper gas-making coal because the use of a coal may be dependent upon mining costs and freight rates as well as upon the inherent value of the coal. The following discussion of the characteristics of gas-making coals on the basis of their composition and inherent value will have to be interpreted later on the basis of the economics of gas making.

Gas Coals and Coking Coals

It is impossible to neglect the characteristics of coking coals when discussing gas coals because next after the value of the gas itself the chief revenue of the manufactured-gas plant comes from the sale of the coke made; hence, the coal used must be of a nature such that the coke produced from it is saleable. In this connection, it must not be forgotten that it is frequently possible to mix two or more coals, no one of which makes a good coke and get an excellent coke from the mixture. In fact, nearly all byproduct coke plants use such mixtures. In such plants the coals used range from 17 to 38% of volatile matter. It has been shown by Mr. David White in Bulletin 29 of the United States Bureau of Mines that American coals which make good metallurgical coke, when coked alone, have a hydrogen--oxygen ratio of 0.59 or more. Those with from 0.59 to 0.55 make fair coke while those below 0.55 make poor coke. However, these limits, of course, depend upon what one means by poor coke, good coke, etc.

Whether a coal will or will not make satisfactory coke depends apparently upon the relative amount of resinous to humic bodies in the coal. As will be seen by referring to the composition of humic bodies and resinic bodies on page 5, the resinic bodies are high in hydrogen compared to the humic bodies. A higher hydrogen-oxygen ratio merely means that the coal is high in resinic bodies compared to humic bodies. It is the resinic bodies in a coal which melt when a coal is heated and cement the coal particles together before decomposing. This leaves a coke after decomposition is complete rather than a powder which would have been left had the resinic bodies been too small in amount to cement the coal particles together on heating. D. H. White, of the United States Geological Survey, thinks the coking or melting is due to the presence of the remains of algal bodies. An experienced coal chemist can make a pretty accurate guess as to whether a coal will coke or not by noting the way in which the coal acts when ground in an agate mortar. If the coal is high in resinic bodies, the surface of the mortar becomes waxy; if the resinic bodies are present in small proportion, the waxing of the mortar is not noticeable.

The amounts and natures of the gas and other byproducts besides coke which a coal yields upon carbonization in a retort are also dependent upon the relative proportions of humic bodies, resinic bodies, nitrogenous bodies, etc., which the coal may contain. If the coal is high in resinic bodies, there will be a large amount of hydrogen in the coal to come off as the rich hydrocarbons and a low amount of oxygen to carry off hydrogen and carbon as H_2O and CO and CO_2, when the coal is retorted. On the other hand, if the coal is high in humic bodies, it will be high in oxygen and low in hydrogen; and consequently there will be a smaller amount of hydrogen to form rich hydrocarbons and a larger amount of oxidized compounds like H_2O, CO and CO_2 in the gas, making the gas lean. It will, therefore, be seen that within limits the higher the resinic bodies and the lower the humic bodies in the coal, the higher will be the hydrogen-oxygen ratio and the better will be both coke and gas-making properties of the coal.

THE CHEMISTRY OF COAL CARBONIZATION

When a ton of coal is retorted the following products will be obtained:

Gas, 10,500 cubic feet.
Coke, 1,350 pounds.
Tar, 10 gallons, or 90 pounds.
Light Oils, 2 gallons.
Ammonia, 5.5 pounds.

The chemical decompositions and reactions which have made these products are quite complex, as would naturally be expected when it it is remembered that the tar alone contains some 200 different compounds. In order to be able to grasp the fundamentals of gas-making, some mental picture should be secured of the chemical and physical changes going on in the coal as its temperature rises in the retort, from the time when the coal is just beginning to be hot enough to give up gas to the time all gas has been yielded and nothing is left in the retort except coke.

In the beginning it must be clearly understood that the compounds as they are distilled out of the coal are very different from the compounds which leave the retort. In other words, the compounds liberated from the coal undergo profound decomposition and unite to form new compounds as they pass over hot coke and against the hot walls of the retort. The compounds which are driven out of the coal do not themselves exist in the coal, in general, but are formed by the destruction of the compounds (resinic bodies, humic bodies, etc.), which, as previously mentioned, make up the combustible materials in the coal. These compounds driven out of the coal are, at ordinary temperatures, solid, liquid and gaseous; but they are all gasified at the temperature of the retort and any solid and liquid, which escape destruction in the hot retort, condense after leaving the retort and form the tar.

It has been shown by many workers, notably by Porter and Taylor, whose work is published in Technical Paper 140 of the United States Bureau of Mines, that when coal is heated in retorts out of contact with air, it begins to decompose perceptibly at 250° C., and the decomposition becomes quite active at 350° C. The younger western coals break down more rapidly and at lower temperatures than the maturer eastern coals and produce more H_2O and CO_2 and CO and less paraffin hydrocarbons. The cellulosic derivatives or humic bodies commence to decompose at a somewhat lower temperature than the resinic bodies, the first gases liberated being CO and CO_2, water and some saturated hydrocarbons, such as methane and ethane, and a considerable amount of tar composed almost entirely of the heavier paraffine hydrocarbons. At this low temperature (350° C.), the amount of tar made from a typical gas coal is larger than that of any other product, the amount of water next in amount and the gas still less. As the temperature rises to 450° C., the rate of decomposition rapidly increases though relative rates are much the same. These are shown by the accompanying graph of Porter and Taylor, Fig. B.

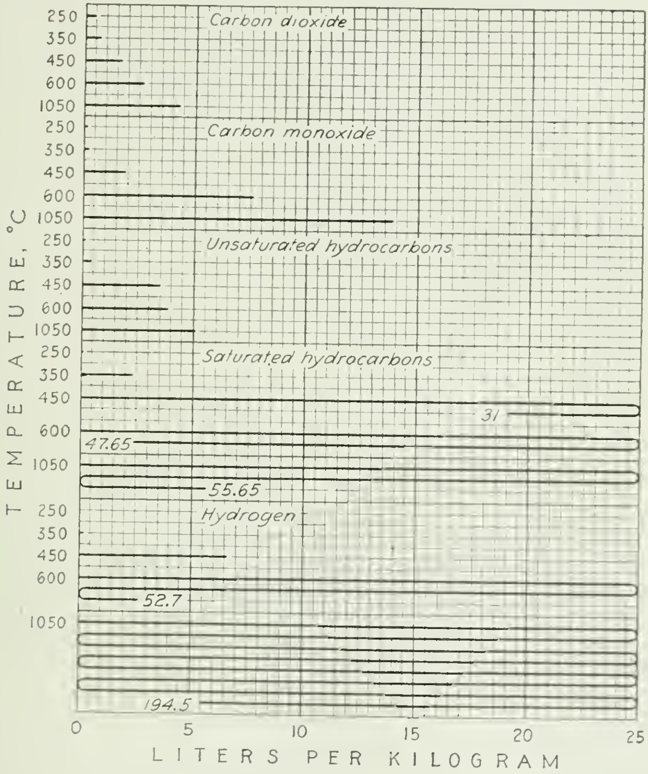

FIG. B

At 450° C. a typical gas coal will yield about 1,500 cu. ft. of a very rich gas, and about 220 pounds of tar consisting almost entirely of paraffin hydrocarbons and almost devoid of benzene and naphthalene. Since the tar obtained at most gas works is almost devoid of paraffin hydrocarbons and very rich in benzene and naphthalene derivatives, it will be seen that the gases and vapors liberated from coal at low temperatures must undergo very profound change as they pass up through the hot retort over the hot coke and against the hot retort walls. The gases liberated at low temperatures are, therefore, seen to be (in the order of their volume) water, saturated hydrocarbons, such as methane and ethane, unsaturated hydrocarbons, such as ethylene, hydrogen, carbon dioxide, hydrogen sulfide and carbon monoxide. More than two-thirds of the volatile matter is driven off below 500° C.

When these gases pass up through the retort, coming into contact with hot walls and coke they undergo reactions somewhat as indicated below:

$H_2O + C = CO + H_2$.
$CO_2 + C = 2CO$.
$CH_4 = C + 2H_2$.
$C_2H_6 = C_2H_4 + H_2$.
$C_2H_6 = 2C + 3H_2$.
$C_2H_6 + H_2 = 2CH_4$.
$3C_2H_6 = C_6H_6 + 6H_2$.

These secondary reactions take place very rapidly at temperatures beginning at 750° C. and with increasing rapidity as the temperature rises. Since the walls of the retort in ordinary operation are maintained at about 1000° C., all those gases which pass along the walls and through the hot coke near the walls get pretty thoroughly "cracked," while the gases which pass through the cool interior largely escape this cracking process. The gas and tar made in a vertical retort is a mixture of the primary low temperature products with the secondary high temperature products. The resultant of all these primary and secondary decompositions is a gas of the following composition:

CO_2	2.0%
C_2H_4,etc.	3.5%
O_2	0.5%
CO	6.5%
CH_4	30.0%
C_2H_6	1.0%
H_2	50.5%
N_2	6.0%

Water amounting to about 10% of the weight of the coal.

In addition, the gas always contains some of the benzene hydrocarbons equal to about 2½ gallons per ton of coal, hydrogen sulfide (H_2S) and other sulfur compounds carrying about half the sulfur which was in the coal, ammonia amounting to about ¼ of the nitrogen that was in the coal, cyanogen and finally naphthalene, anthracene, phenol, cresols and numerous other aromatic derivatives which make up the tar, amounting in all to about 5% of the weight of the coal.

As soon as the coal is dumped into the retort, the walls of which are at about 1000° C., those particles of coal which lie in contact with the hot walls of the retort immediately give off a flash of gas and leave a thin layer of coke in contact with the walls. The heat from the walls is then conducted through the thin layer of coke to the next particles of coal which first give off their loosely held moisture, quickly heat to 250° C., at which temperature the coal commences to decompose and give off gases, the temperature rises and the rate of gas making increases until the temperature of the particles in this layer has risen to about 400° C. At this temperature the resins melt and cement the fragments of coal together. As the temperature rises higher, the pasty mass continues to decompose, gives off gases within the mass, which cause it to swell with innumerable bubbles. The gases continue to be given off until the organic bodies are completely decomposed. The residue, coke, is merely a mass of carbon and mineral matter surrounding the countless cavities which were originally bubbles caused by the gas escaping from the pasty mass. As the gases were leaving the layer during its progressive heating, they passed partly up through the interior of the unchanged mass of coal and partly up through the coke of the previous layer next to the retort wall. That part of the gas which passes through the hot coke undergoes the numerous and profound secondary decompositions already mentioned. As the layer of coal becomes hot and changes to coke it conducts heat on to the next layer. This undergoes the same series of changes except that the layers of coke between the decomposing layer of coal and the walls of the retort are now thicker and that the gas which passs through this coke is longer in contact with it and hence is more severely decomposed. These successive changes take place in each layer of coal as the heat slowly penetrates until the coal is coked clear to the center of the charge. The coking or carbonizing of the coal progresses at the rate of about ½-inch per hour from each wall. About 17 hours, therefore, are required to carbonize the coal in a retort 17 inches wide. The time depends upon the temperature of the retort. In gas house practice, the temperature is higher and the time required is 10 or 12 hours.

It will be seen from the above description that the center of the coal charge remains at a comparatively low temperature for many hours. As a result, a great deal of the liquid products driven out of the outer layers of the coal condense in the inner layers and drips down to the bottom of the retort (in a vertical installation) and out of the drain pipe as tar. This bottom tar, of course, has not been subjected to nearly so much heating as the tar which condenses from the gas from the top of the retort and will, therefore, have quite a different composition.

Hydrogen. The hydrogen of the coal is almost entirely removed as free and combined hydrogen in the gas and tar and as water. About 4/5 of the oxygen in the coal is driven out of the coal in combination with hydrogen as H_2O. A considerable amount of this H_2O undergoes secondary decomposition, thus: $H_2O + C = CO + H_2$.

Oxygen. As indicated in the previous paragraph, most of the oxygen of the coal is driven out combined with hydrogen as H_2O, although small amounts are present in primary products as CO_2 and CO and a small amount goes into the tar as oxidized aromatic compounds, such as phenol, C_6H_5OH. The H_2O formed as shown above partly undergoes decomposition and forms the combustible gases H_2 and CO, hence, it will be seen that the high oxygen in some coals is not a dead loss, as it serves to increase the volume of the combustible gas.

Sulfur. It has been shown that HS_2 is a primary product of coal carbonization. If this H_2S is not allowed to become highly heated in the presence of carbon and hydrocarbons, it will pass out of the retort without change; but if the H_2S becomes highly heated along with carbon and hydrocarbons it reacts with them to form CS_2 and $(C_2H_5)_2S$ and other organic sulfides and hydrosulfides which are not easily removed from the gas. It is important, therefore, to prevent the gas, especially from coal containing high sulfur, from becoming highly heated in the retort. Vertical retorts are advantageous in this respect since a great deal of the gas passes up through the cool interior charge while in horizontal or inclined retorts the gas is nearly all forced to come into contact with the hot top coke and hot top of the retort. This advantage of the vertical retort will make some high sulfur coals useable which might otherwise seem to be too impure. The Sea-board Process of sulfur removal will also help greatly. The time is coming, of course, when it will no longer be possible to depend upon securing coals which are naturally low in sulfur and the practice of dressing coals to remove sulfur will be quite widespread.

Light Oils and Tars. Coal gas made in the ordinary high temperature retort always carries some "light oils," some of which is condensed in the tars and some is recovered from the gas by scrubbing with "straw oil" and then distilling the light oils from the "straw oil." The gas ordinarily contains 0.5% to 1.0% of benzene hydrocarbons, recoverable by scrubbing. The tar made per ton of coal yields on distillation:

Benzol and Toluol	2-4 pounds
Phenol (Carbolic Acid)	1 pound
Cresols	2.5 pounds
Naphthalene	9 pounds
Creosote	15 pounds
Anthracene	1 pound
Pitch	40 pounds

The relative amounts of these vary greatly according to the coal used, the type of gas retort used and the temperature which the retort attains.

Coke. It has already been mentioned that when a coal is heated in a closed retort and the resulting gas, tar, etc., have been driven off there is always left the ash of the coal intimately mixed with the carbon which could not be gasified by combining with the hydrogen and oxygen of the coal. As already explained, this residue from some coals is powdery; from others it is a coke which is or is not strong, depending upon the nature of the organic residue which constitutes the coal. The value of the coke is inversely proportional to its fragility, the amount of ash and sulfur in it and the fusibility of the ash. The best grade of coke is required for metallurgical purposes in the blast furnace and foundry, the next best for operating water gas generators, and for domestic purposes. A very fragile kind of coke is quite satisfactory for steam raising and even gas producer operation.

Coals Tested

Through the offices of the Purchasing Agent of The Ohio State University and the Secretary of the Southern Ohio Coal Operators' Association car load lots of coal representing the important fields and seams of the southeastern Ohio coal regions were shipped to the University siding. In nearly all cases the coal shipped was Run of Mine. Out of each car load there was taken a sample amounting to 5 tons for testing, for this the shipper received no pay, the University paying for the rest of it at prevailing rates. The coals were tested as soon as possible to prevent weathering changes which affect the quality of coke and gas from the coal.

Description of the Plant

Figures 1 and 2 are photographs of the retort structure and figures 3 and 4 are photographs of the byproduct recovery system. Figure 5 is a diagrametric sketch of the plant. Letters used in the text apply both to the photographs and the diagrams.

The retort selected for gasifying the coal is an intermittent vertical type of exactly the same shape and dimensions as those in commercial use. In fact, the retort erected was an extra one of the 250 retorts erected at Rochester, N. Y., for the city gas supply. The United Gas Improvement Contracting Co., which erected the Rochester plant, also erected this experimental retort, generously doing it at cost.

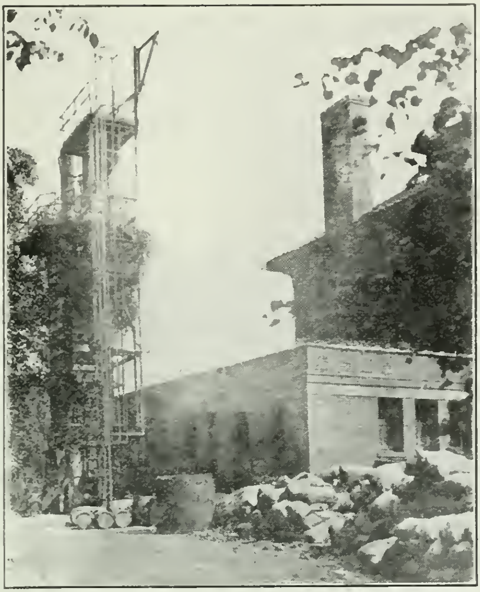

FIG. 2—SIDE VIEW OF RETORT STRUCTURE

The equipment required for recovering tar and ammonia from the gas made, and the arrangement of machinery were designed in the Department of Metallurgy and erected by university mechanics.

The equipment for crushing the coal, washing the coal, and testing and analyzing the products made are all a part of that possessed by the Department of Metallurgy.

The retort carries a standard charge of 1 ton (2,000 lb.) of coal. The charge is all run in a fraction of a minute, and remains in the retort until the carbonization of the entire ton is complete, requiring 10 to 17 hours. In other words, the retort is of the intermittent type of vertical retort—not of the continuous type. The retort is heated for convenience with natural gas, burning at three different levels and giving an exceedingly uniform temperature around the entire setting. The temperature of the setting was recorded by a Brown recording pyrometer with the thermocouple halfway between the top and bottom of the retort. The thermocouple was checked by a standard thermocouple and an optical pyrometer, and was found to be correct.

The gas passes from the top of the retort *A* (see Fig. 5) through the bridge pipe *B* into the foul main *C*, thence down pipe *D* into the bottom of and up through scrubbing tower *E*, thence into the bottom of and up through scrubbing tower *F* and similarly through tower *G*. From tower *G*, the gas is drawn through the exhauster *H* and to the furnace *I*. By means of the pump *J*, a dilute sulphuric acid solution is continuously circulated through towers *E* and *F*. Cold hydrant water passes down through tower *G* and through the cooling coils Z, thence to waste. The cooling coils keep the circulating acid from becoming too warm. These scrubbing towers are filled with wooden grids staggered so that the gas and circulating liquids must come into intimate contact. The circulating acid solution removes all the ammonia from the gas and practically all of the tar not previously condensed. At K, some gas is drawn off continuously for the determination of sulphur; at L, the gas is sampled continuously for its "light oil" contents; at M, a sample is continuously drawn for the determination of the B.t.u. value of the gas by means of a Smith continuously recording gas calorimeter, and at N, the gas is metered by a Bailey orifice gas meter, which continuously records the rate of gas making and integrates the total volume made.

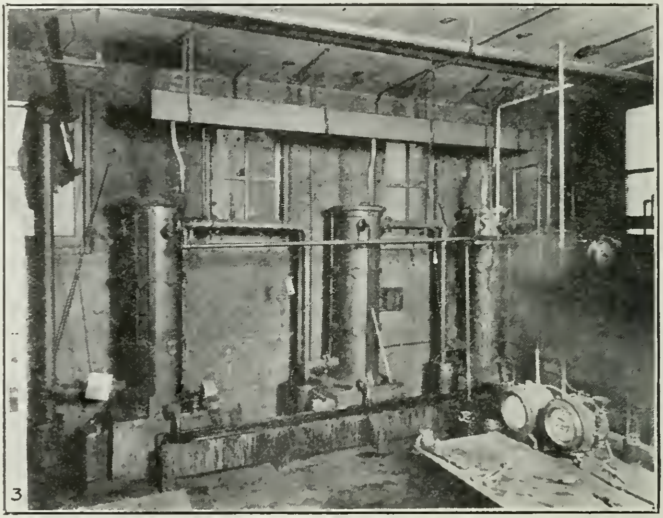

FIG. 3—VIEW OF THE BYPRODUCT RECOVERY SYSTEM

The bottom of the retort setting is a heavy iron casting with an opening the size of the interior of the retort. This opening is closed by a cast iron door, DD, which is hollow and which is alligned with a channel in the iron bottom plate so that the tar which drips down against the door may drain out into tank S. The door is lifted into place before charging the coal and lowered for the discharge of the coke by means of a triplex chain-fall. The coke drops into a portable steel tank and is then dragged from under the retort and quenched with water drawn from an elevated reservoir.

The 5-ton sample of coal previously referred to is put through a set of Jeffrey rolls, which crush it to about ½-inch size. One ton of this crushed coal is then carefully weighed and hoisted by means of a winch into the bin O. All the meters are then inspected to see that they are set at zero, the seals are inspected to see that they are closed, the circulating pump J is started and the recording pyrometer P is read to make sure that the temperature is satisfactory; the door Q at the top of the retort is then opened and the coal slides from *bin* O through shoot R into the retort. While the coal is running it is sampled several times, until a sample weighing abut 10 lbs. is obtained. While the coal is running into the retort, the water seal in the foul main *C* is drained into tank *S*, where it serves to dilute the ammonia liquors yielded by the coal. As soon as all the coal has run into the retort, doors *Q* and *B* are closed, valve *T* is opened, exhauster *H* is started and valve U regulated so that the manometer *V* indicates ⅛-inch of water vacuum and manometer *W* on the foul main indicates 1 10 to 1 16-inch of water vacuum. Thereafter, during the entire run, valve *U* is regulated so as to maintain these vacuums, which are merely sufficient to withdraw the gas from the retort as fast as it is liberated. As soon as gas commences to burn in furnace *I*, the motor of the gas calorimeter *M* is started and the calorimeter put into operation by igniting the gas in it by means of a spark plug. At the same time, the aspirators of the sulphur train *A* (Fig. 6) and light oil train *E* (Fig. 7) are started. These two determinations proceed continuously during the entire run.

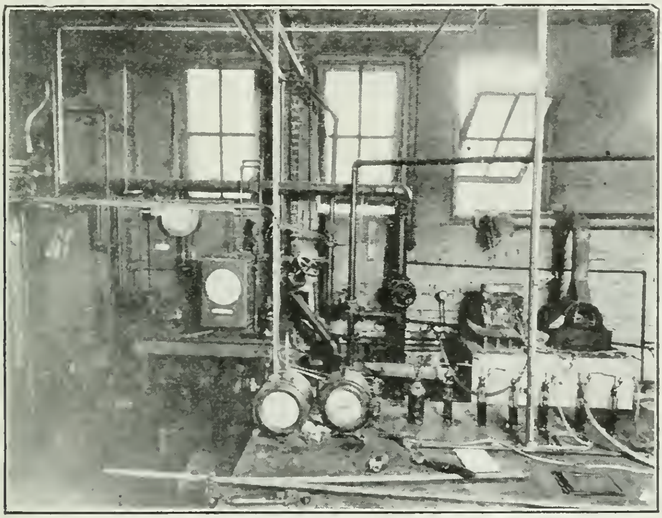

FIG. 4—VIEW OF THE BYPRODUCT RECOVERY SYSTEM

During the 10 to 18 hours which the carbonizaiton of the coal requires, the operator's duties consist merely in seeing that the valve *U* is changed when necessary to maintain the proper vacuum on the retort; that the circulating pump is circulating the sulphuric acid liquor at the proper rate; that the proper excess of sulphuric acid is maintained, and that the temperature of the retort is properly maintained, and that the recording calorimeter, recording meter, sulphur train, light oil train, etc., are properly functioning.

The operator is instructed to regard the test ended when the rate of gas made per hour drops to 300 cubic feet, or when the B.T.U. value of the gas being made drops to 300 B.T.U. net per cubic foot, as it is not likely to be profitable to operate any retort plant beyond this point. It is interesting to note that usually the B.T.U. value drops to 300 at about the same time that the rate of gas making drops to 300 cubic feet per hour.

When this point has arrived, the operator stops the sulphur and light oil determinations, records their meter readings, the total volume of gas made and any other special records that are called for, shuts down the circulating pump, closes valve *U* and stops the exhauster, climbs quickly to the top of the retort and opens bridge-pipe door *B* to permit the remaining gas to escape into the atmosphere. It was very essential that valve *U* and all other openings in the byproduct system be kept tightly closed until the bridge-pipe in foul main *C* is sealed off with water, as otherwise, owing to the chimney effect of the light coal gas in main *D*, air would be sucked into the system, resulting in an explosion in the foul main. This is quite disconcerting, as it generally blows off the water-sealed top of tower *E*—sometimes hitting the ceiling of the room. The towers *E*, *F* and *G* were designed with loose water-sealed tops to take care of this contingency. However, the explosions never take place if valve *U* is closed before opening door *B* and if thereafter no opening is made in the byproduct system (such, for instance, as might be made if the sulphuric acid solution sump *SS* were drained, exposing the bottom of the tower drains *TD*) until the bridge-pipe in the foul main *C* is sealed by water.*

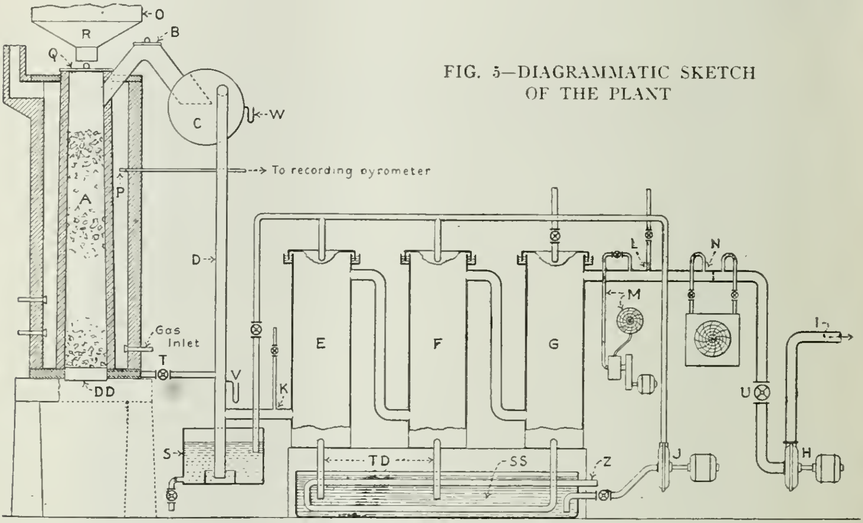

FIG. 5—DIAGRAMMATIC SKETCH OF THE PLANT

The tests were usually started about 8 a. m. so that they were finished before midnight. At this time the operator left for the night.

In the morning, the discharge door *DD* at the bottom of the retort was opened and the coke dropped into the steel tank beneath the retort, the tank pulled under the quencher *WQ* and quickly quenched with the well-distributed spray. The tank *QT* when filled to 1 foot from the top holds just enough water so that when it is all quickly sprayed upon the hot coke, the coke is quenched, but enough heat is retained to dry the coke partly.

While the coke is drying, the tar and ammonia liquor is drained out of the foul main *C* into tank *S*. The main is filled with fresh water. The liquor in the sump *SS* is pumped into tank *S* and the bottom scraped to remove any tar clinging to it. After sufficient time has elapsed to allow the tar and water in tank *S* to separate by gravity, they are drawn off and weighed separately and samples of each set aside for analysis. The coke is then screened through a 1-inch screen, and each size is weighed. About 50 lbs. of the coke comprising oversize and undersize in proper proportions is weighed, dried and weighed again to obtain the moisture in the coke. The dried coke is then sampled for analysis.

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*Since these tests the equipment has been changed and some of these operating details no longer apply.

After oiling the motors, renewing the solutions and putting fresh recording charts on the recording instruments, the equipment is then ready for the next run. Under this schedule it was not feasible to make more than one test in two days if the tar, ammonia, etc., were obtained from each run. With a larger force, a complete run can be easily made each day, and with two shifts two runs could be made in 24 hours.

The methods of chemical analysis used for the coal, coke, tar, ammonia liquor, gas, etc., are the standard methods usually followed by fuel chemists. (See the Gas Engineers' Handbook, Second Edition.) These routine methods will not be described in this article, but it is advisable to indicate somewhat in detail the way in which the "organic sulphur," "light oils' 'and naphthalene are determined. The determinations are started in the beginning of the run and continue until the test is ended.

As soon as the test on a coal sample is started, the determinations here described are begun. For the sulphur determination, the aspirator (Fig. 6) draws gas from the main at *K* through the wash bottles *B* and *C*, which remove the H_2S The gas discharges from the aspirator with the water into the inverted glass tank *D*, thence through meter *E* and into the burner *F*, where it ignites. This burner *F* is enclosed under the chimney *G*, in which is a large supply of ammonium carbonate that slowly vaporizes and passes with the burned gases into the tower *H*. This is filled with broken quartz and down its sides slowly trickles a solution of ammonium carbonate, which accumulates in the vessell *I*, containing all the sulphur of the gas that passes the wash bottles *B* and *C*. Bottles *B* and *C* are filled usually with a NaOH solution, although occasionally a copper sulphate solution is used to remove the H_2S in the gas. Tests show that either solution is satisfactory for this purpose. Therefore, the sulphur in the gas which burns at *F* is only the sulphur which is in the gas combined as CS_2, or other "organic compounds" which are difficult to remove in the purifying processes of commercial gas making. The liquor accumulating in *I* is diluted, an aliquot part taken, bromine added, heated, made acid with HCl, boiled and the sulphur precipitated as barium sulphate and weighed. The result is calculated to grains of sulphur per 100 cubic feet of gas.

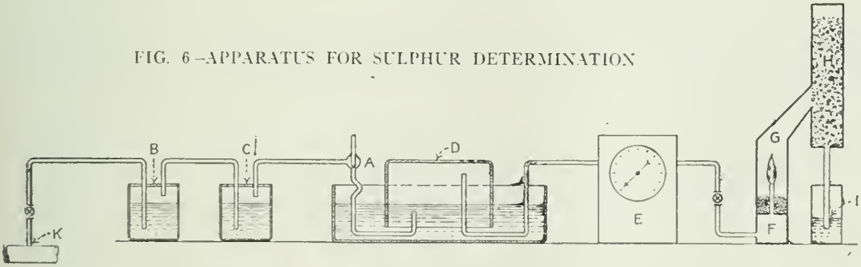
FIG. 6—APPARATUS FOR SULPHUR DETERMINATION

"Light oils" are determined in the gas taken from the main at *L* (see Fig. 7), drawn by aspirator *E* through the NaOH solution in *A*, solid calcium chloride in *B* and the bottles *C* filled with a heavy petroleum oil, and then through the meter *D*. The bottles *C* are weighed before and after the run and the increase in weight in the "light oils."

The naphthalene determination was made by the usual picric acid method on a half dozen samples, the gas being drawn from the main beyond tower *G* (Fig. 5). These determinations, all of which gave practically identical results, showed that the naphthalene in the gases is practically a constant at the temperature of tower *G*, which is cooled by hydrant water. The results average 0.0035 gram per cubic foot, or 5.4 grains per 100 cubit feet.

These results for "organic sulphur" and "light oils" are incorrect in so far as the amount of sample withdrawn per minute was the same throughout the run while the amount of gas made varies from hour to hour. The sample taken should, for strict accuracy, be proportioned to the rate of gas making. The error made is not large, however, and the results are sufficiently accurate for comparative purposes. For the exact work (to be published in a later contribution) a proportioning meter will be used.

Many coals as they are mined contain so much ash and sulphur that the coke would be quite inferior and, of course, the quantity of gas would be correspondingly reduced. It was decided to wash as many of the coals as time and funds permitted, in order to determine any benefits obtained. A large proportion of the coals received, therefore, had at least three tests made on them—two tests on the coal as received and one test on the washed coal.

The washing was carried on under the direction of Prof. W. A. Mueller, of the Department of Metallurgy, and the equipment used was that belonging to the department. The coal was crushed and sized by screening, obtaining two products—through ¼-inch mesh and on ¼- but through ½-inch mesh. The coarse product was jigged in a full-sized Jeffrey jig and the small-sized product was tabled on a Wilfley table. The purified products from these two operations were dried, sampled and mixed. One ton of the mixed material was weighed and retorted exactly as the raw coal was treated.

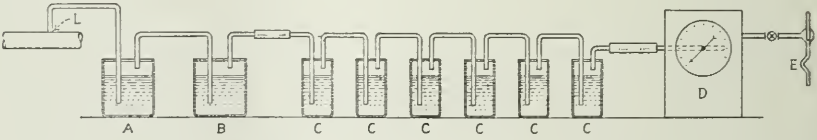

FIG. 7—SET-UP FOR LIGHT-OIL DETERMINATION

It has already been pointed out that every coal has properties that differ from those of other coals and that to get best results from a particular coal it should be retorted at its characteristic temperature. It would have required, however, many thousands of dollars more than were available for this investigation if each coal had been investigated to discover its optimum operating temperature. Hence, 1,000 deg. C. was selected as being a satisfactory average temperature, the temperature being taken at a point about two-thirds of the distance from the bottom and top of the combustion chamber outside the retort. It would probably have been better if 1,100 deg. C. had been taken as the standard temperature. When this temperature (1,100 deg. C.) is maintained, the temperature at the bottom (in the combustion chamber) is 1,325 deg. C.

(2,417 deg. F.) At times, the gas supply was inadequate and the retort temperature fell so low that the retorting results were quite unsatisfactory. For this reason, the results on samples 7, 7W, 8W, 10, 10W, which are shown in the accompanying tables, are not fair to the coals they represent, as far as the gas yields are concerned. The coke results are probably not seriously affected, since the coal was kept in the retort until the carbonization was complete, although the quality of the coke may have been seriously affected by carbonizing at the wrong temperature.

In order to have a check on the results of the retorting tests, however, a standard gas coal (Elkhorn) was run under the same conditions as the Ohio coals were run and also at the high temperatures commonly used in gas plants.

Several tests were made on each coal, but the results reported in this article are not the average results of the several tests. They are the results obtained on the tests when all conditions seemed to work for the highest accuracy.

Quality of Coke Obtained

The quantity and quality of coke obtained in these tests are probably an accurate index of the amounts and natures of the cokes from Ohio coals which will be obtained under commercial conditions, with this modifying statement: The coals shipped in for testing were mostly obtained at the worst period of depression in the mining industry in Ohio, and were dirty and poorly prepared because the normal operating procedures were not functioning at many of the mines from which samples were shipped Hence the analyses of the ash in the coals and coke show abnormally high results for the coals from these regions. The results obtained on the washed coals are probably more nearly representative of the results which will be obtained on Ohio coals in commercial plants in the future, especially since the development of the very efficient machines for removing much of the ash and sulphur by dry-cleaning methods.

The writer used these cokes in a house-heating furnace last winter and found that they make splendid domestic furnace fuels, especially when mixed with about an equal weight of coal. Even when the coal mixed with the coke was of the smokiest type, a practically smokeless furnace was obtained when the coal and coke mixture was used. The coke when used alone tends to give too hot a fire unless there is a great deal of fine coke used. The author freely predicts that the future will see our cities no longer lying dirty and black under a pall of sooty smoke when the manufactured-gas plants which are bound to be erected about Ohio are selling coke regularly to consumers (perhaps under city ordinances) and the householders burn a mixture of coal and coke or coke alone.

The tabulated results list the quality of the cokes under the symbols D, S, W and M, the meanings of which are:

D=Coke suitable for use in domestic furnaces.

S=Coke suitable for steam raising in power plants, etc.

W=Coke suitable for use in water-gas generators.

M=Coke suitable for use in blast furnaces or foundry cupolas.

It is our firm belief, as a result of experience with these cokes, that they will all be very useful for domestic heating. The only exceptions may be those cokes which come from coals having a large amount of fusible or "clinkery" ash. These will be enormously improved by the use of dry cleaning or washing processes before the coal is retorted. Furthermore, it must be remembered that it is always possible to improve a coke by mixing a poor coke-making coal with a good coke-making coal.

All of the cokes made will be entirely satisfactory for steam purposes except those whose ash clinkers too readily. The ash of the Ohio coals and cokes are not more "clinkering" than those from other states. The cokes which are very friable would not do for locomotive use, because too much would be blown out, but the stronger cokes should be very satisfactory. The use of coke in the yard engines will do away with a great deal of the smoke nuisance.

For the manufacture of water gas, the prime essentials are that the coke has as small an amount of ash as possible, that the ash has a high fusion temperature to prevent clinkering in the generator, that the coke has a high reactivity with water for the reaction $C + H_2O = CO + H_2$, and is not too friable, which would cause it to powder and blow out of the generator.

The qualities for a metallurgical coke are: Low ash, low sulphur, great strength and proper reactivity. The fusibility of the ash is of less moment.

Such properties as reactivity and strength of coke are difficult to measure and, in the present preliminary survey, time and funds have not been available for determining them. The results would not, moreover, have been satisfactory unless the cokes were made under the best conditions for each coal. Hence the classification of these cokes has been made entirely by inspection of the coke as it was heaped in a pile after being screened. All cokes were screened through a screen made of rods 1 inch apart, and the percentage that remained on the screen was taken as a measure of the strength of the coke. These screening results, the analysis of the coke, the fusibility of the ash and the appearance of the coke were used to classify the cokes. No claim, however, is made for infallibility in this classification since the personal element enters in. Any coke which is good enough for water-gas making or blast-furnace use will, of course, be excellent for domestic or steam purposes.

TABLE I—COMPARISON OF RESULTS OBTAINED IN THE GASIFICATION OF OHIO COALS*

Test No. †	1RM	2RM	3RM	3W	2 and 3	4RM	5RM
Hours	17	14.5	16	13	15	14.5	15.0
Retort temp. deg. C.	975	975	950	1,100	980	1,000	1,050
Coal analysis:							
Moisture, per cent.	5.49	5.62	6.41	4.30	......	3.61	7.11
Ash, per cent.	9.61	18.05	11.74	6.57	......	11.05	10.45
Volatile, per cent.	37.08	35.86	30.34	42.54	......	41.05	36.15
Fixed carbon, per cent.	47.82	40.47	51.51	46.59	......	44.29	46.29
Sulphur, per cent.	1.29	3.65	2.91	3.43	......	4.22	1.98
Carbon, per cent.	66.00	58.05	65.00	69.21	......	63.87	63.60
Hydrogen, per cent.	4.88	4.45	4.70	4.96	......	4.65	4.40
Oxygen, per cent.	17.02	14.65	15.41	14.59	......	15.07	18.30
Nitrogen, per cent.	1.20	1.15	1.25	1.24	......	1.14	1.27
B.t.u.	12,005	11,053	11,731	12,624	......	12,081	11,943
$H - \frac{1}{8}O$, per cent.	2.76	2.62	2.78	3.14	......	2.77	2.11
Ash fusibility:							
Incipient, deg. F.	2,315	2,475	2,400	2,060	......	2,570	2,140
Complete, deg. F.	2.392	2,510	2,480	2,200	......	2,700	2,580
Coke analysis:							
Moisture, per cent.	1.90	0.51	0.35	0.96	0.50	0.30	0.19
Ash, per cent.	15.80	24.87	17.50	13.04	22.40	17.20	16.09
Volatile, per cent.	3.80	2.22	2.50	3.02	4.10	3.60	3.10
Fixed carbon, per cent.	78.50	72.40	79.65	82.98	73.00	79.90	80.62
Sulphur, per cent.	1.80	3.62	2.49	2.83	3.20	3.48	1.44
Lbs. dry coke	1,220	1,460	1,250	......	1,325	1,322	1,240
Per cent over 1 inch	70	......	70	......	71	85	81
Per cent under 1 inch	30	......	30	......	29	15	19
Quality coke ‡	D.S.	W.D.S.	W.D.S.	M.W.D.S.		M.W.D.	W.D.S.
Cu. ft. gas	7,500	10,900	9,500	11,000	11,300	8,800	9,450
Av. B.t.u., gross	480	485	450	470	472	542	490
Av. B.t.u., net	445	450	415	435	437	507	455

Total B.t.u., gross§	3,642	5,286	4.275	5,170	5,333	4.770	4,626
Organic sulphur	9	12.0	11.5	15	......	......	......
Lb. of ammonia	5.1	5.00	5.15	4.50	5.13	5.40	4.5
Lb. of light oil	2.0	10.0	6.0	......	12.0	9.5	13.0
Lb. of tar	70	80	77	83	72	92	80
Per cent water in tar	45.2	41.7	34.6	15.7	46.1	15.2	20.4
Analysis tar, dry:							
Distil. to 110 deg. C.	1.20	.90	0.0	0.0	1.4	0.3	0.0
110 to 170	2.40	2.90	1.0	0.8	5.2	0.8	0.9
170 to 235	26.60	18.80	15.0	19.9	22.8	19.0	16.4
235 to 270	11.60	8.80	14.8	10.7	10.2	17.4	13.9
270 to 315	15.20	9.60	11.3	6.5	20.6	10.7	8.8
315 to 335	15.50	17.80	19.3	21.0	4.1	20.2	20.1
Pitch	30.40	20.20	39.0	41.0	34.8	31.6	39.6
Sp. gr.	1.11	1.15	1.11	1.13	1.12	1.12	1.12

Test No. †	5W	6RM	6W	7RM	7W	8RM	8W	9RM
Hours	13.5	13	13.5	......	......	17	18	14
Retort temp., deg. C.	1,050	......	1,100	1,000	1,000	1,050	900	1,175
Coal analysis:								
Moisture, per cent.	6.90	6.83	5.74	5.80	6.13	5.75	2.97	3.13
Ash, per cent.	6.35	8.68	6.53	11.29	6.25	11.72	8.30	7.84
Volatile, per cent.	38.65	36.41	38.52	36.21	39.94	38.65	39.81	41.19
Fixed carbon, per cent.	48.10	48.08	49.21	46.70	47.68	43.88	48.92	47.84
Sulphur, per cent.	1.05	1.21	1.00	2.41	1.30	2.23	1.67	2.50
Carbon, per cent.	68.19	66.51	69.48	64.46	68.83	64.66	69.80	69.69
Hydrogen, per cent.	4.88	4.67	4.72	4.48	4.97	4.59	4.99	5.07
Oxygen, per cent.	18.13	17.59	16.91	16.11	17.34	15.55	13.90	13.57
Nitrogen, per cent.	1.40	1.34	1.36	1.25	1.31	1.25	1.34	1.33
B.t.u.	12,500	12,030	12,579	11,705	12,547	11,802	12.623	12,798
H—⅛O, per cent.	2.61	2.47	2.60	2.47	2.81	2.64	3.25	3.37
Ash fusibility:								
Incipient, deg. F.	2,650	2,350	2,375	2,160	2,565	2,440	2 620	2.100
Complete, deg. F.	2,700	2,600	2,625	2,330	2,600	2,500	......	2.510
Coke analysis:								
Moisture, per cent.	1.43	0.65	2.36	1.22	0.95	0.96	0.26	0.18
Ash, per cent.	11.48	16.90	13.24	16.84	15.86	19.35	14.20	15.92
Volatile, per cent.	2.70	2.62	4.60	3.60	2.86	2.95	1.80	3.18
Fixed carbon, per cent.	84.39	79.83	79.80	78.34	80.33	76.73	83.74	80.72
Sulphur, per cent.	1.04	1.33	0.84	1.86	1.53	1.99	1.27	2.53
Lb. dry coke	1,200	1,250	1,120	1,310	......	1,230	1,100	1.280
Per cent over 1 in.	74	57	55	66	......	69	77	80
Per cent under 1 in.	36	43	45	34	......	31	23	20
Quality coke‡	W.D.S.	W.D.S.	W.D.S.	D.S.	W.D.S.	W.D.S.	W.D.S.	M.W.D.
Cu. ft. gas	8,400	7,500	6,100	3,000	3,000	11,950	6.350	8,800
Av. B.t.u., gross	495	490	504	436	very	370	542	500
Av. B.t.u., net	460	456	469	400	low	335	507	466
Total B.t.u., gross§	4,158	3,675	3,074	1.305	......	4.425	3.442	4.150
Organic sulphur	......	......	......	1.5	......	12.0	4.0	9.50
Lb of ammonia	4.60	3.4	3.3	1.6	......	3.5	......	4.65
Lb. of light oil	15.0	......	......	trace	......	13.0	2.00	7.4
Lb. of tar	98	52	45	8	......	68.5	......	96
Per cent water in tar	31.1	14.8	44	43	......	53.5	......	47
Analysis tar, dry:								
Distil. to 110 deg. C.	0.0	0.0	0.0	0.7	......	0.0	......	0.0
110 to 170	0.0	0.62	0.37	1.2	......	2.1	......	1.2
170 to 235	19.5	14.10	20.1	17.7	......	25.3	......	21.8
235 to 270	9.6	14.90	15.3	17.7	......	10.8	......	14.6
270 to 315	7.6	6.40	10.6	9.8	......	11.3	......	10.2
315 to 335	24.9	19.90	21.2	26.1	......	11.0	......	12.6
Pitch	37.9	43.60	32.1	26.8	......	36.9	......	38.8
Sp. gr.	1.11	1.13	1.09	1.10	......	1.11	......	1.21

Test No. †	9W	10RM	10W	11RM	11W	12RM	12A	13RM
Hours	14	15	16	12	13	9	......	12
Retort temp., deg. C.	......	......	low	hot	hot	(1050?)	......	(1100?)
Coal analysis:								
Moisture, per cent	4.08	3.12	3.47	3.85	4.35	0.85	......	3.30
Ash, per cent	6.26	8.08	5.87	11.55	7.70	7.15	......	10.10
Volatile, per cent	42.16	38.03	38.66	37.45	38.55	34.90	......	39.00
Fixed carbon, per cent	47.50	50.77	52.04	47.15	49.40	57.10	......	47.60
Sulphur, per cent	2.25	0.90	0.76	1.31	0.83	1.26	......	2 91
Carbon, per cent	70.36	70.68	72.62	67.91	70.70	78.08	......	68.90
Hydrogen, per cent	5.00	4.99	4.99	4.80	5.06	5.03	......	4.80
Oxygen, per cent	14.73	13.94	14.33	14.04	14.27	6.98	......	12.01
Nitrogen, per cent	1.40	1.41	1.43	1.39	1.44	1.50	......	1.28
B.t.u.	13,002	12,648	12,889	12,270	12,729	14,039	......	12,302
H—⅛O, per cent	3.16	3.24	3.20	3.05	3.28	4.16	......	3.30
Ash fusibility:								
Incipient, deg. F.	2,100	2,750	2,765	2,550	2,700	2,315	......	2,375
Complete, deg. F.	2.510	......	......	2.630	2,800	2.450	......	2,500

Coke analysis:								
Moisture, per cent	0.09	0.22	0.51	0.15	0.09	0.05	......	0.13
Ash, per cent	11.33	13.52	10.87	17.87	12.72	12.30	......	18.54
Volatile, per cent	2.17	2.53	2.53	2.29	0.81	1.12	......	1.31
Fixed carbon, per cent	86.41	83.73	86.09	79.69	86.38	86.53	......	79.97
Sulphur, per cent	1.89	0.81	0.55	1.11	0.66	0.91	......	2.38
Lb. dry coke	1,280	1,250	1,258	1,335	1,300	1,300	......	1,280
Per cent over 1 in.	80	70	66	88	85	90	......	76
Per cent under 1 in.	20	30	34	12	15	10	......	24
Quality coke‡	M.W.D.	W.D.S.	W.D.S.	W.D.S.	W.D.S.	W.D.S.	M.W.D.	W.D.S.
Cu. ft. gas	7,900	5,000	6,000	9,500	11,000	10,000	10,600	9,400
Av. B.t.u., gross	565	500	515	510	495	525	520	500
Av. B.t.u., net	530	465	480	475	460	490	485	465
Total B.t.u., gross§	4,464	2,500	3,090	4,835	5,450	5,250	5,490	4,700
Organic sulphur	9.0	6.0	3.2	5.0	1.0	9.3	9	7.0
Lb. of ammonia	4.95	......	3.8	4.1	4.7	4.6	......	5.34
Lb. of light oil	2.5	......	17.0	17.5	......	15	......	14
Lb. of tar	104	58	99	66	60	100	......	103
Per cent water in tar	40	50	58	30	40	40	......	40
Analysis tar, dry:								
Distil. to 110 deg. C.	0.0	0.1	0.0	0.8	0.3	0.4	......	0.2
110 to 170	0.8	0.5	0.6	0.8	1.8	3.6	......	1.5
170 to 235	22.0	18.2	15.7	16.1	18.3	19.4	......	23.5
235 to 270	9.0	13.1	13.0	10.8	7.8	7.3	......	1.6
270 to 315	11.4	12.6	18.0	20.9	8.7	9.2	......	11.4
315 to 335	9.2	13.1	16.5	19.5	22.6	13.6	......	22.1
Pitch	46.9	42.4	35.5	31.1	40.5	46.5	......	39.7
Sp. gr.	1.13	1.08	1.08	1.12	1.11	1.04	......	1.12

Test No. †	13W	14RM	14W	15RM	16RM	17RM	18RM
Hours	16	14	15	13	11	14½	17½
Retort temp., deg. C.	980	low	too low	hot	1,075	1,100	970
Coal analysis:							
Moisture, per cent	0.05	3.20	3.54	3.90	1.80	3.90	3.15
Ash, per cent	6.30	13.45	6.16	13.50	12.45	12.80	15.00
Volatile, per cent	42.25	37.40	39.84	39.65	36.95	38.50	40.95
Fixed carbon, per cent	51.40	45.95	50.46	42.95	48.80	44.80	40.90
Sulphur, per cent	1.67	2.35	1.55	3.18	1.14	2.88	4.89
Carbon, per cent	74.00	66.27	72.23	64.88	69.78	65.94	63.00
Hydrogen, per cent	4.70	4.62	5.00	4.66	4.77	4.85	5.00
Oxygen, per cent	11.98	11.98	13.71	12.67	10.46	12.21	11.02
Nitrogen, per cent	1.35	1.33	1.35	1.11	1.40	1.32	1.09
B.t.u.	12,987	11,992	13,122	11,842	12,532	11,794	11,632
H—⅛O, per cent	3.20	3.12	3.29	3.08	3.46	3.33	3.62
Ash fusibility:							
Incipient, deg. F.	2,425	2,300	......	2.480	2,670	2,400	1,875
Complete, deg. F.	2,565	2,475	......	2,540	2,750	2,540	2,125
Coal analysis:							
Moisture, per cent	0.20	0.37	0.15	0.42	0.10	0.25	0.20
Ash, per cent	11.10	21.55	12.15	20.74	18.00	20.30	21.95
Volatile, per cent	3.05	2.55	0.74	3.68	1.65	2.95	2.40
Fixed carbon, per cent	35.65	75.53	86.96	75.16	80.25	76.50	75.45
Sulphur, per cent	1.53	2.47	1.29	2.50	0.78	2.36	3.64
Lb. dry coke	1,240	1,290	1,200	1,327	1,370	1,240	1,245
Per cent over 1 in.	76	76	78	82	84	69	69
Per cent under 1 in.	24	24	22	18	26	31	31
Quality coke‡	W.D.S.	W.D.S.	......	W.D.S.	W.D.S.	W.D.S.	W.D.S.
Cu. ft. gas	9,000	8,500	9,500	9,200	8,850	12,100	10,300
Av. B.t.u., gross	475	505	435	535	435	475	420
Av. B.t.u., net	440	470	400	500	400	440	392
Total B.t.u., gross§	4,275	4,293	4,133	4,922	3,850	5,750	4,326
Organic sulphur	6.5	12.0	3.0	22	8	7.5	15
Lb. of ammonia	5.0	3.8	4.6	4.80	4.50	4.8	4.8
Lb. of light oil	13	9.0	......	23	16	13.5	13
Lb. of tar	90	60	86	82	115	73	102
Per cent water in tar	35.3	25.5	33.3	23.50	59.4	23.5	46
Analysis tar, dry:							
Distil. to 110 deg. C.	0.4	0.2	2.5	0.3	2.3	0.10	0.3
110 to 170	2.2	1.0	3.5	2.3	2.5	2.90	0.5
170 to 235	21.4	19.2	26.0	20.4	20.7	29.10	22.8
235 to 270	11.1	11.1	22.0	8.3	9.7	5.80	11.3
270 to 315	13.2	9.1	7.5	8.0	10.7	22.10	24.5
315 to 335	21.9	12.8	......	18.7	17.3	9.40	6.9
Pitch	29.8	76.6	......	42.00	36.8	30.60	30.7
Sp. gr.	1.13	1.06	1.13	1.15	1.13	1.13	1.15

Test No.†	19RM	20RM	21RM	21A	21A
Hours	14.5	14	10	9.08	9.03
Retort temp., deg. C.	1,090	1,000	1,000	1,170	1,200
Coal analysis:					
Moisture, per cent.	4.38	1.83	1.64	.....	.....
Ash, per cent.	12.42	10.60	4.10	.....	.....
Volatile, per cent.	37.10	41.80	34.50	.....	.....

Fixed carb., per cent.	45.10	45.77	59.76	.....	.....
Sulphur, per cent.	2.63	4.47	0.64	.....	.....
Carbon, per cent.	64.00	68.45	80.44	.....	.....
Hydrogen, per cent.	4.82	4.82	5.20	.....	.....
Oxygen, per cent.	14.87	10.55	8.20	.....	.....
Nitrogen, per cent.	1.26	1.13	1.42	.....	.....
B.t.u.	11,978	12,876	14.157	.....	.....
$H-\frac{1}{8}O$, per cent.	2.96	3.50	4.18	.....	.....
Ash fusibility:					
Incipient, deg. F.	.....	.....	.....	.....	.....
Complete, deg. F.	.....	.....	.....	.....	.....
Coke analysis:					
Moisture, per cent.	0.20	0.27	0.22	.....	.....
Ash, per cent.	20.02	24.17	7.76	.....	.....
Volatile, per cent.	3.06	1.96	3.25	.....	.....
Fixed carb., per cent.	76.72	73.60	88.77	.....	.....
Sulphur, per cent.	2.34	3.84	0.66	.....	.....
Lb. dry coke	1,260	1,258	.....	1,280	1,280
Per cent over 1 in.	59	86	.....	.....	.....
Per cent under 1 in.	41	14	.....	.....	.....
Quality coke‡	M.W.D.	M.W.D.	M.W.D.	M.W.D.	M.W.D.
Cu. ft. gas	10,800	8,500	9,500	9,886	11,209
Av. B.t.u., gross	460	493	600	600	552
Av. B.t.u., net	424	458	570	543	503
Total B.t.u., gross§	5,000	4,191	5,700	5.930	6,187
Organic sulphur	10	13	.....	.....	.....
Lbs. of ammonia	3.85	.....	.....	.....	.....
Lbs. of light oil	3.2	9	10.6	.....	.....
Lbs. of tar	61	80	.....	118	73
Per cent water in tar	58.8	45.3	43.5	.....	.....
Analysis tar, dry:					
Distil. to 110 deg. C.	2.0	0.0	3.0	.....	.....
110 to 170	3.0	4.0	3.0	.....	.....
170 to 235	23.00	25.0	19.0	.....	.....
235 to 270	11.00	11.0	19.0	.....	.....
270 to 315	6.00	3.0	2.0	.....	.....
315 to 335	.....	.....	.....	.....	.....
Pitch	.....	.....	.....	.....	.....
Sp. gr.	1.12	1.13	1.11	.....	.....

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*Coke. light oil, tar and ammonia are given in pounds per 2,000 lbs. of coal, gas in cubic feet at 60 deg. F and 30 in. Hg, organic sulphur in the gas in grains per 100 cubic feet.

†Test No. 1—Athens County, Ohio, Middle Kittanning No. 6; seam, 72 in. thick.
Test No. 2—Perry County, Ohio, Middle Kittanning No. 6; seam, 48 in. thick.
Test No. 3—Athens County, Ohio, Upper Freeport No. 7; seam, 48 in. thick.
Test No. 4—Athens County, Ohio, Pittsburgh No. 8; seam, 96 in. thick.
Test No. 5—Perry County, Ohio, No. 6; seam.
Test No. 6—Morgan County, Ohio, Middle Kittanning No. 6; seam, 48 in. thick.
Test No. 7—Athens County, Ohio, Middle Kittanning No. 6; seam, 72 in. thick.
Test No. 8—Perry County, Ohio, Middle Kittanning No. 6; seam, 84 in. thick.
Test No. 9—Perry County, Ohio, Middle Kittanning No. 6; seam, 42 in. thick.
Test No. 10—Athens County, Ohio, Middle Kittanning No. 6; seam, 60 in. thick.
Test No. 11—Athens County, Ohio, Middle Kittanning No. 6; seam, 60 in. thick.
Test No. 12—Smithers Creek, West Va., No. 2, gas. 12A=high temp. run.
Test No. 13—Athens County, Ohio, Upper Freeport No. 7; seam, 48 in. thick.
Test No. 14—Hocking County, Ohio, Middle Kittanning No. 6; seam, 48 in. thick.
Test No. 15—Meigs County, Ohio, Pittsburgh No. 8; seam, 60 in. thick.
Test No. 16—Boone County, West Va., Kanawha No. 5; seam, 52 in. thick.
Test No. 17—Athens County, Ohio, Middle Kittanning No. 6; seam, 66 in. thick.
Test No. 18—Vinton County, Ohio, Jackson No. 4; seam, 48 in. thick.
Test No. 19—Hocking Coal No. 6.
Test No. 20—Belmont Co., No. 8; coal.
Test No. 21—Elkhorn coal. 21A=high temp. runs.

‡Quality of coke.

M=metallurgical, W=water gas, D=domestic, S=steam, from a physical point of view, determined by inspection.

§Total B.t.u. value given in thousands; thus, 3.642=3,642,000.

None of the cokes made in these tests were "stickers." That is, none of them swelled so as to jam in the retort and fail to discharge easily when the discharge door was opened.

Properties of the Gas

The amount and quality of the gas obtained per ton of coal are very powerfully influenced by the conditions of retorting, particularly the temperature. In coke-making practice, the temperature is lower and the amount of gas made is lower per ton of coal than in gas retorts when the gas is the first consideration. For instance, at high temperatures the oxygen from coals tends to react with carbon, thus

$$C+H_2O=CO+H_2$$
$$C+2H_2O=CO_2+2H_2$$

(the oxygen being largely liberated as water). If all the oxygen should be so used, the gas so made would be about 13,000 cubic feet per ton of coal in addition to the hydrocarbons. At low temperatures, the above reactions are very slow and most of the oxygen passes out as steam condensing in the cooling system. As previously mentioned, heating rates were more nearly like those used in coke-oven practice than gas-house practice, hence, these results for gas are probably somewhat low; at the same time low temperatures tend to be favorable to a high yield of light oils and tars.

The amount of sulphur in the gas combined as "organic sulphur" was in all cases far lower than the commonly accepted allowance of 30 grains per 100 cubic feet, even when the highest sulphur coals were used. Hence no fear on the score of sulphur in the gas from these coals need be had. Of course, the raw gas contains a large amount of H_2S, but since the advent of the "Seaboard process" for removing H_2S from gas and the improvement of other methods, the cost of the removal of H_2S is so low as to be but a very minor factor. It is quite possible, indeed, that the recovery of sulphur which is possible by the "Seaboard process" may turn high sulphur into an asset rather than a detriment.

We can, therefore, judge the value of our coals as gas-making coals solely on the basis of the value of the coke, B.t.u. in the gas, and the yield of ammonia, tar, light oils, etc., recoverable from a ton of coal.

Using as a base of comparison any of the items of either proximate or ultimate analysis, plotting the results of gas making against chemical analysis reveals no relationship. The reason is that the influence of varying retorting conditions is much more powerful than the influences of variation in chemical compositions in these coals.

Coals From Different Seams Compared

Coals from seams No. 4, 5, 6, 7 and 8 were included in the tests, and these were checked against Elkhorn coal and Smithers Creek No. 2 gas coal from West Virignia. The results from each seam were averaged to get the figures given in Table II. All results are for products obtained from 1 ton (2,000 lbs.) of coal, except the determination of B.t.u., which is on the basis of a cubic foot of gas.

In Table I test No. 21 shows the results obtained by retorting Elkhorn coal

TABLE II—COMPARISON OF RESULTS OBTAINED WITH COALS FROM DIFFERENT SEAMS

(Yields per short ton of coal)

	Smithers Creek No. 2 Gas	Seam No. 4	5	6	7	8	Elkhorn
Pounds of coke	1,300	1,245	1.370	1,260	1,257	1.302	1,280
Cu. ft. of gas	10.000	10.300	8.850	9,287	9.725	8,833	9,500
Av. gross B.t.u.	525	420	435	484	474	523	600
Total B.t.u.	5,250	4,326	3,850	4.477	4,605	4,627	5,700
Pounds of ammonia	4.8	4.5	4.32	5.00	5.1	......	......
Pounds of light oil	13	13	16	9.64	11	13.8	......
Pounds of tar	102	100	115	73.3	88.3	84.6	118

Only one sample of coal was obtained from each of No. 4 and No. 5 seams. However, when the best results obtained from each seam are compared the figures are:

	Smithers Creek No. 2 Gas	Seam No. 4	5	6	7	8	Elkhorn
Pounds of coke	1,300	1,215	1,370	1,240	......	1,327	1,280
Cu. ft. of gas	10,000	10,300	8,850	12,000	11,000	9,200	9,500
Av. gross B.t.u.	525	420	435	475	470	535	600
Total B.t.u.	5,250	4,326	3,850	5,750	5,170	4,922	5,700
Pounds of ammonia	4.6	4.8	4.5	4.8	4.5	4.8	......
Pounds of light oil	15	13	16	13.5	......	23	......
Pounds of tar	100	102	115	73	83	82	118

at higher temperatures, and from these it would seem probable that improved results would be also obtained by retorting Ohio coals at higher temperatures.

SUMMARY OF THE RESULTS

Coke—The coke obtained from the Ohio coals is equal in quantity to the coke obtained from recognized gas coals. Its quality for metallurgical purposes is decidedly inferior except when certain coals like Pocahontas is mixed with the Ohio coal before retorting. The cokes obtained from the Ohio coals are, in general, quite satisfactory for domestic heating. The more friable cokes could probably be made satisfactory by using a small proportion of Pocahontas or similar coal with the Ohio coal; the exact proportions will be determined by later tests. The ash in the cokes obtained in these tests is higher than is desirable, but this is largely due to the fact that the coals were obtaind when the mines were not properly functioning. The fusibility temperature is in many cases satisfactorily high. Preliminary tests made in domestic furnaces would seem to indicate that these Ohio cokes will be satisfactory for domestic heating, which is by far the most important outlet for coke. When the coke is mixed with about its own weight of coal, the mixture burns with a practically smokeless chimney.

Tar—The tar made is somewhat less in amount when Ohio coals are coked than when a standard gas coal like Elkhorn is coked, although in some cases the tar obtained from Ohio coals was practically as high as from Elkhorn. The value of the tars could not be compared, as time and funds for the complete analysis of the tars were not available.

Ammonia—The results obtained in these tests would indicate that the Ohio coals will yield about the same amount of ammonia as a standard gas coal (No. 2 gas or Elkhorn).

Light Oils—The amount of light oils obtained from the gasification of the Ohio coals is practically the same as that obtained from standard gas coal. However, the composition of the light oils was not investigated.

Gas—The volume of gas obtained by the gasification of Ohio coals at the temperatures used is about the same as that obtained from standard gas coals. The B.t.u. value of the gas from Ohio coals is considerably smaller than that obtained from Elkhorn, although nearly equal to that obtained from the No. 2 gas coal. This is especially true of the No. 8 coal. The total B.t.u. value of the gas from a ton of Ohio coal is in general lower than that from Elkhorn coal.

The impurities such as "organic sulphur" and naphthalene in the gas obtained from Ohio coals are satisfactorily low, being well within the ordinary legal requirements when the gas is made with vertical retort equipment. The large amount of H_2S in the gas before purification need not be regarded as a detriment. In fact, in some cases the recovery of the sulphur may produce revenue.

It is not claimed that Ohio coals are as satisfactory as the accepted standard gas coals for gas-making purposes. However, from all technical points of view, Ohio coals can be satisfactorily used for gas and byproduct making, including the use of the coke for domestic purposes. If economic conditions, such as the cost of mining, freight rates, etc., are such that Ohio coals can be delivered to a gas plant at a lower price than other coals, it is quite possible that they may compete successfully for these purposes.

Since the work reported here was completed new equipment has been added and the general arrangement has been changed as a result of the experience gained in the past year and a half. With these changes it will now be possible to obtain highly accurate data on the carbonization of the different coals. Using the results obtained in the present investigation as a basis for selecting the most promising types of Ohio coal, it is believed that it will be possible to arrive at an exact evaluation of these coals as compared with the standard gas coals of other states.

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Standardization of Telephone Rates

By

C. A. WRIGHT
and
D. B. JUDD

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C. A. WRIGHT
Professor of Electrical Engineering

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STANDARDIZATION OF TELEPHONE RATES

SECTION I. INTRODUCTION

To protect the telephone companies and the telephone-using public, as well as to regulate rates, nearly every state in the Union has a public-utility or railroad commission. These commissions, in the majority of cases, have the authority to prescribe uniform systems of accounts, to authorize the issue of securities and dividends, to fix and enforce standards of service, and to supervise generally the relations of public utilities with the public. The regulation of the telephone companies by these public-utility commissions has not accomplished its full purpose, although it cannot be said that it has been a failure.

FAULTS OF THE PRESENT SYSTEM

Apparent faults of the present "cost plus" system of determining telephone rates are:

1. No incentive is offered the telephone companies to better their service or provide it economically.

2. The system of regulation is laborious and expensive. It is burdensome both to the public-utility commissions, and to the telephone companies.

3. The present system of rate determination is not exact and definite. It involves much guesswork.

4. The present system of regulation does not, in every instance, prevent injustice to telephone subscribers or to telephone companies.

OBJECTS OF INVESTIGATION

The object of the investigation, which is described in this report, was to obtain information and propose methods of correcting some of the deficiencies of the present system which have been most subject to criticism. The system of rate determination which is proposed offers a remedy for (1) the lack of incentive to efficient operation, and a partial remedy for (2) the present labor and expense, without emphasizing (3) the indefiniteness, and (4) the injustice of the present system.

This report sets forth also some original methods of investigation which have been effective in the solution of this problem and which may be found useful by others interested in similar problems.

It does not attempt to decide what elements constitute a fair valuation; what methods should be followed in dealing with depreciation and depreciation funds, or rate of return; whether or not a utility should be taxed or subsidized, or the many other legal and economic problems of rate regulation. It does not attempt to discuss the organization or personnel of the public-utility commissions or primarily to decide whether original or reproduction or normal costs should be recognized as the fair basis for rate determination. The methods suggested, however, *do* base rates upon normal telephone plant costs; that is, average plant costs over a number of years, rather than the cost of any particular telephone plant.

This report proposes a system of rate determination in which the rates for any community are the average rates for communities of the size of the community in question, modified by bonuses or penalties. These bonuses and penalties take account of the natural obstacles presented to the telephone company, and the quality of the telephone service made available to the community.

ASSUMPTIONS

The following assumptions have been made in the development of the proposed plan:

(1) That the purpose of the regulation of the financial policy and service of telephone companies is to obtain the best possible telephone service at the lowest cost which is just to the telephone companies. In order to accomplish this object the state grants the telephone company a monopoly in its community and enforces rules which limit the return on the investment and which prevent speculation. *The incentive for providing good service economically, however, is small because there is no chance to obtain an extraordinary return on investment. The "cost plus" system of rate regulation in its present form is not conducive to efficient management.*

(2) That, if the investment in the telephone company is completely safeguarded from the effects of competition in a given operating field, and from speculation, a small guaranteed return on the investment will attract sufficient new capital to the utility to provide necessary service extensions. Under a competitive system, the return over the cost of giving the serv-

ice would be interest plus a profit which is reasonable for the risk involved. If risk is eliminated by permitting only one company to operate in a given field, rates which consist of cost of giving the service plus interest will not only be just but sufficient to attract new capital to the enterprise. Under such a condition of monopoly, the element of competition must be introduced in some other manner if there is to be any incentive to good management.

(3) That the telephone service in each community should be self-sustaining. This does not necessarily mean that each class of local service in a community should be self-sustaining. It does mean, however, that neither local nor toll service should be supported in part by the other. Exception to the assumption that the telephone service in each community should be self-sustaining might be taken in one possible case—that of very small communities. The allocation of telephone rates to communities and classes of service is discussed in Section II of this report.

(4) That not only *should* the relative values of telephone service to the subscribers in the various communities be considered in fixing rates, in addition to the cost of producing the service, but also that they actually *have* been considered. Telephone companies will contend more strenuously for reasonable rates if the service is good. Under these conditions, the public-utility commissions will more readily grant them, and the public will more willingly pay them. The various indications of the value of the service, however, have not always been considered in their proper relative importance. "Free-service" between communities, and the general impression of the public-utility commissions of plant conditions have been given too much consideration.

Among the factors which affect the *cost* of producing the service are the number of stations served, the calling-rate, and the extent of the territory served. Among those factors which affect the *value* of the service to the subscribers are the number of stations served, and the quality of the service. The use made of the telephone by the average subscriber, and the telephone development are taken as measures of the quality of the telephone service. These indications of the cost and value of service have not been given sufficient consideration.

(5) That the wage-scale will vary only with the size of the community, and that the cost of all telephone plants serv-

ing the same number of stations should be the same, except for the differences due to extent of territory served and the maximum calling-rate.

PROPOSED PLAN

The plan for the standardization of telephone rates proposes:

(1) That if the rates of a sufficiently large number of telephone exchanges serving the same number of stations are averaged, the average rate is the fair one for exchanges of that size. It is the rate which will produce income sufficient to cover the cost of the service, and to provide a reasonable return on the investment. The effects of excessive competition, monopoly, and unfair influence which are in some cases of advantage to the telephone company, and in other cases to the community, will be eliminated if a large number of cases are considered. Typical average rates have been obtained and station-rate curves, showing the relation between these average rates and the numbers of stations served in exchanges are presented in Section VI.

(2) That when any particular telephone exchange is considered, the natural obstacles which the telephone company has to overcome, and the degree of excellence of its service should be recognized by bonuses or penalties applied to the average rates for exchanges of the size of the one in question. There have been taken as the bases for bonuses and penalties (a) the length of pole line per station, which is an approximate measure of the extent of the territory served by a telephone exchange; (b) the maximum busy-hour calling-rate per station, which is an approximate measure of the proper size of the central office; (c) the average number of calls per station per day, which is an approximate measure of the cost of operation and of the value of the telephone service to the individual subscribers; and (d) the development in telephones per thousand people in the area served, which is also an approximate measure of the value of the telephone service to the subscribers. Bonus and penalty bases (b) and (c), are combined in this investigation. The desirability of recognizing plant condition and community wealth by bonuses or penalties should also receive consideration.

The public-utility commissions, the telephone companies and the telephone-using public have consciously or unconsciously recognized the above bases for bonuses and penalties, but not to a sufficient extent. A method of determining the degree to which they *have* been recognized is presented in this report. Analyses have also been made of typical companies, which show the extent to which they *should* have been recognized. Bases for bonuses and penalties are discussed in Sections III, IV, and V.

The proposed plan allows the telephone companies the advantages of monopoly in their respective operating fields. The system of bonuses and penalties introduces the element of competition with its attendant advantages. Each company competes with others in its class for bonuses. For all bonuses that are granted, penalties must be assessed also, as the bonuses and penalties are based upon the average rates for the class of telephone companies which includes the company in question.

PREDICTED RESULTS

The proposed plan for the standardization of telephone rates offers many advantages. It may be predicted generally:

(1) That if the public-utility commissions fix average rates corresponding to the numbers of stations served, modified by the bonuses and penalties suggested, their work will be decreased to such an extent that reasonable attention may be given to fixing standards of service and enforcing them.

(2) That the suggested bonuses and penalties will offer an incentive to the telephone companies to better their service, independent of the efforts of the public-utility commissions.

(3) That the possession by the public-utility commissions, the telephone companies, and the telephone-using public of a knowledge of what are the average, and therefore, just rates, and the just bonuses and penalties will tend to promote harmony in fixing rates.

The above assumptions, proposed plan, and predicted results will be discussed more fully in Sections III, IV, V, and X.

SECTION II. PRESENT METHOD OF DETERMINING TELEPHONE RATES

Public-utility commissions have generally based rates and amounts of securities and dividends authorized on "fair valuations." Ordinarily these fair valuations are estimated costs of reproduction, less depreciation, under normal conditions, checked, and in some cases, modified by original costs and the amounts of outstanding securities issued against the property. An equitable rate has been considered to be one which will enable the telephone company to pay all expenses of operation and provide a return on the invested capital which will attract a sufficient supply of new capital to develop the telephone service to the extent demanded by the public.

EFFECTS OF MONOPOLY

The tendency in the last few years has been to eliminate duplication of telephone equipment in the operating areas by granting the companies monopolies in their communities. National and state laws preventing the purchase of one competing company by another have been repealed in many cases. Many consolidations of competing telephone companies have been effected. There has always been a tendency in small communities on the part of dissatisfied telephone subscribers to organize new and competing telephone companies. Such companies have frequently been unsuccessful financially, and the operation of competing companies has always been harmful to telephone service. The organization of competing companies is now generally prevented by laws which require that a "certificate of convenience and necessity" be granted a new telephone company by the public-utility commission before the newly organized company is allowed to operate. When two companies operate in any community, the resulting duplication of equipment increases the cost of the service. The value of the service to the subscribers is greatly decreased because of the decrease in the number of subscribers to which any one subscriber has access. Monopoly is, therefore, an advantage in the production of telephone service, but under the existing conditions, the elimination of duplication of telephone plant and service has been accompanied by the elimination of competition. The result has been a tendency of the telephone companies to cease trying to provide the best service economically.

LACK OF INCENTIVE TO EFFICIENT MANAGEMENT OFFERED BY PRESENT SYSTEM

This policy of rate-making does not then offer an incentive to the telephone companies to initiate more efficient methods of giving service. If their cost of production is decreased, there is a tendency for rates to be decreased. The result is that efficient management has been penalized. If, through inefficient management, the cost of telephone service is high, the "cost plus" system of rate determination provides the additional revenue necessary, and encourages the management in its inefficiency.

Public-utility commissions are generally authorized to set service standards and enforce them; but because of the expense involved in valuations and similar work, there is little opportunity to set and enforce such standards. There is little incentive for the efficient management to provide the best service, when it gets no credit for it, and when the inefficient management provides poor service more conveniently and with less cost. The existing system of rate determination, by guaranteeing a fixed return on the investment, regardless of whether the telephone service is good or poor, rewards inefficient management. It discourages good management, because if the service is good, it prohibits a return greater than this fixed return.

Each valuation and rate study are an occasion for bargaining and scheming on the part of both the telephone company and the representatives of the public, to persuade the public-utility commission to take action of advantage to one or the other.

COST OF VALUATIONS AND ASSOCIATED WORK

One public-utility commission has in some cases allocated the total expense of producing the service to business, residence, and various other classes of service, and apportioned the cost in accordance with the use made of the service. The allocation of rates to various classes of telephone service is generally determined by other factors than cost. For example, habit or custom, value to the subscriber, and the necessity for controlling the development of certain classes of service influence rate allocation. While it has been argued that an allocation of expense in accordance with cost would give results differing greatly from the allocation determined by the above factors, the cost studies of the commission indicated that this is not the case. It has also been urged that it is difficult or even impossible to allocate correctly the expense. The commission's cost studies indicate that it is not impossible to do this. Further experience will show whether or not the benefits gained from such a cost study justify its expense.

Even if no attempt is made to allocate the expense of operation, and the rates are based upon the total cost of producing the service, the expense and inconvenience of this method of determining rates are great. The entire time of many regulating commissions is given to making valuations and determining rates, based upon the cost of plant and of operation. Little time is left for setting and enforcing standards of service. If, as has happened during the past few years, the commissions grant many rate increases, the suspicion of the telephone-using public that the utilities are favored is aroused. The result is a loss in effectiveness of the commission.

ELEMENTS OF INJUSTICE IN PRESENT RATE SYSTEM

The existing system of rate-determination is frequently unjust to telephone companies or to subscribers. It has already been suggested that it may penalize economical management and good service, or encourage uneconomical management and poor service. The large and powerful telephone company has an advantage over the small company because of its more able and more numerous personnel, because of its more extensive sources of information, and because of its greater influence in pleading its cause with the public-utility commissions. Of course, such an advantage works an injustice to the small company.

Telephone companies, even though the return on their investment is limited by public-utility commission regulation, are not now protected from all risks. In times of rapidly advancing prices, public utilities are frequently in financial distress. There are some of them which in the past few years would have been glad to discontinue their business, and yet they were compelled by public-utility commissions to continue to furnish service at a loss. Public-utility economists have suggested that this condition might warrant the appropriation by the state, in years which are "fat" ones for the utilities, of part of their income, and financial relief by the state in "lean" years. Prompt recognition by the commissions of the condition of the utilities and relief by rate increases would be a more satisfactory and practicable means of eliminating such injustice. It would, however, require a considerable amount of courage on the part of the public-utility commissions, for they would meet with strong popular disapproval.

One source of occasional injustice to public utilities is rate regulation and tax assessment by independent state commissions which frequently act independently of each other and of other state and federal government agencies which perform similar functions.

FAIRNESS OF THE "COST-PLUS" RATE SYSTEM AS APPLIED TO GROUPS OF TELEPHONE COMPANIES

The justice of fixing as fair rates for telephone service those which will cover the average cost of service and allow a reasonable return on the average investment of a large number of telephone companies of the same size and same general character can be well defended. Under a condition of unregulated competition, the rates of telephone companies would be sufficient to cover the cost of producing the service, and to provide an adequate return on the investment. This return would consist of interest equivalent to the return on money invested with the greatest possible safety plus a profit which would vary with the risk of the investment. Under a condition where there was no risk, investors would be willing to invest in the telephone companies, if the rates covered only the cost of service and interest which the safest possible investment would yield. The granting of monopolies to the telephone companies in their communities removes much of the speculative risk from the investment. The greatest risk to the success of any such enterprise is that of competition. If the management is efficient, the risks of obsolescence, fire, wind, flood, and other hazards may be provided for, so that a return on the investment which equals interest plus a very small speculative profit, will be sufficient to attract new capital to the telephone companies. Under such a condition, the "cost plus" system of rate determination would be a just one for fixing average rates for a number of telephone companies of the same size and same general character.

The risk of competition has been in large part removed from the telephone business by the various states in order that they may more effectively provide service. The people of these states rather than the telephone companies or telephone subscribers are entitled to the amount equivalent to the resulting decrease in the telephone rates. Actually, the telephone subscriber gets this saving in the form of decreased rates which partly compensate for the overburdensome taxes on telephone companies. These taxes are assessed by the various states upon the telephone companies and in turn passed along by the companies to the subscribers in the form of increased rates. This gain in economy because of monopoly and the amount of taxation are probably of the same order of magnitude, so that the total effect of these conditions is that the telephone subscribers pay for the monopoly advantages granted by the states with part or all of the taxes assessed upon the telephone companies by the states. The telephone subscriber pays a rate which may equal what he would pay under a competitive system in which the telephone companies are not taxed.

Great social benefit to the state results from the development in small communities of telephone service. These taxes, therefore, might well be used to subsidize the telephone companies in small communities. In these communities it is particularly difficult to obtain rates sufficient to pay for the telephone service needed, because the people have not been educated to a realization of the great economic advantage of the telephone, and because the cost per station of construction of a small telephone system is great on account of the long distances involved. In such communities the need for telephones is great and a valuable social service is performed when a telephone system is installed. In such cases, it is sometimes necessary and desirable that aid be extended in the development of the telephone service and it would be more satisfactory and just to aid the telephone company by state subsidy than by financial help from the telephone companies of the larger towns and cities. The provision of such aid, however, is not to be associated with the plan for rate-determination here proposed to any greater extent than with the methods at present followed.

The present "cost plus" system of rate determination is, therefore, fair for the average company. The average quality of service of a group of telephone companies may be considered to be of acceptable grade, and it may be assumed that the average efficiency of management is good. When the individual company is considered, however, modifications in rate must be made to take account of unusual conditions.

INDEFINITENESS OF PRESENT RATE SYSTEM

An advantage, which may be advanced by some in favor of the present method of determining telephone rates, is that it is exact and definite. If this were so, it would be a strong argument in its favor, but it is not so. Frequently records are not available from which to determine the original costs. It is necessary to guess at many costs in determining the cost of reproduction. The element of "going value" is one which affords much opportunity for differences of opinion. There may sometimes be difficulty in deciding what portion of a telephone plant is used and useful in giving telephone service.

One public-utility commission, arguing in favor of basing telephone rates upon service rendered rather than upon the cost of giving the service, has made the following statement:

> "In the entire scheme of rate-making the valuation of the property is only an aid, an approximation. No matter how expensive or how cheap the property of a utility may be, the public is only interested in service, and for that service, it should pay a fair price; but it is absurd to contend that the enormous sum of money invested in antiquated equipment, furnishing inadequate service by ancient methods discarded in the intelligent progress of the art, entitle a company to higher rates, than better service furnished for less money invested in more cheap, simple and up-to-date equipment.
>
> "The factor of management has more to do with earnings than all the hypothetical reproduction and conjectural depreciation compiled by all the engineers and commissions that have considered them. The public cannot be expected to pay rates on unnecessary property, extravagant construction, improvident investment, or excessive facilities not yet demanded."

In regard to the "reproduction less depreciation" theory of determining rates urged by the telephone company in question, the commission stated:

> "This method of determining value usually included percentages for engineering service never rendered, hypothetical efficiency of unknown labor, conjectural depreciation, opinions as to the condition of the property, the supposed action of the elements, and of course its correctness depends upon whether superintendence was wise or foolish, the investment improvident or frugal. It depends upon probability rather than reality; it is based on such half truths that it bears only remote resemblance to fact, and rises at best to the plane of a dignified guess."

While many public-utility commissioners and telephone men may not entirely agree with this statement, it is a strong indication of the indefiniteness and inexactness of this method of determining rates.

ALLOCATION OF TELEPHONE RATES TO COMMUNITIES AND CLASSES OF SERVICE

If only one class of telephone service were furnished by a telephone company, each subscriber would pay an equal share of the operating revenue of the company; but telephone companies generally have to supply direct or party-line business and residence service at flat rates, or in some cases at measured-service rates, rural or farm-line service, postpayment or prepayment pay-station service, and private-branch exchange service. Extension sets and extension bells must be provided where required and provision must be made for "switching" lines of co-operative or farmers-mutual companies which own and maintain their own lines.

Two-number and particular-person toll or long-distance services which require the adoption of an initial period and rate, and a rate for conversations which extend beyond the initial period must be offered. Special rates must be given at night to induce the public to use long-distance lines during hours of "light load." Report charges must be made to prevent unfair use of telephone service by means of a prearranged code of reports.

It is apparent that it is not an easy problem to determine such a rate for each of the above classes of service, that no injustice will be done any of the subscribers, and that the total operating revenue will be sufficient for the continued operation of the telephone company.

It has been assumed in this plan for rate determination that the telephone service in each community should be self-sustaining.

Each business or industry should be self-supporting. A test of its value to its community is the willingness of consumers to pay enough for the commodity to support the industry. A telephone company should, according to this point of view, be self-supporting. This has not always been the case. There are cases where telephone systems have been supported in part by systems elsewhere, so that competition might be destroyed by providing service at low rates or so that telephone service might be developed in a small community the residents of which have not realized the value of the service.

Rural service, for example, was developed under unfavorable conditions. Most companies found it hard to induce farmers to buy telephone service and, as they were anxious to develop this service, they made rates which were very low. In many cases, service of this kind is not nearly self-sustaining and is being paid for largely by the city subscribers.

It might appear, on first thought, that in any community each class of telephone service should be self-supporting, but this is not always the case. Each class of service should eventually support itself. If a class of subscribers will not pay rates sufficient to cover the cost of the service provided, another class of service should be developed for them which will be self-supporting. However, it is frequently necessary to give a class of service temporarily at such a rate that it is not self-supporting in order that it may be developed to the point where it will pay for itself. In any attempt to apply this principle it must be remembered that it is practically impossible to allocate justly certain elements of cost to the various classes of service. The development of some classes of service, such as residence, increases the value of other classes of services, such as business, so that the cost of such development should be distributed in proportion to the increased value—in so far as it is possible to determine it—over the various classes of service benefited. It is impossible in most cases to determine, except approximately, this increased value. This plan for rate-determination is not in any way dependent upon the possibility of so allocating local telephone rates.

The cost of toll or long-distance service is generally affected by the distance over which the connection is established, the length of time the equipment is in use during the call and the operating cost of establishing the connection. Sometimes the length of the connection or the route affects the operating cost, as when the services of a "through" operator are required, or when supervision by two operators instead of one is necessary.

There has been much discussion as to whether toll rates should cover the entire cost of toll calls from subscriber to subscriber, or only from toll-switchboard to toll-switchboard. If, as has been done by at least one public-utility commission, the subscriber to subscriber cost is assumed, it will be found that in many cases toll service is not self-sustaining.

If toll service is partly supported by exchange rates higher than necessary to cover the cost of exchange service, the development of exchange service may be retarded, or if toll rates are too high, toll development may be retarded. An extreme case of injustice is that of communication in which "free long-distance service" to other communities is provided. The local exchange rates are higher for all subscribers in order that a few subscribers may have the benefit of such free long-distance service.

Toll rates which are not self-sustaining might be justified in some cases because toll service is an asset which increases the value of the telephone service to all exchange subscribers.

Public-utility commissions, acting for the public welfare, have in many cases ordered telephone companies owning and operating toll lines to permit other companies to route calls over these toll lines. In such cases, in justice to the companies owning the toll lines, the commissions have prescribed rental rates which would make this service self-sustaining.

If local and toll service are not each self-sustaining, some communities by paying more than the cost of the toll service they use, might indirectly partly support the local service in other communities. It is assumed in this plan for rate determination that the rates for local and toll service should be mutually independent.

It may be seen that, in addition to cost, the rate for each class of local and toll telephone service is affected by habit and custom, value of the service to the subscriber, competition and unfair influence, the necessity for developing various classes of service and for preventing waste and unfair use of service, and the necessity for making the method of charging simple and easily understandable to the average subscriber.

SECTION III. PROPOSED PLAN FOR DETERMINING TELEPHONE RATES

REQUIREMENTS OF A NEW RATE SYSTEM

The method of determining telephone rates by providing that they shall be high enough for each telephone company to cover the cost of giving the service and provide sufficient return on the investment to attract new capital has been adopted by law in many states. If any other plan is to be adopted, it would have to be shown that it promises to be more logical and fair and definitely better than the plan followed at present.

Any new system of rate determination or any modification of the present system should tend to correct the apparent faults in the present system which have been suggested. It should accomplish the following results:

(1) It should offer an incentive to telephone companies to provide the best possible telephone service and to provide it as economically as possible.

(2) It should make less burdensome and expensive the work of the public-utility commissions and the telephone companies.

(3) It should not increase the number of conditions which work injustice to the telephone companies or telephone subscribers.

(4) It should not involve any more guess work than the present system of rate determination.

PROPOSED SOLUTION OF THE PROBLEM

Various telephone men have suggested that rates for telephone service should be standardized, the number of stations served in a community determining the rate for the various classes of service, and that unusual conditions or service should be recognized by bonuses or penalties applied to individual cases. The population of the exchange area and the calling rate might be suggested also as fundamental bases of standardization. The fundamental basis for the standardization of telephone rates should represent natural conditions which confront the telephone companies and over which they have no control. It should be independent of the effectiveness of the management of the companies. The calling-rates, whether maximum or average, are not independent of the effectiveness of management, but are affected by it. They are, therefore, not a satisfactory basis of comparison. The population of the exchange area represents more nearly than any other single condition the problem presented to the telephone companies; but population is sometimes not easily available, especially in the cases of small towns where a large proportion of the subscribers live in rural districts. The number of stations served in an exchange area while not quite as satisfactory in some respects as total population has the great advantage of being always available. Hence it is the most desirable basis of comparison.

In some communities the telephone company may be able, by ability to bargain which is above the average, to gain the advantage in fixing the rates, while in other cases the community may gain the advantage. Monopoly or unfair influence, the disposition to profiteer, the value of the telephone to the subscribers, and the ability of the subscribers to pay fair rates all affect the rates, sometimes to the advantage and sometimes to the disadvantage of the company. If a sufficient number of cases are considered, the advantage will be on the side of the telephone company as often as it is against it, and the average rates for a group of telephone exchanges of a given size may be assumed to be independent of inequalities in bargaining power. In towns, where there are competing telephone companies, the bargaining power of the companies is greatly decreased and the rates are consequently low. In such cases the value of the service is decreased, and the communities pay rates which are in proportion to the value of the service they receive.

Closely associated with bargaining power are the effects of the character of the community and the grade of service it wants and is willing to buy. Communities of the same size may differ industrially, but this is unusual. Generally, such differences correspond to differences in the sizes of communities. Farming communities are usually small, while the industries are usually confined to the large cities and towns. The grade and amount of service demanded vary regularly with the sizes of communities—rapidly for small communities, and more gradually for large communities. In so far as such conditions are concerned, then, it may be assumed that the average rate is the fair rate for communities of the size under consideration.

FACTORS WHICH JUSTIFY DEVIATION FROM AVERAGE RATES, AND CORRESPONDING BONUSES AND PENALTIES

If the plant and traffic conditions with which a telephone company has to contend and over which it has no control are such as to make the investment and production costs—the sum of which are the total cost of producing the service—higher or lower than the average, the telephone rates should be correspondingly higher or lower than the average. If also the service is very good or very poor, the compensation of the telephone company for that service should be correspondingly high or low. Bonuses or penalties should be applied to the average rates to give rates which are just to individual telephone companies and to the communities they serve. *The value of this rate-standardization plan depends upon the possi-*

bility of obtaining bonuses and penalties which may be weighted properly and fairly, which may be evaluated for individual cases without the expense and inconvenience of the present method of evaluating telephone plants, and which will take account of abnormal plant and service conditions. If, incidentally, these bonuses and penalties are closely associated with the elements of service given the subscribers and emphasize good service, the rate standardization plan will also result in an improvement in the telephone service of all communities.

PROPOSED BASES FOR BONUSES AND PENALTIES

Unusually high plant and operating costs, which are not the fault of the management, peculiar characteristics of the community which make it difficult to give service, and other natural obstacles with which a telephone company has to contend are one natural basis for bonuses, or penalties. The service given is the other basis which should be recognized. Good or poor service may be recognized by analyzing service, by weighting its various elements more or less arbitrarily according to the judgment of representatives of the telephone companies and the public, and by comparing the elements of service of the individual companies with those weighted elements. It may be recognized also by the results of good or poor service as shown by the telephone development in a community, and the average use made of the telephone by the subscribers.

The latter basis for judging service is the best. An evaluation of service by any analysis of service from its elements would set up a standard which would not develop as service conditions change. It would be too rigid and inelastic. The various elements of service would have to be arbitrarily weighted by representatives of the various interests concerned. Such a method would afford much opportunity for difference of opinion both in preparing a standard table of weighted elements of service, and in applying this table to individual cases. The attempted use of such a method of recognizing the degree of excellence of service would result in much friction. Table I shows the various elements of service which would have to be considered. A discussion of these elements of telephone service may be found in Circular No. 9, Engineering Experiment Station, The Ohio State University.

It may be contended that assigning bonuses and penalties on the basis of the above elements of service would be more direct and for that reason more satisfactory. Although it might be practicable to apply bonuses or penalties based on plant and operation elements of service, the results of good management are much more intangible, and it would be difficult to weight and apply the corresponding elements of service directly.

Attention may also be called to cases which will occur in which there will be injustice done to telephone companies, as for example, those of communities where the provision of a high grade of service has not been followed by high development or a high average calling-rate. Such a condition of unfairness is more likely to exist temporarily after the provision of the high grade service than permanently, unless the telephone company has used poor judgment in providing a better grade of service than the community desires or is able to pay for. In such a case the telephone company would not deserve a bonus.

TABLE I.

ANALYSIS OF TELEPHONE SERVICE

ADEQUATE TELEPHONE PLANT	Good Telephone Transmission	Loudness
		Quality
		Freedom from interference
		Education of public in regard to necessity for good equipment
	Adequate Facilities	
	Simplicity of Substation Equipment	
	Reliability of Equipment	
	Safety	
	Permanence	
EFFECTIVE TELEPHONE OPERATION	Accuracy	
	Continuity of Service and Reliability of Operation	
	Speed	
	Assurance of Attention	
	Courtesy and Form	
	Secrecy	
	Service Inspection	
	Education of Public in Regard to the Use of Telephone	
GOOD MANAGEMENT	Business Methods	Accuracy
		Promptness
		Simplicity
	Financial Policy	Adequate depreciation reserve
		Adequate surplus
		Adequate rates
		Proper control of securities
	Care and Training of Employees	
	Citizenship	Progressiveness
		Appearance of plant
		Regard for rights of public
		Local interest and cooperation
	Publicity	Condition of plant
		Methods of operation
		Service objectives and results obtained
		Reasons for rates

The application of development and average use of the telephone by subscribers, as bonus and penalty bases, may be well justified. It makes necessary only two bases for service bonuses and penalties and avoids the complicated process of weighting service which would result from assigning bonuses and penalties on the basis of service elements. The conditions which would warrant bonuses and penalties would be easily obtainable. Development and average calling-rate show the final result of good or poor service, and *it is the result that is important rather than the factors which produce the result.*

BASES FOR BONUSES AND PENALTIES ALREADY RECOGNIZED

The necessity for bonuses or penalties for unusual plant conditions or service has been recognized to a certain extent by public-utility commissions. They have frequently granted rates in excess of those which are normal for communities of the size in question because of a general impression that service is unusually good, because the plant conditions encountered are unusually severe, because the plant has been maintained in exceptional condition, and, in many cases, because of "free service."

The Interstate Commerce Commission and various telephone statisticians in their reports have recognized the importance of pole-line, wire-miles, average-calls, and development. However, no attempt has been made to make use of them as bases for bonuses and penalties in fixing telephone rates.

SPECIFICATIONS FOR BASES FOR BONUSES AND PENALTIES

The following specifications should be observed in choosing bases for bonuses and penalties:

(1) The final results of good or poor telephone service, that is, the amount of service sold and quality of service rather than any specific elements of service, should be used as bases for bonuses and penalties.

(2) Unusual plant conditions over which the telephone company has no control should be used as bases for bonuses and penalties.

(3) The bases for bonuses and penalties should be few in number.

(4) They should be easily determined from existing records, or records which it is otherwise to the advantage of the telephone company to keep.

(5) They should be general in character, and indicative of plant and service conditions as a whole.

The choice of the bases for bonuses and penalties is altogether a matter of judgment. It can be decided only by the vote of large numbers of informed and interested parties. The four bases suggested in this bulletin are those which from this investigation seem most reasonable. However, if additional bonus and penalty bases may be found to be necessary, they will not prevent the successful application of this rate-standardization plan.

The cost of producing telephone service in general determines the lower limit of the rates paid for service. The upper limit is determined by the value of the service to the telephone-using public. The rates obviously cannot be permitted to exceed this upper limit. Attempts to charge "all the traffic will bear," generally result in rates which are greater than the traffic will bear. While the rates have too frequently been so low that they would not cover the cost of service, care should be taken that they are not permitted by means of bonuses in any case to approach the upper limit, or "all the traffic will bear." In such a case, the granting of rate bonuses to a telephone company might defeat the intended purpose by discouraging the use of the telephone. Moderation should be observed in the application of bonuses and penalties.

SECTION IV. PLANT COST

DIFFERENCE BETWEEN CENTRAL-ENERGY AND MAGNETO RATES

The cost of central-office equipment should generally not vary greatly from the average for each group of telephone companies operating the same number of stations. If there is any difference, it would probably result from a difference in the maximum calling-rate or peak-load demand upon the telephone exchange. Exceptions which occur are cases where central-energy and magneto exchanges serve communities of the same size. In an investigation of one large group of telephone companies—Group 4 of the Ohio telephone companies—it was found that the point of change on the station-rate curves—which will be discussed in Section VI—from magneto to central-energy switchboards, is fairly sharp and definite at 1000 stations. If average curves are plotted separately for the magneto and central-energy stations, a discontinuity in the curve at the point of change may reasonably be expected. If the curves are prolonged at the point of discontinuity so that they will overlap as in Figure 1, they might well show what additional rate a central-energy exchange in a magneto group should be allowed, or what should be the decrease in the rate of a magneto exchange in a central-energy group. This difference would be due to the difference in value of the two kinds of service as well as to the difference in cost. In the case of Group 4 of the Ohio telephone companies, the two curves were continuous. This result indicates that there is no difference in cost and value. The same reasoning might be applied to ringing equipment, but in this case also, there were no discontinuities, indicating that the differences in rates due to ringing equipment are not important.

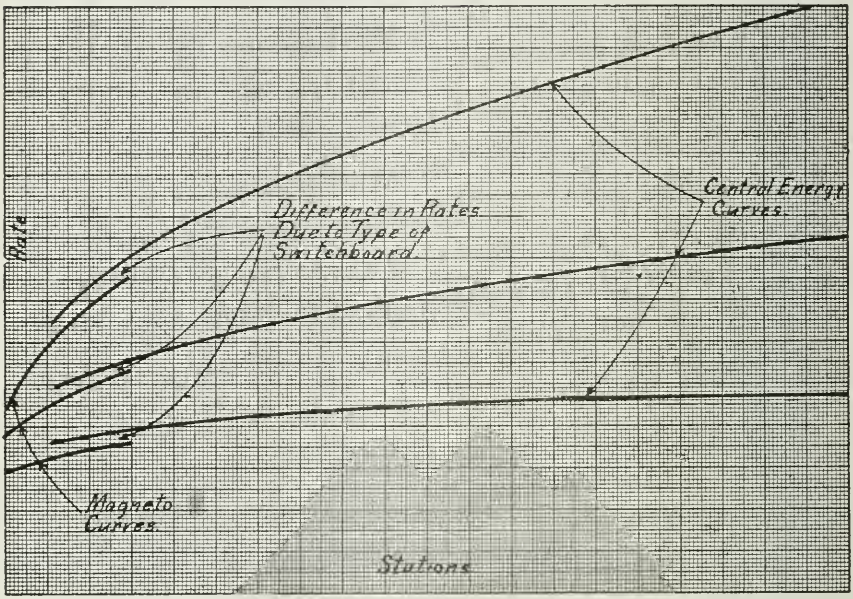

Figure 1

CHANGE IN PLANT COST WITH AREA SERVED

There is a considerable amount of variation in the cost of outside plant within groups of companies operating the same numbers of stations, due to area served and unusual construction difficulties. Miles of pole line per station, or number of poles per station, preferably the former, may be taken as a measure of the area served, and as a basis for a bonus, or penalty. Wire miles has received consideration as a bonus and penalty base, but it is not a good measure of the area served by a telephone company. Area itself might be used directly, but there would be considerable difficulty in defining the area served. One or two stations in an otherwise undeveloped area might serve as an excuse on the part of a telephone company for claiming a bonus because of the large territory it serves. Miles of pole line per station is the best measure of the territory actually served by a telephone company.

In cities where a large proportion of the circuits is underground, miles of underground conduit per station, modified by a factor which will properly weight it relatively to the pole line, should be added to the miles of pole line per station. As this investigation has not considered the largest communities, miles of pole line per station only has been used.

Among the possible unusual construction difficulties may be mentioned the character of the ground. The cost of extending lines through a hilly and rocky country is greater than through a country where the ground is easily workable. It often happens that in cases where the cost of construction is greatest, the people are least able to pay adequate telephone rates, as for example, in a rocky and barren country, where it is difficult to farm successfully and difficult to construct lines.

CHANGE IN PLANT COST WITH CALLING-RATE

Maximum busy-hour load or maximum calling-rate may also be taken as a cost bonus or penalty, independent of the average calls per station which is later suggested as a value bonus or penalty. It determines the cost of part of the central-office equipment and is an indication also of the cost of operating the switchboard. An approximate measure of this factor is the maximum number of operators' positions in use. However, as the number of operators' positions in use is not independent of the efficiency of operation of the telephone plant, it is not as desirable as a bonus and penalty base as the maximum busy-hour load as determined by a peg-count. In the evaluation of bonuses and penalties in this report, both cost and value elements are cared for by one bonus or one penalty which is determined by the average calls per station. This bonus or penalty is approximately the sum of the two bonuses or two penalties obtained by considering the cost and value factors separately.

An examination of the maximum busy-hour calling-rates and average calling-rates of a limited number of exchanges, for which information in regard to maximum busy-hour calling-rates could be obtained, indicates that they vary approximately in the same proportion, or, in other words, that the amplitude but not the form of the average daily traffic-curve for the year varies with the community. This statement is probably more nearly correct when communities of the same size and type are compared than when the comparison is made between those of different sizes and types. In the examination referred to above, all of the communities were of about the same type, that is, served by from 1000 to 3000 stations. If maximum busy-hour load were used as a basis for a bonus or penalty, it should not be difficult for public-utility commissions to induce the telephone companies to inaugurate the use of peg-counts in cases where they were not already in use.

The busy-hour calling-rate and the average calling-rate may both be affected by conditions such as the industrial character of the community or the proportion of foreign-born persons in the population and the nationality of the foreign-born population. For example, the use of the telephone by a French quarter of a city is likely to be much greater than its use by a Scandinavian quarter of the same size.

PLANT CONDITION

A valuable element of telephone service is the assurance to the subscriber that the service will be continuous. The condition of the telephone plant in a large measure determines the extent of this assurance. It may in many cases be possible to estimate the condition of the plant from a statement of its age, and statements of depreciation reserve, and of issues of stocks and bonds. Accrued depreciation may be the basis for a penalty or bonus.

If a plant is over-built to a reasonable extent, it would probably be just to allow a bonus. A plant should be over-built sufficiently to make it possible to furnish service to new subscribers without delay. Records of applications and time necessary to furnish service might be used to advantage as an indication of the condition of the plant. An over-built condition of a plant may be regarded as the opposite of a depreciated condition and one bonus and penalty base may apply to both conditions.

A reasonably over-built condition is more desirable in the case of a telephone company than in the case of an electric light and power company or other type of public utility. If there is an unusual demand for street-railway transportation, it may be met by crowding passengers into the cars, and the additional equipment necessary may be provided after the increased demand has developed. New users of electric light and power, gas, and water service may generally obtain connection to already existing supply circuits or mains. Additional equipment may be provided in the central-station or gas or water plant when opportunity permits, but the additional plant does not generally have to be provided before the new users are supplied with service. However, each telephone subscriber, or in the case of party-line service, each small group of subscribers, requires a circuit to the central office for his sole use. Such a circuit must be available before a new subscriber can be supplied with service. During the extreme conditions which have existed in the past few years many telephone companies have attempted to meet the situation by making party lines of direct lines, or in some cases even by making ground-return circuits of metallic circuits. The results have been most unsatisfactory to both telephone companies and subscribers. In order to supply service promptly to new subscribers, a telephone plant must be over-built.

In this investigation, bonuses or penalties have not been allowed for plant condition or facilities. They may be considered to be included in development bonuses and penalties. People will not subscribe to telephone service if they feel that it will deteriorate because the plant is not maintained and they cannot subscribe to it unless there are adequate plant facilities. However, these elements of service are important and it may be possible that they should be the basis for a separate bonus or penalty.

SECTION V. TELEPHONE SERVICE

The average excellence of telephone service increases as the community increases in size, but in each group of telephone exchanges serving the same number of stations there are individual exchanges to which bonuses or penalties are due because of the quality of their service.

DEVELOPMENT

The degree to which people buy telephone service and the degree to which they use it are approximate measures of its excellence. The development in stations per thousand people shows not only how aggressive the management of a company has been in selling telephone service, but also the quality of service which has been maintained. The better the service the easier it is to sell. It may be difficult in the cases of small communities to ascertain how great a rural population might possibly be served by a telephone company. While not entirely satisfactory, the development within the town or city might be used, as a measure of the total development. In the cases of the larger cities, the proportion of subscribers who are rural subscribers is small, and will not greatly affect the development.

It is much less difficult to obtain the additional stations in a community where the development is not high than in an already well-developed territory. The last subscribers obtained are generally the hardest to obtain.

It might be argued that increasing the development is the same, as far as rates are concerned, as putting the exchange in a higher class, on the station-rate curve. This is not the case. The value element of the service, indicated by the development for which a bonus or a penalty is given, does not result from the number of stations served. This number of stations is taken account of by the position of the exchange on the station-rate curve. The value element of the service results from the higher grade of service produced. Because the service in itself is better, and because the subscriber may, if he desires, obtain connection with a greater number of telephones, the telephone company can sell more telephone service, but the latter element of value is otherwise recognized and the bonus should be applied only for the good service.

If development were used as a basis for a bonus or penalty, it would not be difficult for a public-utility commission to obtain information in regard to the population, the corresponding number of stations, and the development of each exchange area in its state.

The provision of "free service" to neighboring towns has been recognized as having value to the subscribers, and public-utility commissions have allowed increased rates because of free-service. Free-service has the same kind of value as increased development, but the value per given percentage of increase in subscribers with whom it is possible to get connection is not as great as in the case of an increase in development. There is not as free a use of service over the free-service lines as in the case of inter-office trunks in an urban exchange area, because it is less convenient to use the facilities provided for free service than to use those provided for normal local service, and because there is less community interest among subscribers in neighboring communities than among those in the same community. The method followed in weighting free-service is explained in the discussion of the curves of Figures 2 to 7 (Section VI).

When there are two competing telephone exchanges in a community, the rates tend to decrease because of the competition, as mentioned above, and incidentally the service is worth much less. If rates were determined by modifying the average rate in accordance with development, the development of each exchange might be considered either as the total stations of the community divided by the total population (expressed in thousands) of that community, or as the stations of that exchange divided by the total population of the community. In the latter case, which is the more just, the rates will be low because the development is low, and the rate of the new exchange will be the lower of the two if it has fewer stations than the other. In the other case, the rates will be high because, on account of the competition, the development will be high.

CALLING-RATE

Not only may the excellence of service be measured by the proportion of total population who buy telephone service, but also by the extent to which each subscriber uses his telephone. The number of calls per subscriber may be computed from the estimated number of calls per year, or if the distribution of calls throughout the day and year is regular and known, from the busy-hour calling-rate. A rough measure of the busy-hour calling-rate is the number of operators' positions occupied; but, as suggested above, this is not entirely satisfactory because it is not independent of the efficiency of the operating force or management, as is the calling-rate. Even if the calling-rate is low, the number of operators' positions occupied may be high because of inefficiency of operators or of the switchboard in use.

The busy-hour calling-rate is more nearly a measure of the peak-load demand on the telephone exchange because it is approximately proportional to positions of the central-office switchboard and the number of operators required to operate the switchboard, while the average calling-rate per station is not so good a measure of the demand on the exchange but is probably a better measure of the satisfaction given the subscribers by the service. Estimates of the average number of calls per month or per year are generally available, while very few of the small telephone companies have ever taken peg-counts, which would show their busy-hour calling-rates. However, if required by commissions, busy-hour calling-rates could be obtained without great difficulty, and would generally be a more satisfactory means of measuring the use of the telephone by the subscriber than the estimates of numbers of calls.

SECTION VI. STATION-RATE CURVES

SOURCES OF INFORMATION

Copies of the data sheet shown on page 30 were sent to the five hundred and twenty-five Class A, B, C, and D telephone companies in Ohio. The replies gave reliable information in regard to three hundred and six telephone exchange areas many of which, however, provided only a few classes of service. In addition, many data sheets were received on which the information was inadequate for the purpose of this investigation or unreliable because the intention of the questions had not been understood. Portions of this information were checked by information obtained from the 1920 "Annual Report of the Public Utilities Commission of Ohio," "Telephony's Directory of the Telephone Industry," and the records of the Public Utilities Commission of Ohio. When all of this information was arranged and examined, it was found that enough information was in hand to plot reliable curves showing the variation of telephone rates with the number of stations served by telephone exchanges.

There are in Ohio many groups of one city surrounded by a number of small towns and villages which present telephone conditions which differ from the general conditions. In each of these groups, free long-distance service or "free-service" between communities is more highly developed than it is in general. The service is of a higher grade because it is given in cities, and towns and villages, the standards of service of which have been influenced by

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Fill Out One of These Sheets for EACH Town or City in Which Your Telephone Company Operates

Telephone Company..................................Town..................................
Number of stations in town..................Population of town..................
Number of exchanges in town..................Miles of pole line..................
Plant valuation (if available)..................Annual operating expense..................
Capital stock..................................Bonds..................................
Kind of system (magneto, central energy, or automatic)..................
Average number of stations on rural lines..................On party lines..................
Type of ringing..................................
Towns to which free service is given..................................

....

Conditions which make plant valuation or operating expenses high and which would therefore warrant high rates..................................

....

Class of Service	Number of Stations	Rate
Business, 1-party		
Business, 2-party		
Business, 4-party		
Business, Rural		
Residence, 1-party		
Residence, 2-party		
Residence, 4-party		
Residence, Rural		
Switching		
Extensions		
P. B. X. Trunks		
P. B. X. Extensions		
P. B. X. Boards		

Data Sheet

the cities. Business service is relatively more important, and the rates are higher because of the higher grade of service and the more highly developed free-service between communities. These groups of communities have been collected in this investigation in Group 1.

In Group 2 are included cities and towns in which, similarly, the service is of high grade and the rates proportionally high, but in which free-service is not as prevalent as in the cities and towns of Group 1. The equipment and service methods of Group 2 are more thoroughly standardized than those of Group 1.

One group of communities of less than 1000 stations each, under one management, have been included in Group 3, and the remaining telephone exchanges investigated, in Group 4.

Consideration of the larger communities in which message-rate service is provided, and in which much of the outside plant is underground, has been omitted because of lack of data. There is, however, no indication that this plan for rate determination cannot be applied to such communities.

METHOD OF PLOTTING CURVES

The numbers of stations in the various exchanges were plotted as abscissae on sheets of cross section paper against rates as ordinates. The average curves were obtained by finding the center of gravity of each group of points falling in a vertical column on the curve sheets and on each sheet drawing a smooth curve through as many as possible of these centers of gravity.

The points for the average-weighted-rate curves were obtained by adding the products of the various rates and the corresponding numbers of stations and dividing the sum of these products by the total number of stations. The result is the same as would be obtained by dividing the total operating income from local service by the total number of stations. Many of the cases examined were checked in this manner.

Figures 2 to 7 show typical sets of residence-rate curves. One hundred and twenty curves similar to these curves were plotted. Those of Groups 1, 2, and 4 are summarized in Figures 8, 9, and 10. Several of the less important curves were omitted from the summary-sheets for the sake of clearness. The curves of Group 3 are not shown on a summary-sheet because they do not extend beyond 1000 stations, and because they coincide very closely with the lower portions of the curves of Group 2, shown in Figure 9.

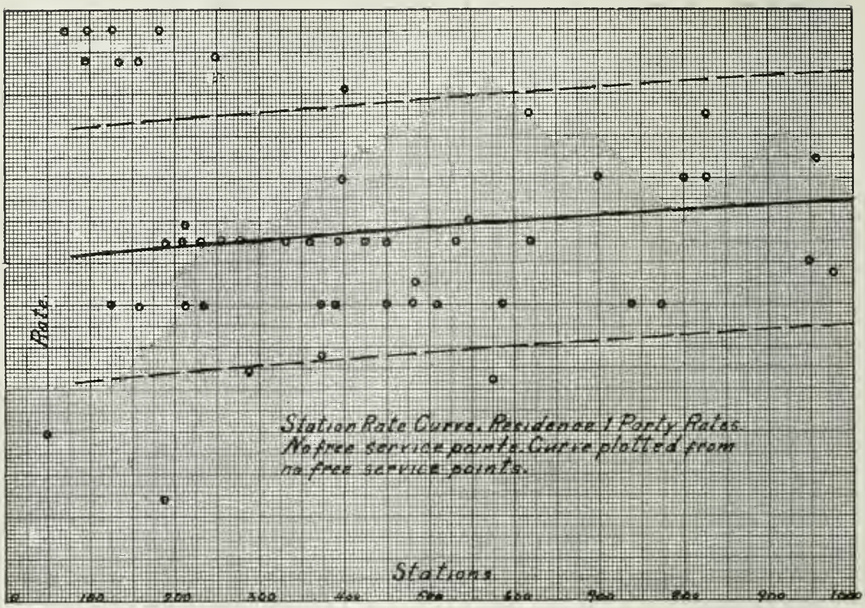

Figure 2

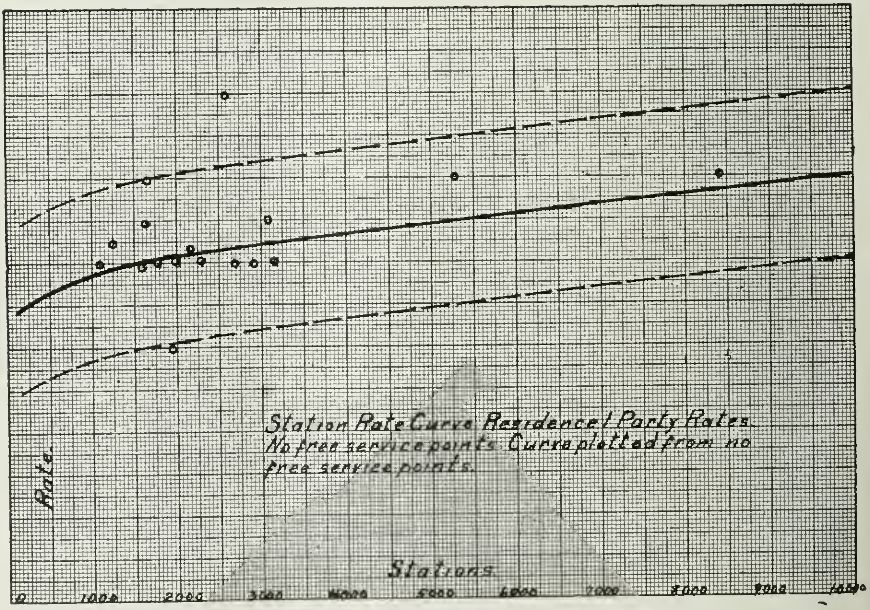

Figure 3

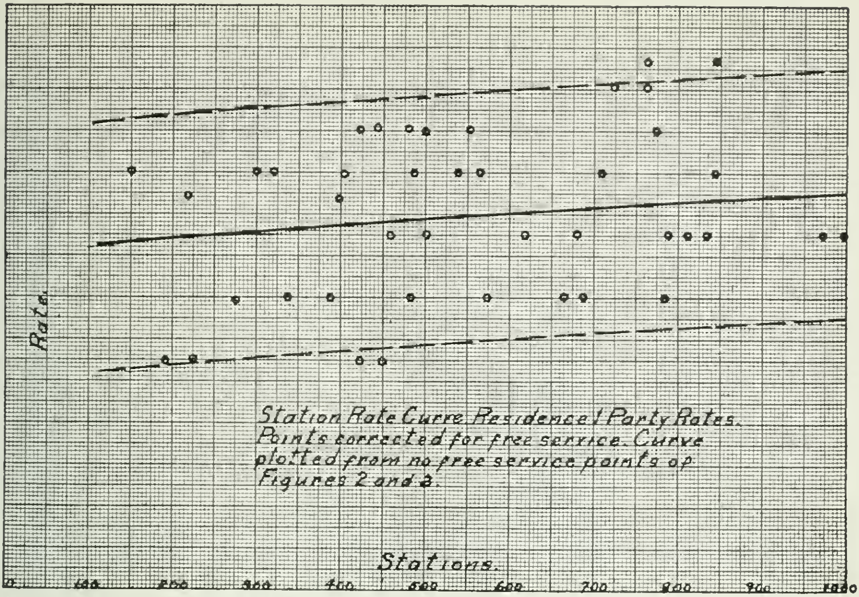

Figure 4

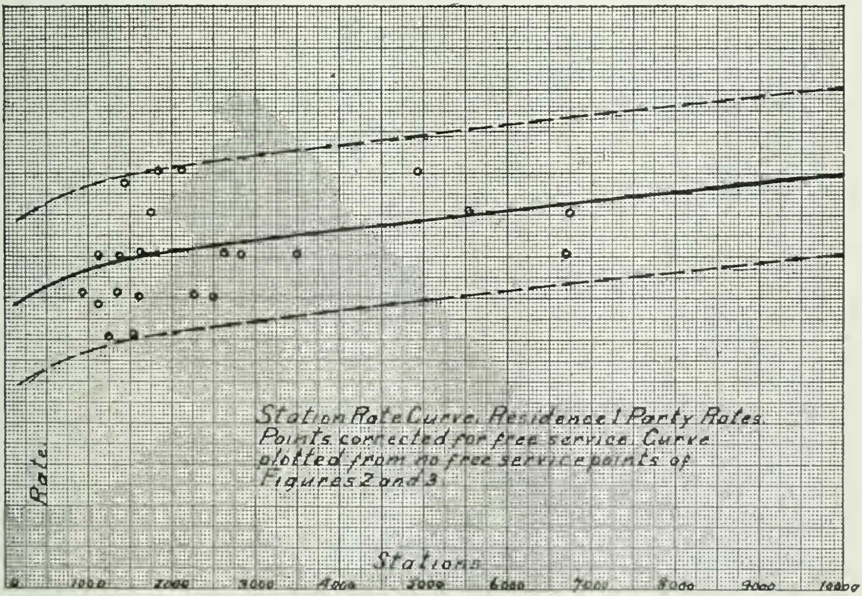

Figure 5

It was found early in the investigation that free long-distance service or "free-service" between communities has had an appreciable effect upon the rates. This relation is to be expected because the value of the service to each subscriber is increased by an increase in the number of subscribers with whom he may, if he wishes, obtain connection.

A sufficient number of exchange areas not affected by free-service were available in the case of the communities of Group 4 to plot reliable curves of business-one-party, residence-one-party, residence-rural, and average-weighted rates against numbers of stations. Figures 2 and 3 show the residence-one-party curves.

The additional stations made available to the subscribers of a community by means of free-service between communities cannot be valued the same per station as the stations in the community itself, because it is less convenient to use the facilities provided for free-service than to use those provided for normal local service, and because there is less community interest among subscribers in neighboring communities than among those in the same community. When the stations of Group 4 made available by free-service facilities are discounted by a factor of .33 and the points corresponding to the corrected exchange areas are plotted on sheets containing the curves of Figures 2 and 3, as in Figures 4 and 5, it will be seen that if curves representing the average of all of the points—those which have been corrected for free-service, and those which are not affected by free-service—were plotted, they would not differ greatly from the curves of Figures 2 and 3. Greater or less correction than that obtained by the use of the factor .33 will result in curves which vary more or less from the original curves.

As the curves of Figures 2 and 3 are plotted with sufficient points to be reliable, and as it may be assumed that the curves of Figures 4 and 5 will have the same shapes if their one point of difference—the effect of free-service—is properly weighted, .33 may be accepted as the weighting-factor which properly indicates the value of the free-service facilities with respect to the facilities provided for local service.

Data sheets were received from many telephone companies which did not state whether or not free-service was provided. Although it may reasonably be assumed that if no mention were made of free-service, no free-service facilities were provided, the points corresponding to these sheets were separated from the rest of the points and plotted as in Figures 6 and 7 with the curves of Figures 2 and 3. It will be seen that their addition to the points of Figures

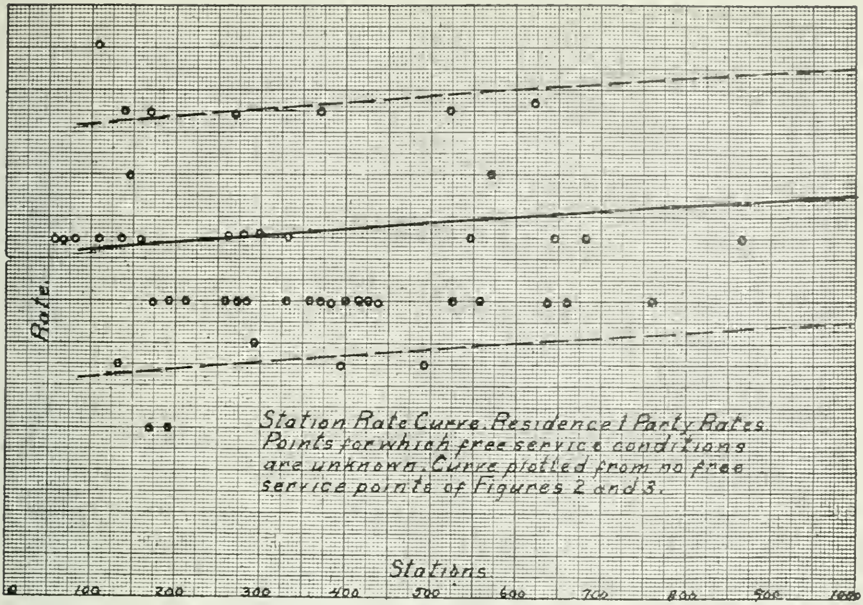

Figure 6

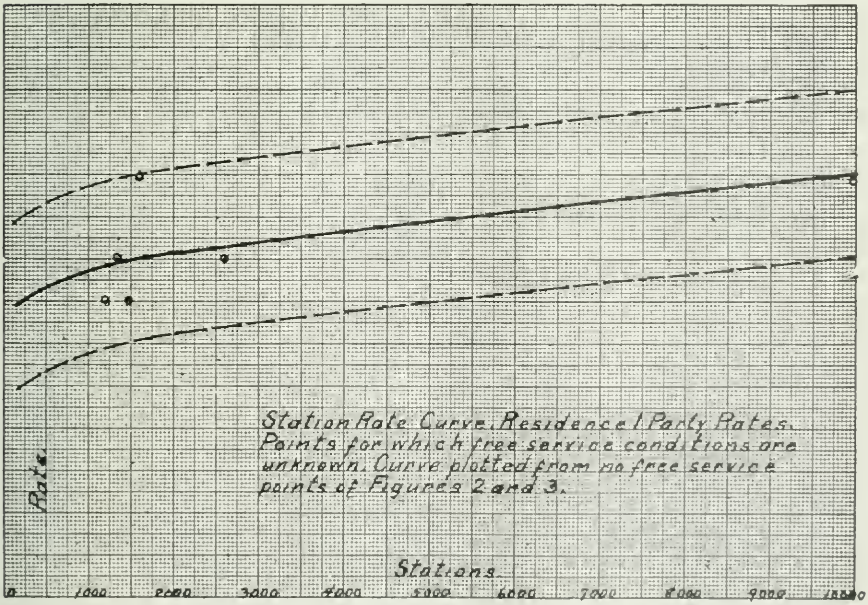

Figure 7

2 and 3 would not noticeably change the shape of the curves. Business-one-party, residence-one-party, residence-rural, and average-weighted rates were treated in this manner. In the cases of business-two and four-party, business-rural, and residence-two-party rates, it was necessary in order to obtain enough points to give reliable curves, to use the points corresponding to both exchange areas with no free-service and exchange-areas with free-service, with the necessary correction. It will be noted that the curve of business-four-party rates bends below 1000 stations in a direction opposite to that of the other curves. This curve is reliable in this respect because the number of points in this region is large. The corresponding curve for Group 1 of exchange-areas, plotted independently, has the same shape.

In the case of residence-four-party rates it was found that the curves did not correspond in shape to the other curves plotted. This curve is one of slight slope, so that only a small correction should be allowed for free-service in fixing rates. If too great allowance for free-service has been made, no correction for free-service, however great, which might be made, will make the points fall on the average curves, because the correction for free-service is made by moving the points horizontally along the axis of stations. For this reason, it was decided that a misleading result would be obtained if points corrected for free-service were used in plotting the curve. These points were accordingly rejected and the curve plotted from "no free-service" points and from points for which no free-service conditions had been reported. In these cases it was assumed, as stated above, that there was no free-service.

The numbers of points available with which to plot curves of business-extension, residence-extension, and switching rates, and private-branch-exchange-board, trunk and extension rates were small so that points corrected for free-service as well as "no free-service" points and those for which no free-service conditions had been reported were used in obtaining the average curves.

The telephone service of the telephone companies of Group 1 provides generally for free-service so that points corrected for free-service had to be used in plotting the curves of the various classes of service. It is assumed that if the points were properly corrected for free-service, the curves would have the shapes of the curves obtained by plotting the "no free-service" points of Group 4. When the business-one-party curve is plotted without correction for free-service, and corrected by factors of .33 and .10, the latter curve is similar in shape to the curve of "no free-service" points of Group 4, while the curves plotted without correction and with a correction

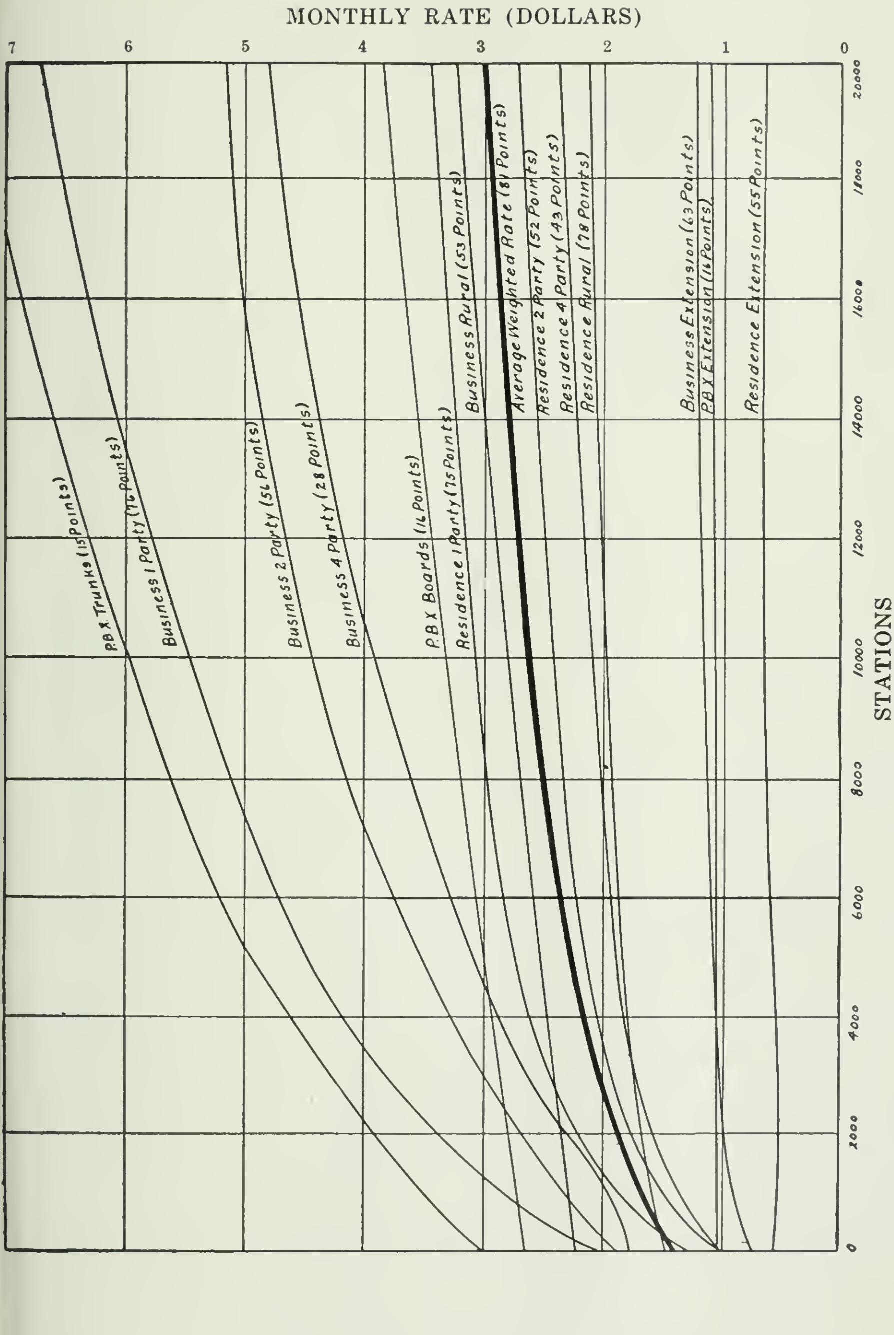

Figure 8. Station-Rate Curves. Group 1

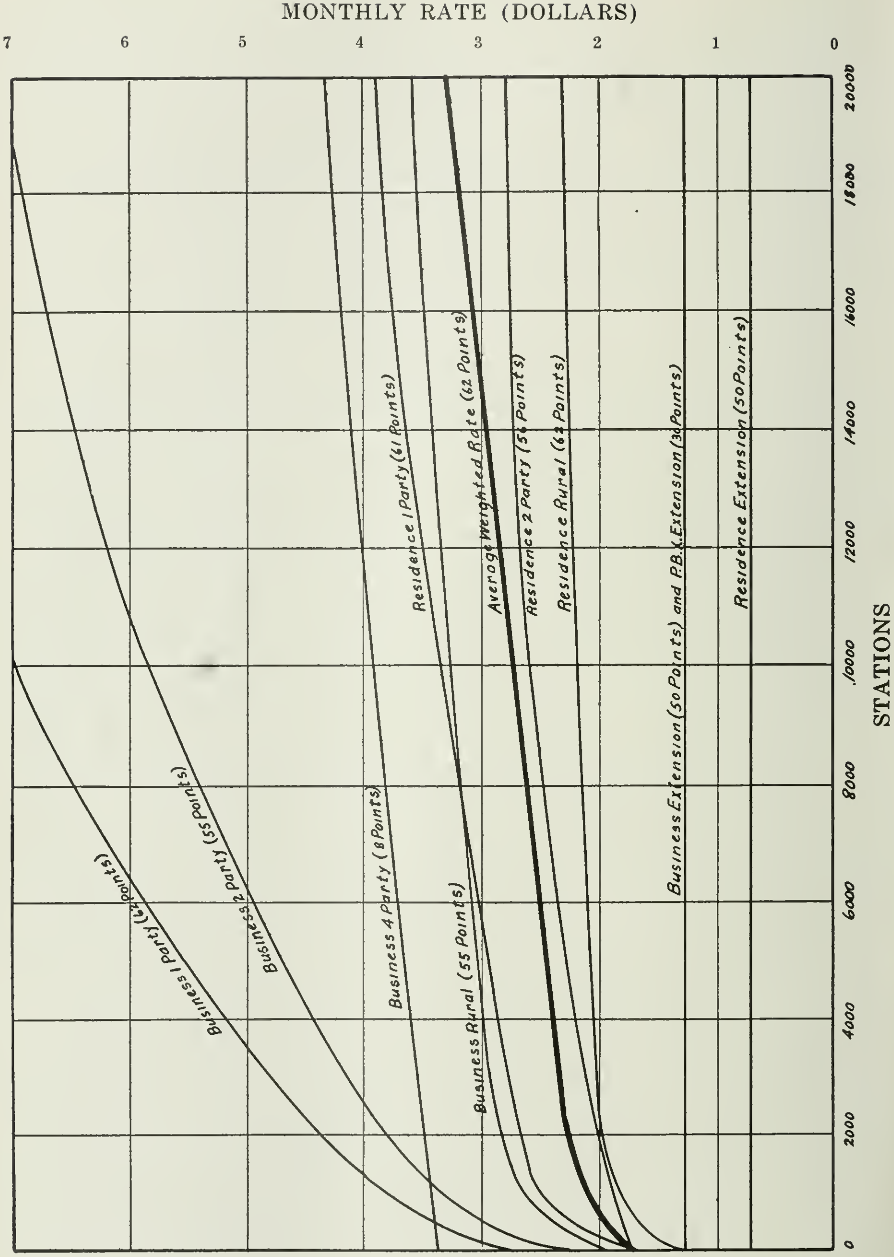

Figure 9. Station-Rate Curves. Group 2

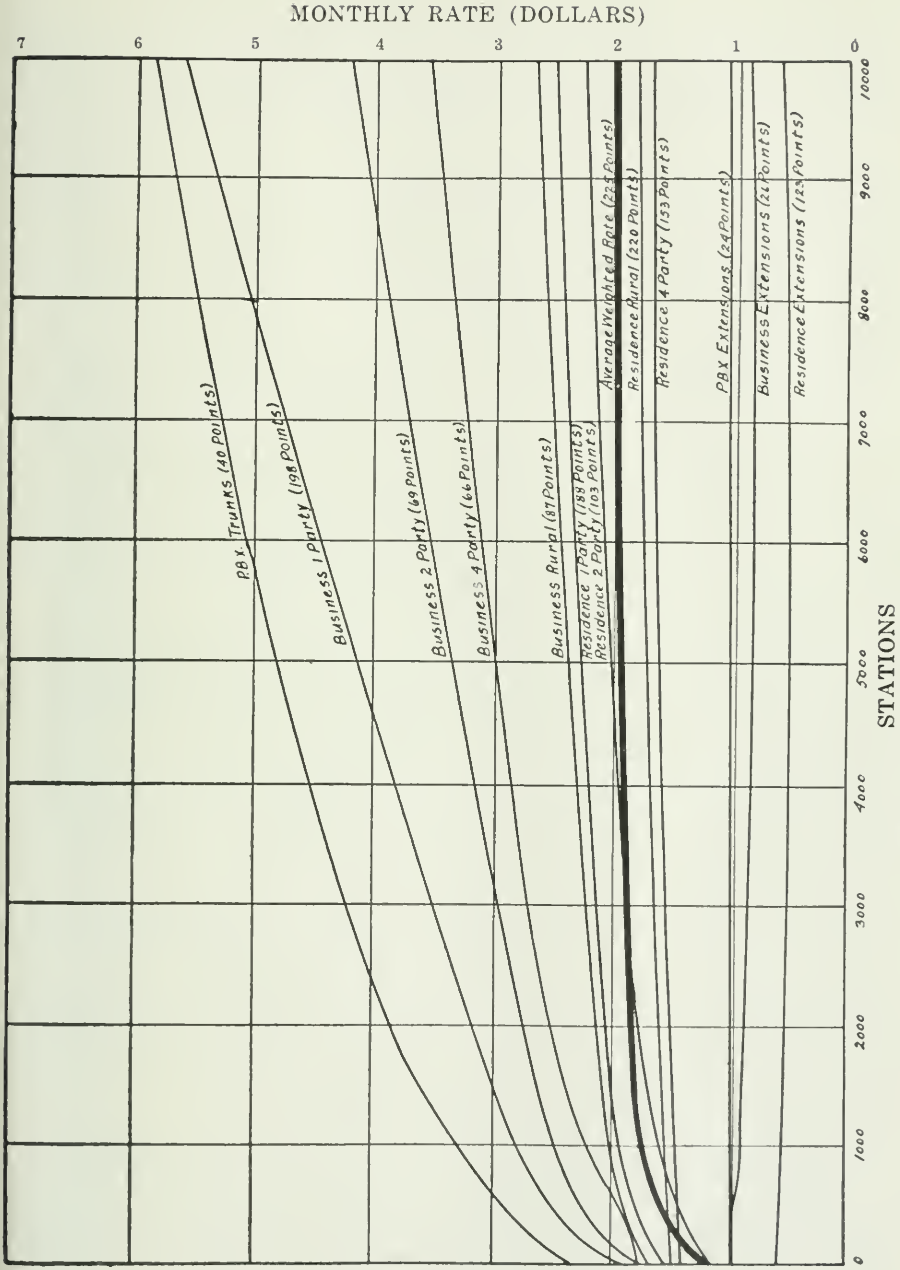

Figure 10. Station-Rate Curves. Group 4

factor of .33 give curves which deviate in opposite directions from the curves of Group 4. This result indicates that multiplying the number of additional stations made accessible to a subscriber in any community by free-service by a factor of .10 weights these stations properly in comparison with the stations in the community itself. In the case of the points of Group 4, the corresponding factor was .33.

The method used in determining these factors does not make it possible to obtain exact values or make sharp distinctions. However, the difference in factors indicates that, in fixing rates, greater account has been taken of free-service in the cases of the telephone companies of Group 4 than of Group 1. As a further check on this factor for the companies of Group 1, the business-two-party points were corrected for free-service with factors of .10 and .33, with a like result. The curves of business-four-party, and rural rates, residence-one, two- and four-party and rural rates, business, residence and private-branch-exchange extension rates, private-branch-exchange-board and trunk rates, and average-weighted rates were plotted, with a correction factor of .10. Similarly, a correction factor of .10 was found to be the most satisfactory one for the curves of Group 2.

The values of the correction factors for these groups of communities indicate that less importance has been attached to free-service by the more highly developed telephone companies than by the smaller ones. More skillful consideration of the relative values of the various elements of service has brought a realization of the over-emphasis which has been placed upon free-service by the telephone companies and the public-utility commissions. The development of other phases of the service has placed free-service in a relatively less important position.

Free-service works an injustice to many of the local exchange subscribers. The cost of maintaining free-service between communities which is in reality used by a small proportion of the local exchange subscribers, must be distributed over the local exchange rates so that the telephone rates of all telephone subscribers in a community are higher in order that a few subscribers may have the benefit of a special kind of telephone service.

INCREASE IN TELEPHONE RATES WITH SIZE OF COMMUNITIES

The rates for telephone service in all cases increase rapidly with the number of stations for the small communities and less rapidly for the large ones. There are two important reasons for this increase: First, the increase in the cost of the necessary telephone plant, and the cost of operating it per station for a given grade of telephone service increase with the number of stations served. Second,—a point which is frequently not realized by telephone subscribers—the quality of service demanded is higher and the use made of it per station is greater in the large communities. The people in the large communities have become more accustomed to telephone service and more dependent upon it than those in the small communities.

This increased cost of telephone service in the large communities is accompanied by an increased value to the subscriber, because of the possibility of obtaining connection with a greater number of other persons in the community. This increased value of the telephone service to the subscriber is not the reason for the higher telephone rates in the large communities, but it has been referred to so frequently that there is a large proportion of the telephone-using public who think that the telephone companies have made the increased value an excuse for higher telephone rates.

Let us consider two communities having telephone companies which operate respectively 20,000 and 40,000 stations. If the average calling-rate per station during the busy-hour were .55, 11,000 calls would be initiated during the busy-hour in the first telephone exchange. A higher average calling-rate per line may be assumed for the second exchange, because the service is more valuable to each subscriber. If the average calling-rate were .65, 26,000 calls would be initiated during the busy-hour, or a considerably greater number of calls per subscriber. The increase in operating force needed in the larger exchange over the smaller one would be in still greater proportion because the average call would consume more operators' time. A larger proportion of calls would be trunked from one central office to another, and a greater length of time would be consumed in completing trunked calls than those calls in which both the called and calling subscribers are connected in the same central office. The cost of operating the switchboards per station would, therefore, be greater in the larger exchange.

The cost of the switchboard per station would be greater in the exchange of 40,000 lines because of the increased switchboard space necessary to accommodate the greater number of operators, and because of the greater complexity of equipment.

Not only will the cost of central-office building and land be greater because of the greater space required, but also because of the higher real-estate values in the larger communities. Some of the fixed charges, such as taxes, are much higher proportionally in the larger communities.

The outside-plant cost per station is higher in the city than the small town, because of the high cost of underground construction, and the delicate construction problems made necessary by the congestion of aerial equipment in the streets, and consequent high cost of such construction. In many cases also the average distance from the subscribers' station to the central office is greater, and the plant cost per station is greater for this reason.

The investment cost per station of the large telephone system would thus be greater than that for the small system, and maintenance and other operating and fixed costs also increase rapidly with the size of the community. If a merger of two competing telephone companies in a city, each operating 20,000 stations, had been effected, the same increase in investment and operating cost would have resulted, but the elimination of duplication of stations would to some extent, though probably only slightly, compensate for the increased costs mentioned above.

COMMENTS ON STATION-RATE CURVES

In order to check the relative reliability of the curves it was noted what percentages of points fell within and without given bands on each side of the curves. In the curves of Group 4, the residence curves are apparently better established than the business curves; also the business curves of Groups 1 and 2 showed greater reliability than the business curves of Group 4.

While an exact proportionality in variation among the curves on each curve sheet cannot be expected, still it may be seen by an inspection of Figures 8, 9, and 10 that they are approximately proportional, so that, for a given variation in the average-weighted-rate curves, a proportional variation in the remaining curves may be assumed without serious error.

The curves of Groups 1, 2, 3, and 4 have the same general shape, but those of Groups 1, 2, and 3 indicate higher rates than those of Group 4, as would be expected from the higher grade of service.

The average-weighted-rate curves of Groups 1, 2, and 3 have greater slopes than that of Group 4 because they are affected to a greater extent in Groups 1, 2, and 3 by the steep-sloping business-rate curves than is the average-weighted-rate curve of Group 4.

The more rapid rise in the lower portions of the curves of Groups 2 and 3, and the higher rates are probably accounted for by the more aggressive managements of Groups 2 and 3.

In Groups 1, 2, and 4 it may be noticed that the station-rate curve for private-branch-exchange trunks exceeds the business-party curve by an approximately constant difference. This difference would be expected since the first private-branch-exchange trunk installed usually includes one private-branch-exchange switchboard. In Group 3, the rate for private-branch-exchange trunks is quoted as the business-party rate plus the rental of one switchboard. The constant difference, then, probably represents the rental of a private-branch-exchange switchboard.

In all groups, the rate for extension service is practically independent of stations as shown by the station-rate curves. An extension makes the telephone more accessible to the subscriber but does not necessarily greatly increase his use of the telephone. The cost to the company is merely the rental of the subset and associated equipment plus a small cost of operation and maintenance regardless of the size of the exchange.

NORMAL-COST BASIS OF STATION-RATE CURVES

There has been much discussion of the relative merits of "original cost," "reproduction cost," and other costs as a fair and practicable rate-base. "Normal cost" has of late been considered with favor by many public-utility commissions, because it is based upon average costs over a number of years, and eliminates abnormal costs which would make the rate base abnormally high or low. It has a disadvantage, however, of making the determination of the rate-base more dependent upon the soundness of the judgment of the appraiser than the other "costs."

The rates of the station-rate curves are based upon normal costs. In any one group of communities having the same number of stations, some of the telephone systems will have been constructed in times of high cost, and others in times of low cost. The rates in each community resulting from the bargaining between the telephone company and the subscribers will tend towards those which include costs of operation and maintenance plus a return on the actual investment, whether this investment was made in time of high or of low prices. The average rates for these communities, or the rates shown by the station-rate curves, therefore, include average costs of operation and maintenance of all of the companies in a group, plus a return upon the average of investment costs of the telephone systems over a number of years, or the normal cost.

DEPENDENCE OF RELIABILITY OF STATION-RATE CURVES UPON FREE COMPETITION

The fairness of the station-rate curves as a basis from which to determine rates for individual telephone companies depends upon freedom of bargaining in the various communities between telephone companies and subscribers. When the rates are arbitrarily determined by rate specialists and not by free bargaining, the average rates are not necessarily fair ones from which to determine the rates for individual communities.

SECTION VII. EVALUATION OF BONUSES AND PENALTIES WHICH HAVE BEEN APPLIED

BONUS AND PENALTY BASES

It is assumed in this report that the bases upon which bonuses and penalties should be granted are (a) miles of pole line per station, (b) maximum busy-hour calling-rate, (c) average calls per day per station, and (d) development. It is assumed further that these bonus and penalty bases are so fundamental that they have been recognized and taken into account by public-utility commissions, telephone companies, and the telephone-using public, so that if their values are averaged in a sufficient number of cases in a sufficient number of states of the same general character, values will be obtained which are a measure of what has been the average judgments of those organizations and of those people interested in the determination of telephone rates. In this investigation bonus and penalty bases (b) and (c) are combined into one cost and value bonus and penalty base. The value obtained for this base will be the sum of those which would be obtained if separate bases were assumed. Other bonuses and penalties, as for example, one for plant condition, may be added if desirable, without changing the methods of evaluation. The actual inconvenience and labor of the evaluation will, however, increase with the number of bonus and penalty bases assumed.

THE RATE DEVIATION EQUATION

It is assumed that a deviation from the average for a telephone rate is accounted for by deviation from the average of miles of pole line per station, average calls per day per station, and development, or

$$K_1P + K_2C + K_3D = R,$$

which we may call a rate deviation equation, where

P = percentage deviation from the average miles of pole line per station.

C = percentage deviation from the mean of the average calls per day per station.

D = percentage deviation from the average development.

R = percentage deviation from the mean of the average-weighted rate for an exchange of the size in question.

K_1 = percentage bonus or penalty per percent of deviation from average miles of pole line per station.

K_2 = percentage bonus or penalty per percent of deviation from the mean of the average calls per day per station.

K_3 = percentage bonus or penalty per percent of deviation from average development.

LIMITATIONS ON SCOPE AND EXACTNESS OF EVALUATION OF BONUSES AND PENALTIES

It cannot be assumed that the constants should be the same for exchanges of all sizes, so that the exchanges from which any set of constants is evaluated should all be of approximately the same size. In this report, only those exchanges operating from one thousand to three thousand stations were considered. If it were possible to obtain the necessary information in regard to a sufficient number of exchanges, including a smaller range of stations, the results would be more reliable.

In the evaluation of bonuses and penalties which have been applied by various public-utility commissions and telephone companies, no account has been taken of free-service except in so far as it has changed development.

It would be desirable to correct rates in cases where increased rates have been granted because of free-service, and, in the formation of the equations for solution, to omit in the development factors the correction for free-service. As information with which to make the correction for rates was not available, the equations for those exchanges in which free-service was appreciable were omitted. There is no reason to suppose that the error, resulting from the inclusion of the equations for exchanges in which the amount of free-service given is small, is serious.

Because of the difficulty of explaining what information was desired, and because the records examined were not intended to give this information, the information in regard to the bases for bonuses and penalties of many of the cases were insufficient or unreliable; hence these cases had to be rejected.

POSSIBLE METHODS OF EVALUATING BONUSES AND PENALTIES

Several methods may be followed in evaluating the constants K_1, K_2, and K_3. They are as follows:

(1) Write the equations for the various individual cases using the unknown constants K_1, K_2, and K_3 for each case, but for each individual case using its own numerical values for P, C, D, and R. Solve algebraically for the unknown constants, K_1, K_2, and K_3, for each possible combination of three equations, and average the values of K_1, K_2, and K_3 obtained. This method is laborious and its reliability is doubtful.

(2) Write the equations for the various individual cases as for (1). Then separate them into three equal groups, those with large positive rate deviations, R, in one group, those with large negative rate deviations in another group, and those with

zero and small positive and negative rate deviations in the third group. Obtain averages of miles of pole line per station, average calls per day per station, development, and rate deviations in each group without changing the signs of the negative deviations. With the three sets of average values, and K_1, K_2, and K_3, write three deviation equations, and solve for K_1, K_2, and K_3. While this method is not laborious, it was not used in this report because the number of cases, small at the start, had to be separated into three groups, and average deviations obtained for each group. This would render the results unreliable. If a large number of cases is available, the results obtained by this method should be reasonably reliable.

(3) Write the equations for the various individual cases as for (1). Then find the most probable values of K_1, K_2, and K_3, and the probable errors in each case, by means of the Method of Least Squares. The details of applying this method to the solution of a group of independent observations of equal weight such as those in Table 5 may be obtained from many sources, among which may be mentioned Chapters 3, 4, and 10 of "Method of Least Squares" by Mansfield Merriman. This is the method of solution which was used in this investigation. It is the most satisfactory method because it depends upon no doubtful assumptions. By it may be determined not only the most probable values of K_1, K_2, and K_3, but also the probable errors in each case. The probable error in any case is such that the number of errors greater than it equals the number which are smaller than it. This method of solution thus gives a definite measure of the reliability of the values of K_1, K_2, and K_3 obtained.

SOURCES OF INFORMATION

Tables 2, 3, and 4 show the method of deriving from fundamental data the percentages of deviation from the average. Table 2 shows the data and deviations for thirty-five exchanges in Ohio. This information was obtained from records of the Public Utility Commission of Ohio and the Bulletin on "Population of Ohio" of the "Fourteenth Census of the United States." The population of the territory served by the exchange, or the "exchange-area," was calculated by assuming that the populations of the "exchange-city," and the whole "exchange-area" are to each other as the stations in the exchange-city and the total stations.

Table 3 shows the data and deviations for forty-four exchanges in Ohio, Indiana, Illinois, Wisconsin, and Iowa. These data were obtained from "Memorandum Concerning Telephone Companies and

Telegraph Companies Reporting to the Interstate Commerce Commission for 1917," and bulletins on populations of the states under consideration of the "Fourteenth Census of the United States." It was necessary in this case, because of lack of information in regard to division of stations between the exchange-city and its surrounding exchange-area, to use total stations and population of the city or town in obtaining development. There is very little error in the development obtained by this method for cities, but considerable error in the cases of small towns and villages.

Table 4 shows the data and deviations for twenty-three of the exchanges considered in Table 3. Letters were written to all of the exchanges in Table 3, requesting accurate information. Incidentally, this information afforded a means of checking the accuracy of the populations of the exchange-areas as obtained for Table 2. Estimates of population of the exchange-areas were requested, and the deviations based upon these estimates agree fairly well with those calculated by the method of Table 2.

In Table 5 are collected the values of P, C, D, and R of selected cases of Tables 2, 3, and 4 and some additional ones received in answer to the letters to Middle Western telephone companies, referred to in the discussion of Table 4. These selected cases were used in the solution by the Method of Least Squares. Only those cases were included in this table which were free from any apparent possibility of error.

BONUSES AND PENALTIES WHICH HAVE BEEN APPLIED

The solution of the deviation equations gives the following probable values of the constant with the corresponding probable errors:

$$K_1 = .08 \pm .6$$
$$K_2 = .16 \pm .8$$
$$K_3 = .02 \pm 1.3$$

That is, for a one percent variation in miles of pole line per station a bonus or penalty of eight hundredths of one percent has been applied to the rate, and the probable error of this observation is plus or minus six-tenths of one percent or nearly eight times the probable value of K_1. The values of K_2 and K_3 may be interpreted similarly. The bonuses or penalties based upon P, C, and D which have unconsciously been applied have been small, and in the case of D, development, negligible. Development has apparently received little or no recognition as a proper basis for a bonus or penalty.

Table 2. Data and Deviations (1921) for Ohio Telephone Companies

		Pole Line			Calls per Day			Development			Average Weighted Rate			
No.	Working Stations	Miles of Pole Line	Miles of Pole Line per Working Station	Percent Variation of Column 3	Number of Calls per Day	Calls per Day per Working Station	Percent Variation of Column 6	Population of Exchange Area Calculated from Population of Exchange City	Stations in Exchange per Thousand People in the Exchange Area	Percent Variation of Column 9	Gross Operating Income per Month from Local Service	Gross Operating Income per Month per Working Station	Percent Variation in Column 12	No.
	1	2	3	4	5	6	7	8	9	10	11	12	13	
1	1804	72	.040	—47	9330	5.16	3	6700	269	18	3190	1.77	— 5	1
2	2956	175	.059	—21	28500	9.66	93	13400	220	— 4	5590	1.89	1	2
3	3099	279	.090	20	13100	4.23	—16	16600	187	—18	5390	1.74	— 7	3
4	1154	54	.047	—37	4700	4.07	—19	5350	216	— 6	2145	1.86	— 1	4
5	1375	77	.056	—25	5100	3.72	—26	8550	161	—30	2390	1.74	— 7	5
6	2331	125	.054	—28	4880	2.10	—58	9100	256	12	3330	1.43	—24	6
7	1713	162	.094	25	8000	4.66	— 7	10000	171	—25	3820	2.23	19	7
8	1550	103	.067	—11	11200	7.23	44	5200	298	30	2540	1.64	—12	8
9	1562	134	.086	15				7700	203	—11	3110	1.99	6	9
10	2309	131	.057	—24	13300	5.76	15	9200	251	10	4160	1.80	— 4	10
11	1977	52	.026	—65	8830	4.46	—11	11800	167	—27	3100	1.57	—16	11
12	2300	51	.022	—71	4000	1.74	—65	9600	240	5	4450	1.93	3	12
13	1620	94	.058	—23	33300	20.56 ?		7350	221	— 4	3095	1.91	2	13
14	2365	160	.067	—11	8690	3.66	—27	8500	278	21	4220	1.78	— 5	14
15	2748	240	.088	17	14700	5.36	7	11900	231	1	5320	1.93	3	15
16	3175	124	.039	—48	15500	4.89	— 3	9100	349	52	5390	1.70	— 9	16
17	3340							15600	214	— 7	5890	1.76	— 6	17
18	3265	532	.163	117	13200	4.04	—20	15500	211	— 8	6960	2.13	14	18
19	2327	287	.123	64	10600	4.56	— 9	13100	178	—22	5270	2.26	21	19
20	3590	480	.134	79	18800	5.24	4	15100	238	4	6330	1.76	— 6	20
21	2610	99	.038	—49	20500	7.86	57	12900	202	—12	5280	2.02	8	21
22	2300	235	.117	56	9600	4.80	— 4	11600	173	—24	5020	2.51	34	22
23	1138	32	.028	—63	6000	5.27	5	5900	193	—16	2215	1.95	4	23
24	1679	123	.073	— 3	4710	2.81	—44	5950	282	23	2905	1.73	— 8	24
25	1470	132	.090	20				8300	177	—23	2480	1.69	—10	25
26	1851	239	.129	72	9700	5.24	4	9900	187	—18	3350	1.81	3	26
27	1540	150	.097	29				7300	211	— 8	2530	1.64	—12	27
28	1603	98	.061	—19	7900	4.93	— 2	5500	292	28	3365	2.10	12	28
29	1595	195	.123	64				3500	455 ?		2560	1.60	—14	29
30	1501	57	.038	—49	6760	4.50	—10	4950	303	32	3060	2.04	9	30
31	1702	200	.117	56	5430	3.19	—37	26500	64 ?		2570	1.51	—19	31
32	1995	136	.068	— 9	12300	6.17	23	7700	259	13	3700	1.85	— 1	32
33	2539	211	.083	11	22100	8.70	74	10600	240	5	4550	1.79	— 4	33
34	2982	123	.041	—45	19100	6.42	28	15900	187	—18	6750	2.26	21	34
35	1085	87	.080	7	26600	24.50 ?		3800	286	25	2090	1.93	3	35
Averages			.075	38		5.02	25		229	17		1.87	10	

Table 3. Data and Deviations (1917) for Middle West Telephone Companies

		Pole Line			Calls per Day			Development			Average Weighted Rate			
No.	Working Stations	Miles of Pole Line	Miles of Pole Line per Working Station	Percent Variation of Column 3	Number of Calls per Day	Calls per Day per Working Station	Percent Variation of Column 6	Population of Exchange City	Stations in Exchange per Thousand People in the Exchange Area	Percent Variation of Column 9	Gross Operating Income per Month from Local Service	Gross Operating Income per Month per Working Station	Percent Variation in Column 12	No.
	1	2	3	4	5	6	7	8	9	10	11	12	13	
1	1604	82	.051	—45	1333 ?	.83 ?	——	7804	206	—19	2420	1.51	14	1
2	1712	25 ?	.015 ?	——	8580	5.00	—14	10928	157	—38	2035	1.19	—10	2
3	1201	260	.216	132	7500	6.25	8	5212	230	— 9	1659	1.38	— 5	3
4	2509	274	.109	17	11667	4.65	—20	8191	307	21	3790	1.51	14	4
5	1035	99	.096	3	6267	6.05	4	5451	189	—26	1672	1.62	23	5
6	2674	122	.046	—51	145925 ?	54.5 ?	——	8116	329	29	3405	1.27	— 4	6
7	1418	166	.117	26	……	……	……	3000	472 ?	……	1606	1.13	—14	7
8	1002	198	.197	112	5000	4.99	—14	2275	440 ?	……	1004	1.00	—24	8
9	1765	202	.114	23	12367	7.00	21	9901	178	—30	2590	1.46	11	9
10	1378	128	.093	0	6890	5.00	—14	4168	331	30	1642	1.19	—10	10
11	1292	37	.029	—69	5333	4.14	—29	4758	272	7	1271	.98	—26	11
12	1638	234	.143	54	5833	3.56	—39	5958	276	9	2021	1.23	— 7	12
13	1312	173	.132	42	6333	4.84	—16	3720	353	39	1525	1.16	—12	13
14	1152	58	.050	—46	5751	4.99	—14	4507	255	0	1369	1.18	—11	14
15	2450	174	.071	—24	17388	7.11	23	9872	248	— 2	3280	1.34	2	15
16	1557	95	.061	—34	6831	4.40	—24	5478	284	12	2021	1.30	— 2	16
17	1131	7 ?	.006 ?	——	4500	3.98	—31	4511	251	— 1	1095	.97	—26	17
18	2775	232	.083	—11	15419	5.57	— 4	5362	518 ?	……	2701	.98	—26	18
19	1991	270	.136	46	7571	3.79	—35	4683	426 ?	……	2469	1.24	— 6	19
20	2190	161	.074	—20	8033	3.66	—37	6627	331	30	2283	1.04	—21	20
21	2076	166	.080	—14	10052	4.84	—16	4697	443 ?	……	2433	1.17	—11	21
21	1030	53	.052	—44	4333	4.19	—28	5776	179	—30	1435	1.39	5	22
23	2377	175	.074	—20	32025	13.55	134	10425	228	— 9	3481	1.46	11	23
24	2410	97	.040	—57	18887	7.83	35	9343	258	2	3442	1.43	8	24
25	1389	163	.117	26	7869	5.67	— 2	8876	156	—38	2393	1.72	30	25

Table 3—Continued

		Pole Line			Calls per Day			Development			Average Weighted Rate			
No.	Working Stations	Miles of Pole Line	Miles of Pole Line per Working Station	Percent Variation of Column 3	Number of Calls per Day	Calls per Day per Working Station	Percent Variation of Column 6	Population of Exchange City	Stations in Exchange per Thousand People in the Exchange Area	Percent Variation of Column 9	Gross Operating Income per Month from Local Service	Gross Operating Income per Month per Working Station	Percent Variation in Column 12	No.
	1	2	3	4	5	6	7	8	9	10	11	12	13	
26	1070	123	.115	24	2500	2.34	—60	3210	334	31	1838	1.72	30	26
27	1633	269	.165	77	21667	13.30	129	5842	279	10	2855	1.75	32	27
28	2472	238	.096	3	14666	5.92	2	7690	322	27	3556	1.43	8	28
29	1438	95	.066	—29	8147	5.65	— 2	3430	420 ?	....	1921	1.33	1	29
30	1045	36	.034	—63	13129	12.55	116	4294	243	— 4	1488	1.42	8	30
31	2070	132	.064	—31	10000	4.83	—17	7260	285	12	2907	1.40	6	31
32	1388	128	.092	— 1	6435	4.63	—20	5679	244	— 4	1746	1.26	— 5	32
33	1907	130	.068	—27	933 ?	.49 ?	....	8100	236	— 7	2695	1.41	7	33
34	1329	87	.066	—29	6143	4.62	—20	5295	251	— 1	1646	1.24	— 6	34
35	1483	318 ?	.215 ?	....	6660	4.48	—23	3035	489 ?	....	1760	1.18	—11	35
36	1002	83	.082	—12	6333	6.33	9	2245	445 ?	....	1377	1.37	4	36
37	1373	88	.064	—31	12000	8.75	51	8451	163	—36	2030	1.48	12	37
38	2174	147	.068	—27	11623	5.33	— 8	18293	119	—53	3055	1.41	7	38
39	1520	876 ?	.576 ?	....	7095	4.66	—20	4353	349	37	1525	1.00	—24	39
40	1331	150	.112	20	7571	5.69	— 2	5582	238	— 6	1794	1.35	2	40
41	1039	180	.173	86	4000	3.86	—33	2997	348	37	1533	1.48	12	41
42	1157	42	.036	—61	7900	6.83	18	6654	174	—32	1447	1.25	— 5	42
43	1026	131	.128	38	6400	6.23	7	3929	261	3	1356	1.32	0	43
44	1118	325 ?	.290 ?	....	5044	4.52	—22	4466	251	— 1	1464	1.31	— 1	44
Averages.........		....	.093	38	....	5.79	28	....	254	19	....	1.32	12	

Table 4. Data and Deviations (1921) for Middle West Telephone Companies

		Pole Line			Calls per Day			Development			Average Weighted Rate			
No.	Working Stations	Miles of Pole Line	Miles of Pole Line per Working Station	Percent Variation of Column 3	Number of Calls per Day	Calls per Day per Working Station	Percent Variation of Column 6	Population of Exchange Area Calculated from Population of Exchange City	Stations in Exchange per Thousand People in the Exchange Area	Percent Variation of Column 9	Gross Operating Income per Month from Local Service	Gross Operating Income per Month per Working Station	Percent Variation in Column 12	No.
	1	2	3	4	5	6	7	8	9	10	11	12	13	
1	1125	50	.044	—49	13000	11.55	131	6000	188	—15	—	—	—	1
2	3300	1380	.418 ?	—	15761	4.78	— 4	23000	144	—35	—	—	—	2
3	1000	120	.120	40	10517	10.52	110	4200	238	8	2080	2.08	20	3
4	1415	185	.131	52	3500	2.47	—51	7500	258	17	2162	1.53	—12	4
5	1114	38	.034	— 60	6575	5.90	18	7800	143	—35	2150	1.93	11	5
6	1303	89	.068	—21	8456	6.49	30	6000	217	— 1	2671	2.05	—18	6
7	1834	87	.047	—45	5000	2.72	—45	10900	168	—24	2800	1.53	—12	7
8	2900	185	.064	—26	15000	5.17	3	15600	186	—15	5850	2.02	16	8
9	3000	—	—	—	14000	4.67	— 7	10000	300	36	6000	2.00	15	9
10	2930	138	.047	—45	14796	5.05	1	60000 ?	49 ?	—	6350	2.16	24	10
11	1600	195	.122	42	6500	4.06	—19	6000	267	21	2415	1.51	—13	11
12	900	160	.178	107	6000	6.67	33	3600	250	14	830 ?	—	—	12
13	1700	175	.103	20	7500	4.42	—12	8000	212	— 4	3500	2.06	18	13
14	1025	140	.136	58	5000	4.88	— 2	3300	311	41	1500	1.46	—16	14
15	2303	273	.118	37	7487	3.25	—35	7100	324	52	3300	1.43	—17	15
16	918	56	.061	—29	2500	2.72	—46	3850	238	8	1810	1.97	13	16
17	1999	89	.045	—48	10880	5.45	9	6500	308	40	3410	1.71	— 2	17
18	1531	110	.072	—16	6500	4.25	—15	6500	236	7	3100	2.02	16	18
19	1239	170	.137	59	4270	3.45	—31	5300	234	6	1645	1.33	—24	19
20	1566	52	.033	—62	8000	5.11	2	7400	212	— 4	2760 ?	1.76 ?	1	20
21	1369	269	.196	128	4632	3.38	—32	8400	163	—26	1810 ?	1.32 ?	—24	21
22	1050	25	.024	—72	3152	3.00	—40	8800	119	—46	1550 ?	1.47 ?	—15	22
23	2368	29	.012	—86	12000	5.07	1	18400	129	—41	3510	1.48	—15	23
Averages		—	.086	53	—	5.00	30	—	220	22	—	1.74	15	

TABLE 5

DEVIATIONS USED IN OBTAINING CONSTANTS

Values of the percentages of deviation, P, C, D, and R, from the observation equations of the form, $K_1 P + K_2 C + K_3 D = R$, applying to the consistent cases chosen for solution by Method of Least Squares.

No. of Case	P	C	D	R	No. of Case	P	C	D	R
1	—47	+ 3	+18	— 5	36	+77	+129	+10	+32
2	—21	+93	— 4	— 1	37	+ 3	+ 2	+27	+ 8
3	+20	—16	—18	— 7	38	—63	+116	— 4	+ 8
4	—37	—19	— 6	— 1	39	—31	—17	+12	+ 6
5	—25	—26	—30	— 7	40	— 1	—20	— 4	— 5
6	—28	—58	+12	—24	41	—29	—20	— 1	— 6
7	+25	— 7	—25	+19	42	—31	+51	—36	+12
8	—24	+15	+10	— 4	43	+20	— 2	— 6	+ 2
9	—65	—11	—27	—16	44	+86	—33	+37	+12
10	—71	—65	+ 5	+ 3	45	—61	+18	—32	— 5
11	—11	—27	+21	— 5	46	+38	+ 7	+ 3	+ 0
12	+17	+ 7	+ 1	+ 3	47	+40	+110	+ 8	+20
13	—48	— 3	+52	— 9	48	+52	—51	+17	—12
14	+117	—20	— 8	+14	49	—21	+30	— 1	—18
15	+64	— 9	—22	+21	50	—45	—45	—24	—12
16	—49	+57	—12	+ 8	51	+20	—12	— 4	+18
17	+56	— 4	—24	+34	52	+37	—35	+52	—17
18	— 3	—44	+23	— 8	53	—48	+ 9	+40	— 2
19	+72	+ 4	—18	+ 3	54	+59	—31	+ 6	—24
20	—19	— 2	+28	+12	55	—62	+ 2	— 4	+ 1
21	—49	—10	+32	+ 9	56	+128	—36	—22	—24
22	—45	+28	—18	+21	57	—72	—40	—46	—15
23	+17	—20	+21	+14	58	—86	+ 1	—129	—15
24	+ 3	+ 4	—26	+23	59	—21	+28	— 3	— 1
25	+23	+21	—30	+11	60	+21	—15	—30	— 4
26	+ 0	—14	+30	—10	61	—46	+37	—16	+22
27	—69	—29	+ 7	—26	62	+19	—12	—10	— 6
28	+54	—39	+ 9	— 7	63	—67	+34	+ 9	+25
29	—46	—14	+ 0	—11	64	—72	—24	+ 7	—14
30	—24	+23	— 2	+ 2	65	+250	+21	+22	+37
31	—34	—24	+12	— 2	66	+16	— 5	— 6	— 6
32	—20	—37	+30	—21	67	—16	—13	—10	— 8
33	—20	+134	— 9	+11	68	+23	—20	+ 9	— 2
34	—57	+35	+ 2	+ 8	69	—57	+24	— 5	+ 5
35	+24	—60	+31	+30					

$K_1 = .08 \pm .6$ $K_2 = .16 \pm .8$ $K_3 = .02 \pm 1.3$

The high degree of uncertainty in the values of these constants is due to the fact that if bonuses or penalties P, C, or D were applied, it was done unconsciously and, therefore, in an irregular and uncertain manner. The injustice of the present system of rate determination, which has, in many cases, penalized some telephone companies for the same conditions for which it has rewarded others, is strongly indicated by these results.

SECTION VIII. APPROXIMATELY JUST BONUSES AND PENALTIES

The values of the bonus and penalty constants which should justly be applied may be approximately computed as follows: Assume a division of plant valuation as shown in Table 6, which is a reasonable average estimate for telephone plants of the size considered—one thousand to three thousand stations—and average percentages which the elements of production and investment expenses are of plant valuation, as shown in Table 7. Then analyze the effects on rates of variations in miles of pole line per station, average calls per day per station, and development. Such analyses will indicate that the constants should have larger values than those obtained above.

TABLE 6

DIVISION OF TELEPHONE PLANT VALUATION

Division of Telephone Plant	Percentage of Total Plant Valuation
Outside plant	60
Substation equipment	20
Central office equipment	10
Miscellaneous equipment	10
Total	100

TABLE 7

TELEPHONE PRODUCTION AND INVESTMENT COSTS

Division of Cost	Percentage of Plant Valuation
Production Costs:	
Traffic	8.0
Maintenance	4.5
Commercial	2.5
General	1.5
Investment Costs:	
Taxes	1.5
Insurance	.5
Depreciation	7.5
Interest	8.0
Total Costs	34.0

EFFECT OF INCREASING MILES OF POLE LINE PER STATION

If it is assumed that "A" is the original plant valuation, and that the miles of pole line per station are increased one hundred percent, the average calls per station and development remaining unchanged, the original and increased costs of operation are as follows:

TABLE 8

EFFECT OF INCREASING MILES OF POLE LINE PER STATION

Production Costs:	Original Cost of Operation	Increased Cost of Operation
Traffic (8 percent)	.08 × A	(.08 × A)
Maintenance (4.5 percent)	.045 × A	(.045 × A) + (.0166 × .6 × A)
Commercial (2.5 percent)	.025 × A	(.025 × A)
General (1.5 percent)	.015 × A	(.015 × A)
Investment Costs:		
Taxes (1.5 percent	.015 × A	(.015 × A) + (.015 × .6 × A)
Insurance (.5 percent)	.005 × A	(.005 × A) + (.005 × .6 × A)
Depreciation (7.5 percent)	.075 × A	(.075 × A) + (.075 × .6 × A)
Interest (8 percent)	.08 × A	(.08 × A) + (.08 × .6 × A)
Total Cost (34 percent)	.34 × A	(.34 × A) + (.115 × A)

$$\text{Percent increase} = 100 \left(\frac{.115}{.34}\right) = 33.8$$

It is assumed that maintenance increases but not in proportion to increased length of pole line. An increased cost of operation of 33.8 percent for the above one hundred percent increase or .338 percent per percent increase in miles of pole line per station results. Miles of pole line per station is entirely a cost bonus-and-penalty base. Therefore, the bonus-and-penalty constant should be of the order of magnitude of .34 instead of .08, the constant at present applied.

EFFECT OF INCREASING AVERAGE CALLS PER DAY PER STATION

If the average calls per station are increased one hundred percent, the miles of pole line per station, and development remaining unchanged, the cost of operation increases as follows:

TABLE 9

EFFECT OF INCREASING AVERAGE CALLS PER DAY PER STATION

Production Costs:	Original Cost of Operation	Increased Cost of Operation
Traffic (8 percent)	.08 × A	(.08 × A) + (.08 × A)
Maintenance (4.5 percent)	.045 × A	(.045 × A) + (.045 × .1 × A)
Commercial (2.5 percent)	.025 × A	(.025 × A)
General (1.5 percent)	.015 × A	(.015 × A)
Investment Costs:		
Taxes (1.5 percent)	.015 × A	(.015 × A) + (.015 × .1 × A)
Insurance (.5 percent)	.005 × A	(.005 × A) + (.005 × .1 × A)
Depreciation (7.5 percent)	.075 × A	(.075 × A) + (.075 × .1 × A)
Interest (8 percent)	.08 × A	(.08 × A) + (.08 × .1 × A)
Total Cost (34 percent)	.34 × A	(.34 × A) + (.102 × A)

$$\text{Percent increase} = 100 \left(\frac{.102}{.34}\right) = 30$$

The increase in the cost of operation for the one hundred percent increase in average calls per station per day is 30.0 percent or .30 percent per percent increase in average calls per station per day.

As each subscriber uses his telephone twice as much as he did originally, it may reasonably be assumed that the value of the service has been increased 100 percent or one percent per percent increase in calls per station. If the value of the bonus and penalty base is the average of the above percentages, it should be .65 instead of .16, the constant at present applied.

EFFECT OF INCREASING DEVELOPMENT

If the development were increased one hundred percent, the miles of pole line per station, and the average calls per day per station remaining unchanged, the cost of operation would increase as follows:

TABLE 10

EFFECT OF INCREASING DEVELOPMENT

	Original Cost of Operation	Increased Cost of Operation
Production Costs:		
Traffic (8 percent)	$.08 \times A$	$(.08 \times A) + (.08 \times A)$
Maintenance (4.5 percent)	$.045 \times A$	$(.045 \times A) + (.045 \times A)$
Commercial (2.5 percent)	$.025 \times A$	$(.025 \times A) + (.025 \times A)$
General (1.5 percent)	$.015 \times A$	$(.015 \times A) + (.015 \times A)$
Investment Costs:		
Taxes (1.5 percent	$.015 \times A$	$(.015 \times A) + (.015 \times A)$
Insurance (.5 percent)	$.005 \times A$	$(.005 \times A) + (.005 \times A)$
Depreciation (7.5 percent)	$.075 \times A$	$(.075 \times A) + (.075 \times A)$
Interest (8 percent)	$.08 \times A$	$(.08 \times A) + (.08 \times A)$
Total Cost (34 percent)	$.34 \times A$	$(.34 \times A) + (.34 \times A)$

$$\text{Percent increase} = 100 \left(\frac{.34}{.34}\right) = 100$$

The increase in cost of operation is one hundred percent, but as there is also an increase of one hundred percent in the number of stations, there is no increase in the cost of operation per station. However, each subscriber has access to twice as many other subscribers as originally, so that as a first approximation the value of the service to each subscriber may be assumed to have increased one hundred percent. If the bonus and penalty base is the average of the percentages of increase of cost and value, it should be .50 instead of .02, the constant at present applied.

SECTION IX. JUST BONUSES AND PENALTIES

The above method of computing the proposed bonus and penalty constants is such an approximate one that they cannot be accepted without a further analysis of their effects upon a telephone company, which attempts to obtain a bonus by increasing P, C, or D, and upon its subscribers. In order to make such an analysis, a suggested classification of telephone companies in accordance with rates will be made, a typical telephone exchange chosen, and an analysis made of the effect of increasing P, C, and D, separately, the other two constants remaining unchanged in each case.

Throughout this discussion, non-essential digits and fractions will be omitted.

CLASSIFICATION OF TELEPHONE EXCHANGES IN ACCORDANCE WITH RATES AND NUMBERS OF STATIONS OPERATED

In classifying the telephone exchanges with respect to rates, the most practicable plan will be to define arbitrarily the classes of telephone exchanges with respect to the numbers of stations operated, and to make a separate rate schedule for each class. As the smallest change in rate which is ordinarily considered is 25 cents a month, and as the business-one-party station-rate curve has the greatest slope and the greatest reliability of the station-rate curves, the basis for classification used is a change of 25 cents a month on the business-one-party curve. By a comparision of the slopes of the business-one-party and average-weighted-rate curves, between 1000 and 3000 stations, it will be seen that the ratio of slopes is approximately 5 to 1; that is, a change of 25 cents a month on the business-one-party curve corresponds to a change of approximately 5 cents a month on the average-weighted-rate curve.

Marking off 25-cent intervals from $2.00 to $6.00 on the business-one-party curve of Group 4, seventeen classes are obtained as shown in Table 11. The corresponding numbers of stations may be obtained by drawing vertical lines from the points on the curve to the axis of stations.

TABLE 11

STATION-RATE CLASSIFICATION

Rate Class	Business-One-Party Rate	Corresponding Stations	Limits in Stations	Station Class	Limits in Stations
1	$2.00	0	1-100		
2	2.25	220	101-300	A	1-700
3	2.50	500	301-700		
4	2.75	1000	701-1300	B	301-1800
5	3.00	1550	1301-1800		
6	3.25	2200	1801-2600	C	701-3400
7	3.50	3000	2601-3400		
8	3.75	3800	3401-4200		
9	4.00	4600	4201-5000	D	1801-7400
10	4.25	5400	5001-5800		
11	4.50	6200	5801-6600		
12	4.75	7000	6601-7400	E	3401-9900
13	5.00	7850	7401-8200		
14	5.25	8700	8201-9100		
15	5.50	9500	9101-9900	F	7401-11700
16	5.75	10350	9901-10700		
17	6.00	11200	10701-11700		

The second and third columns show the rates and corresponding numbers of stations for the rate classes. The fourth column shows the numbers of stations limiting the rate classes. The station-class limits, in the sixth column, are chosen so that each exchange will be placed in two classes. To the classification of business-one-party rates correspond similar classifications of all of the other rates.

It will probably not be desirable that the telephone companies adhere rigidly to these classifications. The income of each exchange will be determined by the average weighted rate of the rate-class in which the exchange is placed, modified by bonuses or penalties which may be applied to it.

The telephone companies can generally be depended upon to allocate their rates so that the service will be developed in the manner most satisfactory to the community, for this will generally be the allocation which will be most profitable to the telephone company. If cases occur where the telephone company, having been assigned an average-weighted rate and bonuses or penalties, attempts by manipulating its rate classification to increase its income at the expense of the community, it will be made apparent to the public-utility commission by comparison with the average rate classifications of all exchanges. The telephone company may then be given an opportunity to justify its classification. Cases will, of course, occur where abnormal rate-classifications are justified, but they should always be subjected to examination.

Each telephone exchange shall be assigned to two station-classes. By the application of bonuses, a telephone company might possibly obtain the highest rate in the upper class, or by the application of penalties, it may have to accept the lowest rate in the lower class. Thus, an exchange operating 1500 stations would be placed in Classes B and C. It might be able by bonuses to obtain a rate of $3.50 a month for business-one-party service, or it might have to accept $2.50 a month because of penalties. The correction by bonuses or penalties shall be made from the rate of the non-overlapping, or rate classes. Thus the bonuses and penalties of the exchange of 1500 stations would be applied to a rate of $3.00 a month. If sufficient bonus were applied to amount to 25 cents a month, the rate schedule corresponding to $3.25 a month would apply. The above classification will amply provide for bonuses or penalties which are likely to be earned by abnormal bonus and penalty bases.

THE TYPICAL TELEPHONE EXCHANGE

In order to obtain further information in regard to the fairness of the present and proposed bonuses and penalties, they were applied to a typical exchange of 2000 stations, which is an average of the range of numbers of stations in the exchanges to which the present constants apply. Since all of the data from which the present bonus and penalty constants were obtained refer to the exchanges of Group 4, the station-rate curves of this group were used in assigning limits to the various classes. Referring to Table 11, it may be seen that the typical exchange of 2000 stations may fall in station-classes C or D.

Making use of the average values which apply to the Ohio telephone exchanges of Table 2, the plant and service statistics of Table 12 would apply to the typical telephone exchange of 2000 stations.

TABLE 12

STATISTICS OF TYPICAL TELEPHONE EXCHANGE OF TWO THOUSAND STATIONS

1.	Working stations	2,000
2.	Miles of pole line (2000 x .075)	150
3.	Average calls per day (2000 x 5.02)	10,040
4.	Maximum busy-hour load (2000 x .710)	1,420
5.	Population of field for development (2000/229) (1000)	8,750
6.	Plant valuation (44880/0.34)	$132,000
7.	Gross annual income (2000 x 1.87 x 12)	$44,880
8.	Rate for business-one-party service (Class 6)	$3.25

RATE AND BONUS AND PENALTY CHANGES REQUIRED TO RAISE AN EXCHANGE FROM ONE RATE CLASS TO ANOTHER

As has been pointed out, a change of 5 cents a month on the average-weighted-rate station-rate curve corresponds to a change of 25 cents a month on the business-one-party curve. Then from the definitions of P, C, D, and R, it follows that the percent change in average-weighted rate necessary to raise the exchange from one rate class to another is $.05/1.87 = 2.7$ percent, where $1.87 is the average average-weighted rate of Table 2. It follows also that the corresponding percent changes in the bonus and penalty bases with the present and proposed constants are as shown in Table 13.

TABLE 13

PERCENT CHANGE IN P, C, OR D REQUIRED TO RAISE EXCHANGE TO HIGHER CLASS

Constant	Basis	Present Constant	Percent Change to Raise to Higher Rate Class	Proposed Constant	Percent Change to Raise to Higher Rate Class
K_1	Pole Line	.08	$2.7/.08 = 34$	.34	$2.7/.34 = 8$
K_2	Calls per Day	.16	$2.7/.16 = 17$	.65	$2.7/.65 = 4$
K_3	Development	.02	$2.7/.02 = 135$	.50	$2.7/.50 = 5$

Thus, in order to raise the telephone company from one rate class to another, the development must be increased by 135 percent, an impossible increase with the present constants. With the proposed constants, an increase in development of 5 percent will accomplish a like result.

EFFECT OF INCREASING P; HOLDING C AND D UNCHANGED

Miles of pole line per station is a measure of conditions against which a telephone company must contend, rather than a condition which it would be expected to better. Thus, a telephone company would obtain a bonus if it were necessary for the pole line per station to be higher than the average for exchanges of the same size in order that it might reach its subscribers. The company would not, however, be expected to extend lines into sparsely settled regions merely to obtain a bonus.

In order that the telephone company described in Table 12 may obtain an increase in its business-one-party rate of 25 cents a month, or in its average-weighted rate of 5 cents a month, by increasing the miles of pole line per station, an increase of pole line per station of 8 percent (with proposed constant) or of 34 percent (with present constant) is necessary as is shown in Table 13. Accordingly, the miles of pole line per subscriber must be increased from .075 to .081 or .100. If it is assumed that the new territory is one-tenth as thickly settled as the original territory, and if x represents the additional subscribers requiring, each, 0.75 miles of pole line to connect them to the central office,

$$(150 + 0.75\,x)\ /\ 2000 = .081, \text{ and } x = 16, \text{ or}$$
$$(150 + 0.75\,x)\ /\ 2000 = .100, \text{ and } x = 66$$

That is, with the present constants there will be required 66 subscribers with 50 additional miles of pole line, and with the proposed constants there will be required 16 subscribers with 12 additional miles of pole line.

Assuming the average values of Table 2, the plant statistics with the proposed and present constants would then be as follows:

	With Proposed Constants	With Present Constants
1. Working stations	2,000	2,000
2. Miles of pole line	162	200
3. Average calls per day	10,040	10,040
4. Population of field for development	8,810	9,020
5. Development	227	222
6. Gross annual income	$46,080	$46,080
7. Original gross annual income	$44,880	$44,880
8. Gain in gross annual income	$1,200	$1,200
9. Cost of installing necessary additional well-constructed pole line	$7,800	$32,500
10. Cost of installing cheapest additional pole line	$1,200	$5,000
11. Increase in production and investment costs other than interest corresponding to "9"	$870	$3,640
12. Increase in production and investment costs other than interest corresponding to "10"	$130	$560
13. Net gain in gross income corresponding to "9"	$330	—$2,440
14. Net gain in gross income corresponding to "10"	$1,070	$640
15. Maximum profitable investment corresponding to "9"	$4,130	—$30,500
16. Maximum profitable investment corresponding to "10"	$13,380	$8,000
17. Business-one-party rate	$3.50	$3.50

An explanation of each item and a discussion of the plant statistics as a whole follow:

(1) The number of working stations in each case is 2000, rather than 2016, and 2066, because when the additional pole line was installed, it was not assumed that any new subscribers were obtained. Obtaining new subscribers relates to development, and will be considered under development.

(2) As stated above, 12 and 50 miles of pole line were added. Hence the total miles of pole line are 162 and 200.

(3) The calls per day per station, and therefore calls per day, remain unchanged.

(4) As miles of pole line is assumed to be a measure of area served, and as it has been assumed that the additional territory to be served is only one-tenth as thickly populated as the original territory, the populations of the original and increased fields for development will be proportional to the miles of pole line, with the additional pole line properly weighted. Then, if x equals the population of the increased field,

$$x \cdot \frac{229}{1000 \times 2000} = \frac{150 + 1.2}{150}, \text{ and } x = 8810, \text{ and}$$

$$x \cdot \frac{229}{1000 \times 2000} = \frac{150 + 5}{150}, \text{ and } x = 9020$$

(5) Development is obtained by dividing the stations by the population of the fields for development, and multiplying by 1000

$$\frac{2000}{8810} \times 1000 = 227, \text{ and } \frac{2000}{9020} \times 1000 = 222$$

These values represent decreases of .87 percent and 3.06 percent from the average development, 229. As changes of 5 percent and 135 percent respectively are required to move a telephone exchange from one class to another, the above decreases in development may, in the case under discussion, be neglected.

Development, although closely associated with miles of pole line per station is thus found to be negligible with regard to rate correction. This follows from the fact that they are measures of two entirely different, although dependent, conditions. Miles of pole line per station is a measure of sparseness of population, while development is a measure of the number of people in a given population who use the telephone, whether this population is thin and rural or thick and urban.

The interdependence of the two lies in the fact that there is a natural and easily explained tendency to construct pole lines first in thickly settled communities or along the highways. In this early stage development would be low and pole line per station low. As the more sparsely settled regions are reached, development and miles of pole line per station would both increase. However, in the case of a telephone exchange of the size under consideration, development within the field will bring no increase in pole line per station, but probably a decrease, since it does not involve the construction of extra pole line, so much as it does persuading unwilling customers to buy telephone service. Thus, in order to raise miles of pole line per station the telephone company must exploit new territory. This will increase the field for development rapidly, but the stations only gradually. Therefore, as pole line per station increases, development will decrease.

(6) and (7) The original gross annual income is $1.87×12×2000=$44,880. The gross annual income is ($1.87+.05)×12×2000=$46,080 where .05 is the bonus, corresponding to the increase of 25 cents a month in the business-one-party rate.

(8) The difference between (6) and (7) gives the increase in gross annual income—in each case, $1200—due to the bonus allowed for 12 or 50 additional miles of pole line constructed. It makes no allowance, however, for the increased investment and production costs due to the additional pole line.

(9) and (10) If the telephone company desired to construct a permanent pole line with which to give good service, the cost of the pole line might be assumed to be $650 a mile. If, on the other hand, the additional pole line were constructed merely to obtain a bonus, it would be constructed as cheaply as possible, and the cost might be as low as $100 a mile. The corresponding costs of the 12 miles of pole line would be $7800 and $1200, and of the 50 miles of pole line, $32,500 and $5000.

(11) and (12) An increased investment results in increased production and investment costs. If pole line per station is increased—average calls per station, and development remaining the same—the production and investment costs, other than interest, will be increased by 11.5/.6, or 19.2 percent, less 8 percent interest, or by 11.2 percent of the additional plant valuation as is shown in Table 8. If this factor is applied

to the pole line costs of (9) and (10), the increased production and investment costs other than interest will be obtained. They are approximately $870, $130, $3640, and $560.

(13) and (14) The difference between these sums and the gain in gross annual income of (8) gives the approximate net gains in gross annual income, $330, $1070,—$2440, and $640. The negative sign indicates that the increase in gross annual income produced by the increase in rates will not be sufficient to cover increase in production and the investment costs, other than interest, chargeable to the increased investment. This increase in production and investment costs would then have to be paid from other sources.

(15) and (16) If the above amounts are capitalized at 8 percent, it will be found that the increase in rates obtained by a bonus, will justify expenditures of $4130, $13,380, —$30,500, and $8000 in order to obtain the bonus.

(17) This business-one-party rate, $3.50, is the old rate, $3.25 plus a bonus of 25 cents obtained by the additional miles of pole line per station.

An examination of the above results leads to the following conclusions:

(a) The proposed constant, .34, would not quite produce conditions under which the development of new territory by the construction of the best telephone lines is made profitable and self-supporting. The cost of 12 miles of such a line would be $7800, and the expenditure which would be justified after production and investment costs, other than interest, were paid would be only $4130.

(b) The proposed constants would produce conditions under which the construction of the cheapest telephone lines is easily justified in so far as expenditure of money is concerned. The cost of 12 miles of such a line would be $1200 and the expenditure justified is $13,380.

((c) It would appear that the proposed constant would produce conditions under which the development of new territory by the construction of reasonably good telephone lines (costing $525 per mile) would be justified, and that such lines would in general be satisfactory in new territory. If good judgment is used in the selection of territory to be developed, not only will the pole line bonus immediately give an increase in rates, but after some years, development and average calls per station will increase, and additional revenue due to more subscribers will add value to the investment. These additional sources of revenue follow from judicious selection of the field for development, careful advertisement of service, and efficient management, rather than from the construction of new pole line. The effect of the pole line bonus with respect to the development of new fields, however, is to encourage such development by making communities already established, and in direct connection with the new territory, pay for it in advance.

(d) The present constant, .08, which has unconsciously been applied would justify only the cheapest development. The expenditure of money which would be justified to obtain a bonus would be $8000, and the cost of the additional fifty miles of pole line would be $5000.

(e) The present constant greatly discourages the better types of new pole line construction. The increased production and investment costs attendant upon the construction of fifty miles of good pole line would, even after the increase in rates were granted, result in an annual loss of $2440. This loss would result from the investment of $30,500. In other words, the present system would force the telephone company under discussion, in order to develop new territory, to expend over $30,000, and then accept an annual loss of $2440 for several years until the service is made self-supporting. This affords an explanation of why only a large and wealthy telephone company can exploit new territory advantageously.

(f) It would thus appear that the proposed constant is conservative. It might to advantage be made even greater.

EFFECT OF INCREASING C; HOLDING P AND D UNCHANGED

The average calls per day is a measure both of cost of plant and cost of operation, and of the value of the service to the subscriber. From an examination of a limited number of exchanges it was indicated that the maximum busy-hour load is proportional to the average calls per day. The busy-hour load or peak load on the telephone exchange determines in part the size and cost of the central office. In this investigation, because of the difficulty of obtaining information as to the busy-hour load, busy-hour load was represented by average calls per station per day. As the average calls per station per day determines also the cost of operation, this factor is a measure of cost.

The use of the telephone will be greater if transmission is good, if service is fast and accurate and generally satisfactory, and if the management is effective. Average calls per station per day, then, is also a measure of the value of telephone service to the subscriber.

If the management of the typical telephone exchange described in Table 13 should desire to obtain a bonus for average calls per station per day, an increase of 4 percent (with proposed constant) or 17 percent (with present constant) would be necessary in order to raise the telephone company to the next rate-class as is shown in Table 13. The central-office equipment would have to be augmented, the operating force added to, the service improved, and advertised, and the good will of the public gained in order that the use of the telephone might be increased. The increases of 4 percent and of 17 percent amount to 402 calls per day and 1710 calls per day. The plant statistics with the proposed or present constants would then be:

	With Proposed Constant	With Present Constant
1. Working stations	2,000	2,000
2. Average calls per day	10,442	11,750
3. Maximum busy-hour load	1,477	1,662
4. Business-one-party rate	$3.50	$3.50
5. Gross annual income	$46,080	$46,080
6. Original gross annual income	$44,880	$44,880
7. Gain in gross annual income	$1,200	$1,200
8. Estimated increase in cost of central-office equipment	$530	$2,240
9. Increase in production and investment costs other than interest	$490	$2,110
10. Net gain in gross annual income	$710	—$910
11. Maximum profitable investment	$8,880	—$11,380
12. Surplus available for advertising and improvement of service	$8,350	$13,620

An explanation of each item and a discussion of the plant statistics as a whole follow:

(1) Working stations have not increased, as any change in the number of working stations would affect development, and development has been assumed to be constant.

(2) These numbers of calls are obtained by adding the additional numbers necessary to raise the telephone company into the next higher rate class to the average calls per station per day of the typical telephone exchange described in Table 12.

(3) The maximum busy-hour load is 1420. This is obtained by multiplying 2000 working stations by .710, a factor obtained from an examination of statistics of twenty telephone exchanges operating between 1000 and 3000 stations. Because

of the fact that data in regard to only twenty telephone exchanges were available, this factor amounts to no more than an indication of the relation between maximum busy-hour calls and stations. The examination of these telephone exchanges indicated also that maximum busy-hour load and average calls per station per day vary in the same proportion. Therefore, the probable maximum busy-hour loads are 1420 + (.04 × 1420), and 1420 + (.17 × 1420), or 1477, and 1662.

(4), (5), (6), and (7) Business-one-party rate, gross annual income, original gross annual income, and gain in gross annual income are the same as for the previous discussion in regard to miles of pole line per station and are obtained in the same way.

(8) The increase in maximum busy-hour load will require a proportional increase in central-office equipment. According to assumption, the gross annual income will amount to 34 percent of the plant valuation. Then, if the original gross annual income were $44,880, the original plant valuation must have been $44,880/.34, or $132,000. If 10 percent of this plant valuation is in the central office equipment, this part of the plant valuation, or $13,200, will increase in proportion to the maximum busy-hour loads in the two cases considered. The additional amounts invested, then, are .04 × $13,200, or $530, and .17 × $13,200, or $2240.

(9) An increase in the production and investment costs other than interest will result from the increased traffic cost which increases in proportion to the average calls per station per day, and from the increased maintenance and investment costs on the additional central-office equipment which is required.

The traffic cost is 8 percent of the plant valuation, as is shown in Table 9, or in this case, $10,560. The approximate increase in traffic costs in the two cases considered, with proposed constant, and with present constant, are .04 × $10,560, or $420, and .17 × $10,560, or $1800.

As may be seen from Table 9 the increase in maintenance and investment costs other than interest on the additional required central-office equipment is 14 percent of the additional investment in central-office equipment, or in this case, $70 and $310. The total production and investment costs are then $420 plus $70, or $490, and $1800 plus $310, or $2110.

(10) The net gain in gross annual income is obtained by subtracting the above increases in production and investment costs from the gain in gross annual income, $1200 ,resulting in the amounts $710 and —$910. The fact that the latter of these amounts is negative shows that too little importance has been placed upon average calls per day per station or the service factors of which it is an indication, since the increase in production and investment costs alone is greater than the gain in income resulting from the rate bonus. Hence, with the present constants, a betterment in transmission, speed and accuracy of service, and efficiency of management which would result in increased calls would be penalized.

(11) If the net gains in gross annual income are capitalized at 8 percent, there will be obtained the maximum expenditures which would result in profit to the telephone company. These are $8880, and $11,380.

(12) The sums available for advertising and improving service are the difference between the maximum profitable expenditures and the costs of additional required central-office equipment. These are $8880 — $530, or $8350, and — $11,380 — $2240, or — $13,620.

If a proposed bonus and penalty constant of 0.40 instead of 0.65 is used, a 7 percent increase in average calls per station per day will be required to raise the exchange to the next higher rate class. The net gain in gross annual income will be $1200 —$880, or $320, which would justify an investment of $4000. As the increase in the cost of the central-office equipment is $920, $3080 would be left for advertising and service improvement.

An examination of the above results leads to the following conclusions:

(a) The proposed constant, 0.65, is too liberal. The amount available for advertising and improving the service should be more than ample. It will probably be found that a smaller constant as 0.40 will be sufficient to encourage telephone companies to better their service and incur expenses of advertising in order to obtain bonuses.

(b) The present constant 0.16 is too small. It will be seen that an effort to obtain a bonus will make it necessary for the telephone company to make an annual expenditure in production and investment costs which exceeds the gain in gross annual income by nearly $1000. An improvement in service would thus result in a financial loss, and the telephone company would be penalized for improving its service.

In the above discussion, it has been assumed that no subscribers would be added. Any advertising campaign, or improvement of service would obtain new subscribers for the telephone company, and, therefore, increase development. However, unless the advertising campaign were especially directed toward this end, an increase in the number of subscribers sufficient to cause an increase to a higher rate due to an increase in development only would not be probable. In actual practice, such a campaign would not be directed toward either end to the exclusion of the other, but the rate increase would finally result from the summation of the effects of both. However, in this discussion, the purpose of which is to determine the justice of the values assigned to the constants, it is, of course, necessary to consider them separately rather than as operating simultaneously.

EFFECT OF INCREASING D; HOLDING P AND C UNCHANGED

Development is primarily a measure of the value of the telephone service to the subscriber, but to obtain a development which is greater than the average, the telephone company must incur expenses of operation and management; hence cost is also involved.

In order to obtain greater than the average development, transmission must be improved, and plant troubles which directly affect the subscriber decreased, greater speed and accuracy of service developed, business methods adapted to the needs of the community, the telephone service advertised, and the service bettered in many other ways.

In order, by improving the service, so to increase development that a bonus of 25 cents a month is justified, there will be required an increase of 5 percent in development (with proposed constant) or 135 percent (with present constant) as shown by Table 13. This increase will be applied to the business-one-party rate $3.25 a month of the typical telephone exchange described in Table 12. Assuming that the population of the field for development is constant, these increases in development would involve changes in stations of 100 or 2700, or from 2000 to 2100, or 4700.

Statistics in regard to the telephone exchange after the development with proposed and present constants necessary to earn the bonus had been reached, and the rate increase granted would be as follows:

	With Proposed Constant	With Present Constant
1. Working stations	2,100	4,700
2. Population of field for development	8,750	8,750
3. Rate for business-one-party service	$3.50	$4.25
4. Gross annual income	$48,400	$116,700
5. Original gross annual income	$44,880	$44,880
6. Gain in gross annual income	$3,520	$71,820
7. Increase in cost of equipment	$6,600	$178,200
8. Increase in production and investment costs other than interest	$1,720	$46,400
9. Net gain in gross annual income	$1,800	$25,420
10. Maximum profitable investment	$22,500	$317,750
11. Justified expenditure for service improvement and advertising	$15,900	$139,550
12. Justified yearly expenditure for service improvement and advertising	$1,270	$11,160
13. Justified yearly expenditure for service improvement and advertising per additional station	$13	$4

An explanation of each item and a discussion of the statistics as a whole follow:

(1) and (2) The percentages of increase in development required to obtain a bonus, 5 and 135 percent, represent 100 and 2700 stations. The total stations then are 2100 and 4700. The population of the field for development has been assumed constant.

(3) The rate after a bonus has been obtained with the proposed constant is $3.25 plus $.25, or $3.50 a month. The 4700 stations required to obtain a bonus with the present constant, place the telephone company in Class No. 9, the business-one-party rate of which is $4.00 a month as is shown by Table 11. Then, the bonus for development would raise this rate to $4.25 a month. The corresponding average-weighted rate is $2.07 a month.

This number of stations in a community of this size is improbable. The development would be 538 telephones per thousand people. Such a development would require that there be a telephone, and in many cases, several telephones in every home and place of business. Such a condition could not be found, but it would be necessary with the present constant in order that a bonus might be obtained for development.

(4), (5), and (6) The original gross annual income is the average weighted rate before the bonus was granted times the number of stations, or \$1.87 × 12 × 2000, or \$44,880. After granting the bonus, the gross annual income is \$1.92 × 12 × 2100, with the proposed constant, and \$2.07 × 12 × 4700, with the present constant, or \$48,400, and \$116,700. The differences between the gross annual incomes and the original gross annual incomes, or \$3520, and \$71,820, are the gains in gross annual income.

(7) and (8) As the original gross annual income is \$44,880, and as this, as shown in Table 7 is 34 percent of the plant valuation, the plant valuation is \$132,000. As may be seen from Table 10, the production and investment costs other than interest which were originally 26 percent of the plant valuation \$132,000, or \$34,300 increase in proportion to development. The increases in production and investment costs other than interest then are \$1720 and \$46,400. The plant valuations also increase in like proportion. The increases in plant valuation are \$6600 and \$178,200.

(9) The net gain in gross annual income is the difference between the gain in gross annual income, and the increase in production and investment expenses other than interest. In this case with proposed and present constants, the net gains in gross annual income are \$1800 and \$25,420.

(10) and (11) The net gains in gross annual income capitalized at 8 percent will produce the amounts which may profitably be invested in obtaining the bonus. These amounts are \$22,500 and \$317,750. These amounts less the increases in cost of equipment of (7) are the amounts, \$15,900 and \$139,550, the justified expenditure for service improvement and advertising.

(12) and (13) Assuming that the interest at 8 percent on the justified expenditure for service improvement and advertising is spent for these purposes, the justified yearly expenditures for service improvement and advertising with the proposed and present constants will be \$1270, and \$11,160, and the corresponding justified yearly expenditures per additional station for service improvement and advertising will be \$13 and \$4. As the cost per station of obtaining 2700 additional stations in a field for development with a population of 8750 will be very high, \$4 per station will certainly be inadequate. As it will probably not be difficult to obtain 100 addi-

tional stations in such a field, $13 will probably be more than sufficient.

An examination of the above results leads to the following conclusions:

(a) The proposed constant 0.50 is adequate, and probably too liberal.

(b) The present constant, 0.02, is too low and makes it unwise for a telephone company to attempt to obtain a bonus by increasing development.

(c) A constant less than 0.50 and much greater than .02 would probably be just. If the cost of improving and advertising the service to the extent necessary to obtain the additional stations were known, a positive determination of the proper constant could be made. In the absence of such data, the most reasonable assumption is that the best value of the constant would be an average value between the present and proposed constants, or 0.26. It is not to be expected that a value constant can be determined as definitely as a cost constant.

THE PROPOSED BONUS AND PENALTY CONSTANTS

It would appear, then, that the best values of the constants would be:

$K_1 = 0.34$, $K_2 = 0.40$, and $K_3 = 0.26$,

and the deviation equation becomes:

$$0.34P + 0.40C + 0.26D = R.$$

It is to be noted that although these constants are larger than the present ones, 0.08, 0.16, and 0.02, their order of importance is the same.

SECTION X. APPLICATION OF RATE-STANDARDIZATION PLAN

The proposed rate standardization plan will provide rates which are in the average case just to telephone companies and subscribers at the present time, because in communities of any given size they are based upon average rates, which, as explained above in Section VI, are average production costs plus investment costs on normal plant costs.

ADJUSTMENTS NECESSARY TO CARE FOR CHANGES IN PRICE LEVEL AND DEVELOPMENTS IN THE ART OF TELEPHONY

The proposed plan does not, however, make any adjustments for rapid changes in price level, such as those of the past five years, nor does it make any adjustments for new developments in telephone equipment or methods of operation.

If the general price level of all commodities should rise rapidly, rate increases would be granted only after considerable delay. The production and investment costs of the telephone companies would increase rapidly following the change in price level, and the income of the telephone companies would not be sufficient to cover the costs. If, on the other hand, the price level should fall rapidly, the result would be an income to the telephone company greater than sufficient to cover the production and investment costs. The surplus income should fairly be distributed to the telephone subscribers by decreases in rates.

If developments in the art of telephony make the production of telephone service more economical as well as more satisfactory, the telephone companies should not derive all of the benefit from them, and rate decreases should follow; although, on the other hand, the rate decreases should not be so great that the telephone subscribers will derive all of the benefit from the development.

In order to forestall these possible disadvantages, the public-utility commissions must at times make arbitrary adjustments in the average rates. Some plan must be provided by which the public-utility commissions may be informed at all times of the average rates and of the conditions which would justify bonuses or penalties ,and, having information in regard to changes in average rates, they must make any necessary arbitrary adjustment promptly.

METHODS OF APPLICATION OF RATE-STANDARDIZATION PLAN

One plan which would be satisfactory would be for the public-utility commissions to keep continuous records of production and investment costs, and the statistics from which may be obtained, miles of pole line and conduit per station, maximum busy-hour calling-rate, average calls per station per day, and development. From these data, the bonuses and penalties and new average rates to be applied may be obtained. These data in the form of annual or semi-annual reports should be obtained separately for each exchange in the state, whether or not the various telephone companies operate one or a number of exchanges.

The work on the part of the telephone companies necessary to prepare these reports would not be burdensome. The advantage to the telephone company of keeping financial accounts has been conceded. Similar advantages would result from plant and traffic accounts which would be made necessary by this plan. They would suggest to the telephone companies many improvements and economies.

From these data may be obtained the average curves, constants, and statistics for groups of exchanges of various sizes. These will be the data for average typical exchanges, similar to the data of the typical exchange of 2000 stations discussed in this report. The knowledge on the part of those interested in the production of telephone service of what are typical telephone exchanges will be valuable for comparative purposes.

The allowable average-weighted rate will be assigned by the public-utility commission and the choice of a rate schedule should be left to the telephone company, as suggested in Section IX.

A public-utility commission obviously cannot make an arbitrary adjustment of rates for each small change in production and investment costs. It is desirable, in fact, to make such adjustments as infrequently as possible. A minimum change in production and investment costs may be chosen, such as 10 percent, below which arbitrary adjustments will not be made. If it should become apparent to a public-utility commission that this minimum change may be exceeded, all necessary information may be collected and examined, and other preparations made to adjust the rates without delay when the minimum change has been exceeded. Such a procedure may well apply to changes in production and investment costs resulting from any causes such as changes in price level, or economies due to developments in the art of telephony.

NECESSITY FOR PERIODICAL VALUATIONS OF TELEPHONE PLANT

It is not reasonable to suppose that this plan for rate standardization could be applied indefinitely without such a check upon plant valuation and, therefore, upon investment costs, as would result from reappraisals of the telephone plants. Such reappraisals should be made on a state-wide basis at chosen intervals—every ten years for instance—and the continuous records modified accordingly if necessary. If such reappraisals were made periodically in this way, they would be sufficient check upon the continuous records, and would be much less costly than the unending series of appraisals and reappraisals and valuations of individual telephone plants which are necessary under the present system of rate determination. The work of the public-utility commissions would be decreased to such an extent that attention could be given to service standards and their enforcement.

ADDITIONAL PROBABLE REQUIREMENTS IN APPLICATION OF RATE-STANDARDIZATION PLAN

It will be necessary to apply this rate-standardization plan to sections of the country which are homogeneous. Confusion might be caused by an attempt to apply the average rate curves and constants of one section to one of a widely different character. However, more just results might be obtained in some cases by considering a number of states of the same general character together rather than separately, for, in this way, a tendency of one public-utility commission to be unfair to one or the other of the parties concerned might be balanced by an opposite tendency of another public-utility commission.

While in the preceding discussion of proposed bonus and penalty constants only the average station-rate curves of Group 4 of Ohio telephone companies with the corresponding data of Table 2 were considered, if the rate standardization plan were applied to Ohio, an average of the curves of Groups 1, 2, 3, and 4, and the corresponding data would be used. The average of the curves for exchanges of more than from 1000 to 1500 stations will probably provide rates which cover the cost of producing the telephone service and which will, therefore, be a just basis for rate determination at the present time. The average of the curves for exchanges of less than from 1000 to 1500 stations will probably provide rates which are not compensatory, and will have to be arbitrarily adjusted by the public-utility commissions.

If communities in which the telephone service is on a message-rate basis are included in the group of communities to which this rate standardization plan is applied, its average operating income per station will determine its position on the curve of average-weighted rates. Bonuses and penalties must then be applied to average operating income per station.

It should be noted that an advantage of this rate-standardization plan is that its adoption would not result in any radical change in rates, as the average rates are averages of those rates now in force.

APPLICATION TO OTHER PUBLIC-UTILITY RATES

While this investigation has been confined entirely to the standardization of telephone rates, the proposed plan can be applied also wholly or partially to the standardization of electric light and power rates, water, gas, and street railway rates, or, in other words, to all public-utility rates.

SECTION XI. SUMMARY

The faults of the present "cost plus" system of rate determination have been pointed out (Section II). A plan for rate standardization which will eliminate the worst of these faults, and which is practical has been proposed and discussed (Sections III, IV, and V). It has been shown that station-rate curves may be made which show the relation between numbers of stations served by exchanges and rates. These station-rate curves are based upon normal costs which are receiving serious consideration as a basis for rate determination. Typical station-rate curves for Ohio have been plotted and discussed (Section VI). The rate-deviation equation has been presented (Section VII), and applied to present telephone rate conditions in Ohio and other Middle West states. The solution of this equation shows what importance has been attached to the proposed bonus and penalty bases. Just bonus and penalty constants have been proposed (Section VIII) and carefully analyzed and modified (Section IX). The rate-deviation equation with bonus and penalty constants which should have been applied is presented (Section IX). The method of applying the proposed plan for rate standardization has been discussed (Section X).

The proposed plan offers an incentive to telephone companies to provide the best possible telephone service, and thus corrects the worst fault of the present system of rate determination. The adoption of the proposed plan would decrease the present burden upon the utility commissions and the telephone companies, and would eliminate some of the injustice and indefiniteness of the present system. Under the proposed system, quality of telephone service rather than cost of service is emphasized and a general improvement of telephone service may, therefore, be predicted as a result of the application of the proposed plan for rate standardization.

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EFFECT OF PULSATIONS ON FLOW OF GASES

BY
HORACE JUDD
AND
DONAL B. PHELEY

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BY

HORACE JUDD

PROFESSOR OF HYDRAULIC ENGINEERING,

AND

DONAL B. PHELEY

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EFFECT OF PULSATIONS ON FLOW OF GASES

By Horace Judd, Columbus, Ohio
Member of the Society
and
Donal B. Pheley,[1] Columbus, Ohio
Non-Member

The movement of gases and liquids when calculated by the existing hydraulic formulas presupposes a steady or continuous flow of the fluid. Anything which causes this flow to proceed in puffs, waves, or pulsations will result, by the action of metering devices, in errors often of great magnitude which generally do not admit of any adjustment, or of any definite knowledge of the amount of the error.

The present paper discusses work undertaken under the joint direction of the Engineering Experiment Station of the Ohio State University and the Research Sub-Committee on Fluid Meters of The American Society of Mechanical Engineers, which had for its object (a) *the study of the nature of the pulsation and* (b) *the discovery of some practical means of reducing or eliminating the pulsation or of compensating for its effects on the devices used for measuring fluid flow. The investigation was confined to the venturi meter, the orifice meter, the flange nozzle meter, and the pitot meter, using air flow from a small compressor discharging into a 3-in. line. It is believed, however, that the basic principles established by the experiments are fundamental for pulsating-flow conditions for gas, steam, and water as well as for air, and also for other sizes and kinds of installations.*

ONE of the most disturbing factors encountered in recent years in the metering of air, gas, steam, and water, especially in connection with all forms of power engineering, has been that due to turbulent or pulsating flow. This has not been confined to any one class or type of meter, but is present to a more or less degree with all forms of metering devices.

2 The measurement of gases and liquids when calculated by the existing hydraulic formulas presupposes a steady or continuous

[1] Robinson Research Fellow, The Ohio State University.

Presented at the Annual Meeting, New York, December 4 to 7, 1922, of The American Society of Mechanical Engineers.

flow of the fluid. Anything which causes this flow to proceed in puffs, waves, or pulsations will result, by the action of metering devices, in errors often of great magnitude which generally do not admit of any adjustment, or of any definite knowledge of the amount of the error.

3 The work described in the present paper was undertaken under the joint direction of the Engineering Experiment Station of The Ohio State University, and the Research Sub-Committee on Fluid Meters of The American Society of Mechanical Engineers. A sub-committee of the Fluid Meters Committee consisting of A. R. Dodge, H. N. Packard, and H. Judd was selected to take direct charge of the research work.

PURPOSE OF THE INVESTIGATION

4 The object of the investigation as outlined by the sub-committee in direct charge was twofold:

a To study the nature of the pulsation

b To discover some practical means of reducing or eliminating the pulsation, or of compensating for its effects on the devices used for measuring fluid flow.

5 As the work has proceeded it has been necessary to use a specific installation involving the flow of air only, and also to limit somewhat the scope of the investigation. The work is by no means considered to be complete, and it is the intention in the future to continue with the research so as to shed more light on many doubtful points that have arisen, as well as to try out a number of new suggestions.

6 For convenience, flow meters have been classified in two main divisions which may be called (1) positive meters, and (2) inferential meters. The domestic gas, or water, meter is an example of the first class. It is a displacement meter in which an actual volume of gas is introduced into a container of known size, and the quantity thus measured is registered on the meter.

7 In commercial installations of even moderate size, inferential meters are used almost entirely. In meters of this class some function of the quantity of fluid passing a given cross-section of pipe is measured and from this observation the actual flow is deduced or " inferred." Perhaps the most common function observed

in such meters is a pressure difference which is a quadratic function of the velocity of flow. This method can be made to give accurate results under steady-flow conditions, but when the flow is pulsating the accuracy of the measurement is seriously affected, if, indeed, not entirely destroyed.

8 Three general cases may be mentioned where this problem is of great importance: (1) The measurement of natural gas, both entering and leaving a compressor station where reciprocating compressors are used; (2) the measurement of air both entering and leaving large reciprocating air compressors, or blowing engines; and (3) the measurement of steam supplied to reciprocating steam engines. The steam flow is pulsating in character because the engine cuts off the steam supply during a considerable part of each stroke. In each of these cases the flow of the fluid has a regular, comparatively rapid, rhythmical pulsation, which occasions serious errors in measurement, especially where the measuring element is of the inferential type.

9 Similar pulsating conditions are present in water flow where reciprocating pumps are used. The problem, however, is more easily solved by the proper use of air chambers and surge tanks. Water hammer in pipe lines from whatever cause bears a striking similarity to the pulsating effect produced by an air-compressor valve.

10 We have confined our investigations to inferential meters. These meter elements as selected are the venturi meter, the orifice meter, the flange nozzle meter, and the pitot meter. Furthermore, we have been limited to air flow from a small compressor discharging into a 3-in. line; and hence our findings, strictly speaking, would be applicable only to installations of similar character. However, it would seem highly probable that the basic principles established by these experiments would be fundamental for pulsating-flow conditions for gas, steam, and water as well as for air, and also for other sizes and kinds of installations.

EQUIPMENT EMPLOYED IN THE INVESTIGATION

11 The experimental work was carried on in the Mechanical Engineering Laboratories of the Ohio State University, and was begun in May, 1920. A large part of the work was done during the summer vacations of 1920–1921. During the remainder of the two years devoted to the work such time was put in as could be spared

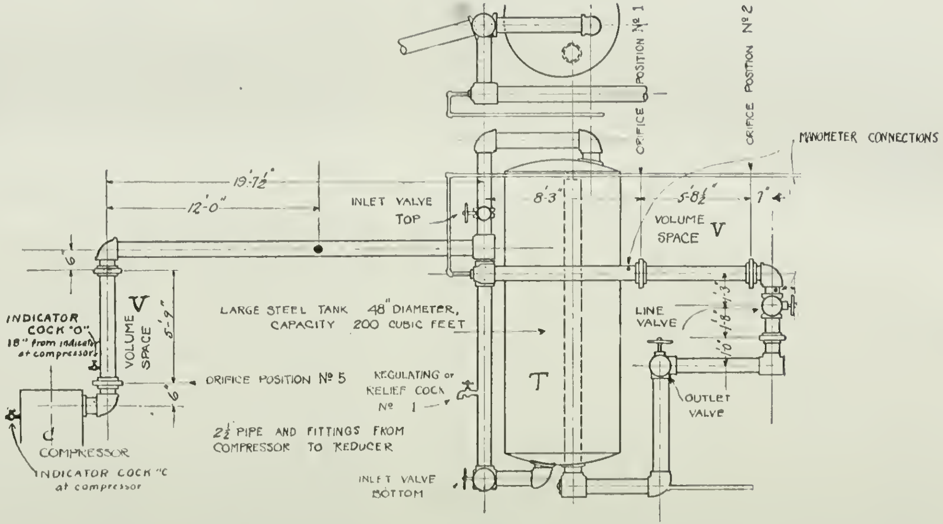

Fig. 1 General Layout of Apparatus — Left-Hand Portion

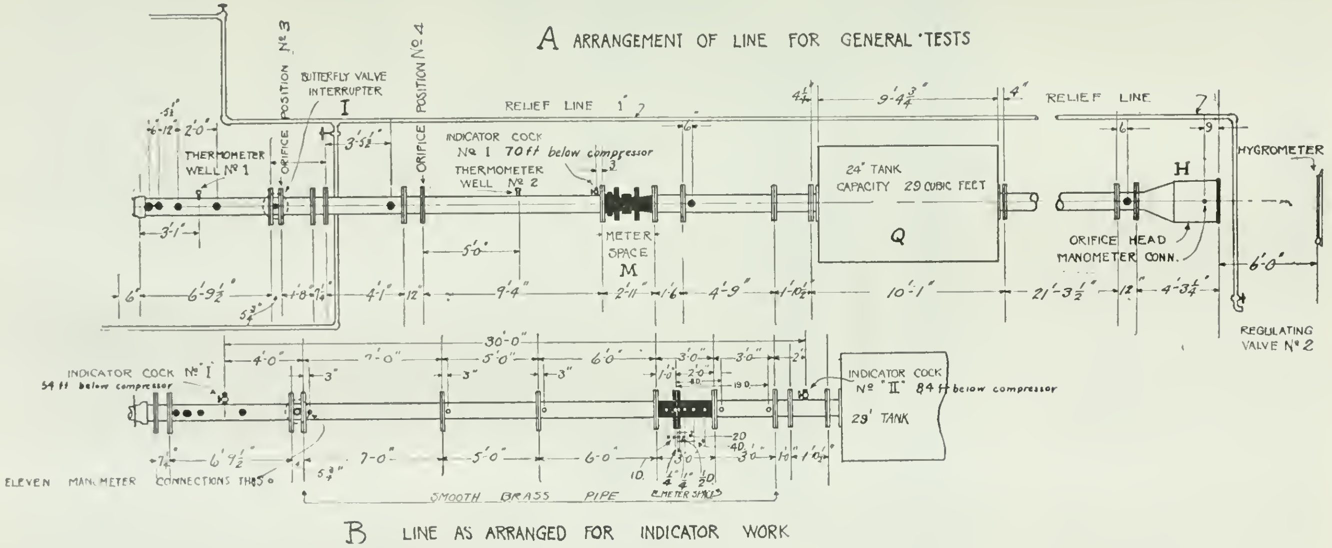
A ARRANGEMENT OF LINE FOR GENERAL TESTS
RELIEF LINE 1"
RELIEF LINE
BUTTERFLY VALVE INTERRUPTER
I
ORIFICE POSITION Nº 3
ORIFICE POSITION Nº 4
THERMOMETER WELL Nº 1
INDICATOR COCK Nº 1 70 ft below compressor
THERMOMETER WELL Nº 2
METER SPACE
M
24" TANK CAPACITY 29 CUBIC FEET
Q
H
ORIFICE HEAD MANOMETER CONN.
HYGROMETER
REGULATING VALVE Nº 2
6'-9½"
4'-1"
9'-4"
2'-11"
4'-9"
10'-1"
21'-3½"
4'-3¼"
9'-4¾"
6'-0"
5'-0"
3'-1"
30'-0"
INDICATOR COCK Nº I 54 ft below compressor
INDICATOR COCK Nº "II" 84 ft below compressor
29' TANK
ELEVEN MANOMETER CONNECTIONS THUS
SMOOTH BRASS PIPE
METER SPACE
B LINE AS ARRANGED FOR INDICATOR WORK

FIG. 1 GENERAL LAYOUT OF APPARATUS, RIGHT-HAND PORTION

from the university schedule. Acknowledgment should be made of the valuable service rendered by Mr. Paul Bucher,[1] especially during the vacation periods.

12 The essential elements for carrying on our project were: (1) a disturbing element to produce the pulsating flow; (2) a quieting element, or elements, to eliminate or modify the pulsations; and (3) a measuring element, to indicate constant flow conditions and also to indicate the effect of the pulsating flow.

13 *Disturbing Elements.* Fig. 1 shows the general layout of the apparatus, at the extreme left hand of which is located the air compressor *C,* a 9-in. by 9-in. single-stage, single-acting, gas engine-driven machine running at 293 r.p.m. This supplied air to a line about 120 ft. in total length of which 50 ft. was made up of $2\frac{1}{2}$-in. pipe containing several short lengths and fittings. The remaining 70 ft. comprised the 3-in. test line of straight continuous length. This test line was at first made up of standard 3-in. black pipe of commercial quality; later 24 ft. of 3-in. brass pipe was substituted for that portion of the test line preceding the meter and extending 3 ft. below the meter section. (See Fig. 1, *B.*)

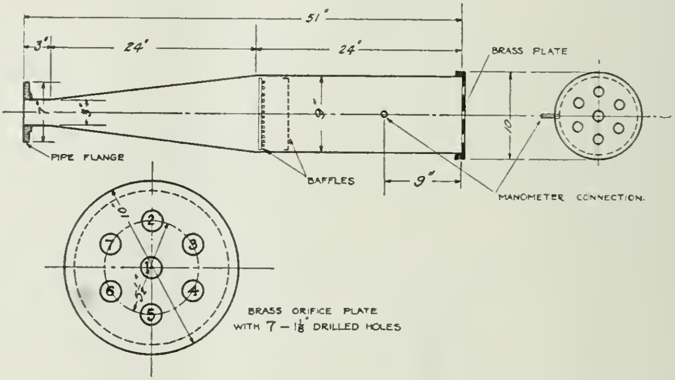

Fig. 2 Orifice Head

14 This air supply with its full pulsating effect could be admitted directly to the test line or could be first discharged through a large tank before entering the test line. A second disturbing element for producing pulsations artificially is shown at *I,* Fig. 1, the butterfly-valve interrupter. This butterfly valve could be

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[1] Assistant Professor of Steam Engineering, The Ohio State University.

driven at speeds ranging from 180 r.p.m. to 800 r.p.m., producing thereby a variation in the number of pulsations per second.

15 *Quieting Elements.* Considerable study was made of this essential feature and many trials were made to satisfy ourselves that we were securing pulsationless flow where and when needed. The tank *T*, Fig. 1, was used to quiet the pulsation before the meter station, *M*, was reached. This was a 48-in. vertical tank of 200 cu. ft. capacity. Air could be admitted either at the bottom or at the top and released from the tank through the internal pipe which reached nearly to the top. This tank when fitted with $1\frac{1}{4}$-in. orifices at top and bottom enabled us to get air flow free from pulsations before the test meters were reached.

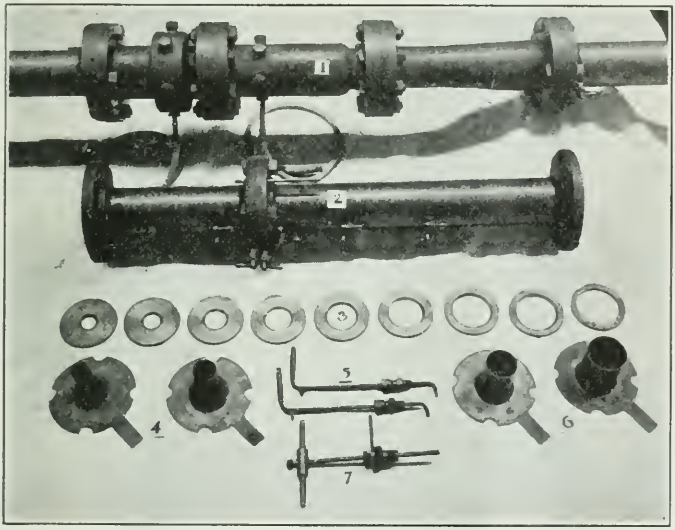

Fig. 3 Flow-Meter Units

16 A second quieting tank was inserted at *Q*, Fig. 1, at a point 8 ft. below the meter station. This was necessary in order to secure pulsationless flow at the orifice head for the purpose of establishing standard flow conditions. This tank was selected almost by chance and afterward was proved by test to be of sufficient capacity to eliminate practically all of the effect of the pulsating flow, and with the insertion of a $1\frac{1}{4}$ in. orifice at the exit from the tank we were entirely successful in securing pulsationless-flow conditions.

17 There are two positions marked, *V*, Fig. 1, one near the compressor and one just beyond the large quieting tank, where volumes of different sizes were inserted for the purpose of studying their quieting effect. Because the term volume seems to apply better we have perverted the word " volume " from a term meaning capacity to a special designation, and have used it altogether to denote the various tanks of different dimensions which have been used as quieting elements. Most of these volumes were used in the second volume space, *V*, beyond the large quieting tank.

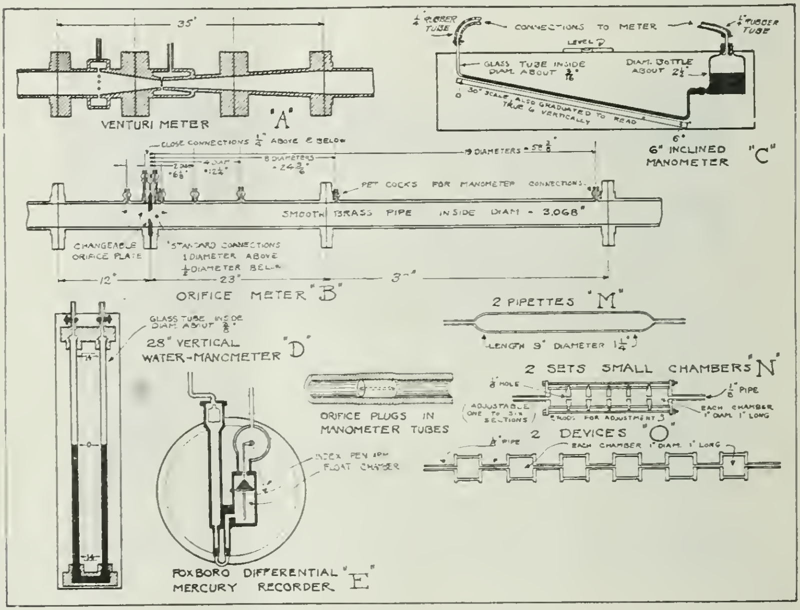
35"
VENTURI METER
"A"
¼" RUBBER TUBE
CONNECTIONS TO METER
¼" RUBBER TUBE
LEVEL
GLASS TUBE INSIDE DIAM. ABOUT 3/16"
DIAM. BOTTLE ABOUT 2½"
0
30" SCALE ALSO GRADUATED TO READ TRUE 6 VERTICALLY
6"
6" INCLINED MANOMETER
"C"
CLOSE CONNECTIONS ¼" ABOVE & BELOW
2 DIAM. = 6⅛"
4 DIAM. = 12¼"
8 DIAMETERS = 24 9/16"
19 DIAMETERS = 58 3/8"
PET COCKS FOR MANOMETER CONNECTIONS
SMOOTH BRASS PIPE INSIDE DIAM - 3.068"
CHANGEABLE ORIFICE PLATE
STANDARD CONNECTIONS 1 DIAMETER ABOVE ½ DIAMETER BELOW
12"
23"
37"
ORIFICE METER "B"
2 PIPETTES "M"
LENGTH 9" DIAMETER 1¼"
GLASS TUBE INSIDE DIAM. ABOUT 3/8"
28" VERTICAL WATER-MANOMETER "D"
0
2 SETS SMALL CHAMBERS "N"
⅛" HOLE
⅛" PIPE
ADJUSTABLE ONE TO SIX SECTIONS
RODS FOR ADJUSTMENT
EACH CHAMBER 1" DIAM. 1" LONG
ORIFICE PLUGS IN MANOMETER TUBES
2 DEVICES "O"
⅛" PIPE
EACH CHAMBER 1" DIAM. 1" LONG
INDEX PEN
FLOAT CHAMBER
FOXBORO DIFFERENTIAL MERCURY RECORDER "E"

Fig. 4 Section Sketches of Meters, Manometers, and Connections

18 The line valve near the entrance to the 3-in. test line was used as a quieting element when employed as a throttling device. Both a gate valve and a globe valve and in some cases orifices were thus used as throttling devices.

19 The combination of throttling with volumes constitute the muffler type of quieting device. Fig. 7 shows an 8-section pipe-flange muffler which was also inserted in the line at the second volume station. This muffler is located in a by-pass in front of the line which runs directly from the compressor. All the volumes used

as quieting devices were placed in direct line, but later it was found that the by-pass position answered just as well for the muffler or the other quieting devices. Fig. 9, Nos. 1, 2, 3, 5, 6, shows some of the disks which were tried in the muffler. Fig. 8 shows sectional views of the pipe-flange muffler and also other forms of mufflers, including two funnel mufflers. A form of pulsating bag, Fig. 6, was used as a quieting volume and was connected to the compressor line in a way similar to an air chamber on a reciprocating pump. Another form of device used was a system of revolving fans or baffles. (See Fig. 9, No. 7.)

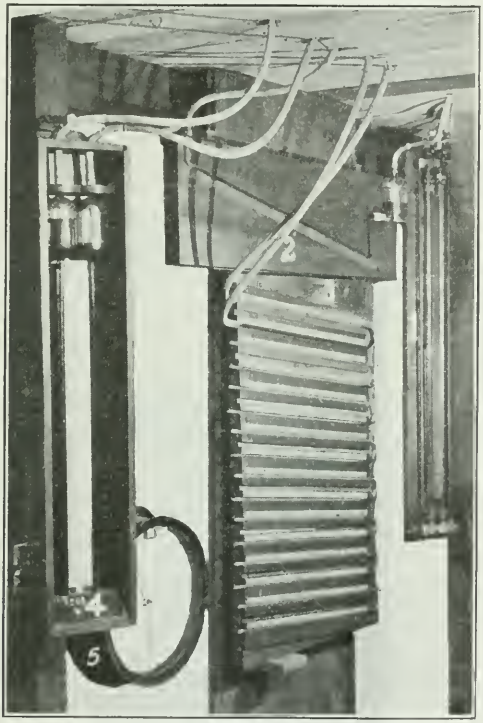

Fig. 5 Manometers Used with Flow-Meter Units

20 *Measuring Elements.* Under this heading will be taken up in order: (1) the orifice-head meter, (2) meter elements used, and (3) manometers used.

21 *Orifice-Head Meter.* It was recognized at the outset that one of the indispensable features was an accurate method of measur-

ing the discharge of the pipe, or (its equivalent) an accurate means of indicating the velocity of the air in the pipe. This was effected by means of an orifice head at the end of the pipe line, Fig. 1, *H*. Fig 2 is a detailed sketch of the orifice head. Its outer diameter is 9 in. for a distance of 2 ft., followed by a tapered section to meet the 3-in. line. This is given a taper of 7 deg., and was so chosen as being the limiting[1] angle for preventing as far as possible the swirling and eddying of the air as it passes into the orifice head from the line. Seven holes, reamed to $1\frac{1}{8}$ in., were provided in the head plate ($\frac{3}{16}$ in. thick), although during the tests not more than

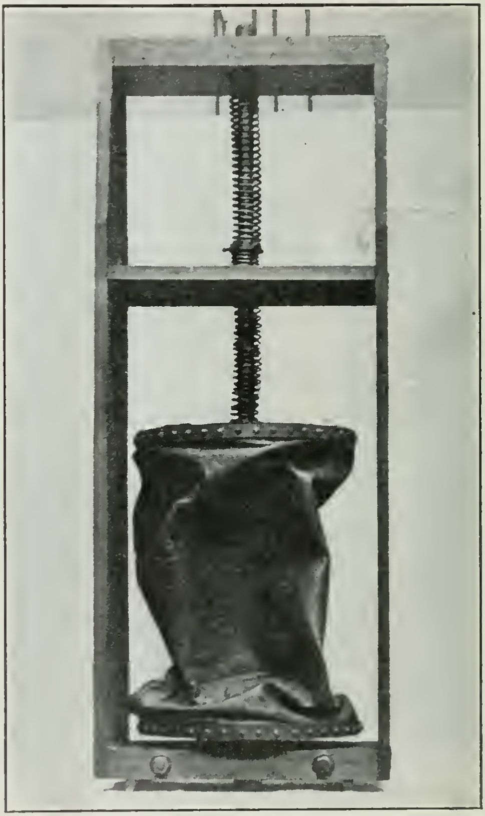

Fig. 6 Pulsating Bag

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[1] Trans. Am.Soc.M.E., vol. 42, 1920, p. 26: Physical Basis of Air Propeller Design, F. W. Caldwell and E. N. Fales.

five holes were used at one time. In general the capacity of the compressor was reached with four holes open with a standard static discharge head of 0.9 in. of water at the orifice head. The orifice head was calibrated by discharging air through it from the calibrated tank T under a constant static head at the orifice head. Readings every fifteen seconds were taken of this static head at the orifice head, giving 25 to 45 readings per run. The formula for a single orifice is:

$$q = KAV = 0.1261\,K\sqrt{h/W_a}$$

where q = air flowing cu. ft. per sec.
K = coefficient of discharge
h = static head at orifice head, inches of water
W_a = weight of 1 cu. ft. of air at temperature of flowing air at orifice plate.

For separate, single orifices the maximum value for $K = 0.6269$, the minimum, $K = 0.6037$; average for 13 runs, $K_m = 0.6086$. For orifices in combinations of twos, threes, fours, and fives, average of K for 22 runs including the single orifices = 0.6115.

22 The orifice head was also checked against a second orifice to show how uniformly the air was distributed in its cross-section. A variation was found between the center orifice and the surrounding single orifices ranging from 0.1 per cent below to 0.2 per cent above the ratio for the center orifice. The perforated baffles were then put in. These were two in number, the first being made up of a ¼-in. wooden strip screen with ¼-in. openings. Two inches in front of this wooden screen was placed a second perforated iron plate with 600 ¼-in. holes (see Fig. 2). All possible combinations of the orifices including single orifices and 5-hole orifices with and without the center orifice gave a variation not exceeding 0.1 per cent. This established uniform flow conditions in the orifice head regardless of the number of orifices open in the head plate.

23 For the standard 3-in. pipe (inside diameter, 3.068 in.) with a manometer head at the orifice head of 1 in. of water, barometer 29.50 in., humidity at 75 per cent, the velocity of air in the pipe for each orifice has been calculated as follows:

Temperature of air, deg. fahr.	Velocity of air in pipe, ft. per sec.
50	5.49
60	5.55
70	5.61
80	5.67
90	5.73

Wherever mention is made of the velocity of air in the pipe we have assumed 5.35 ft. per sec. for each orifice for a standard orifice head of 0.9 in. of water at 75 deg. fahr.

24 *Meter Elements Used.* The point of insertion of the meter elements in the line, Fig. 1, *M*, was about 70 ft. from the compressor, 50 ft. above the orifice head, and 25 ft. from the entrance to the 3-in. test line.

25 The venturi meter was a standard unit with 3-in. entry and 1-in. throat. It is shown attached to the pipe line in Fig. 3, No. 1, and in sectional view in Fig. 4, *A*.

26 The orifice meter, Fig. 3, No. 2, was made up of a flanged section of 3-in. brass pipe of the same length, 35 in., as the venturi section. The orifice flange, Fig. 4, *B*, was placed 12 in. from the upstream end and was counterbored to receive the set of orifice plates and to center them accurately. The downstream side of the hole in each plate was chamfered to $\frac{1}{32}$ in. in thickness, Fig. 3, No. 3.

Ratio of orifice and pipe diameters, per cent	Size of orifice plates: Diameter of orifice, in.
33	1.098
40	1.224
50	1.530
60	1.836
70	2.142
80	2.448
90	2.754
100	3.060

The manometer connections were made at one diameter above and one-half diameter below the orifice plate as the standard points of connection.

27 The flange nozzle meter was made by inserting in the orifice-meter section a special-shaped rounded-edge orifice with projecting cylindrical end, as in Fig. 3, Nos. 4 and 6. This nozzle

approximates the converging part of the venturi tube in shape. The manometer connections were made at one diameter above and one-half diameter below the flange, corresponding to the standard points of connection of the orifice meter.

28 Two forms of pitot tips were used. Fig. 3, No. 5, shows the hatchet-edge static tip (pitot No. 1) with $\frac{1}{8}$-in. side openings. This is used with the accompanying open-ended impact tip with $\frac{3}{16}$-in. opening. Both tubes are made of $\frac{1}{4}$-in. seamless brass tubing. Fig. 3, No. 7, shows a modified form of pitot tip (pitot No. 2) having $\frac{5}{16}$-in. brass tubing and an impact or leading opening facing the direction of flow and a static or trailing opening directly opposite. The diameter of each opening is $\frac{1}{8}$ in.

TABLE 1 SIZE OF FLANGE NOZZLES

Diam. of nozzle, in.	Inlet diameter at rounded edge, in.	Length, in.		
		Rounded edge	Cylindrical end	Total
$1\frac{1}{8}$	3	1	2.375	3.375
$1\frac{1}{2}$	3	1	2.375	3.375
2	3	1	2.375	3.375
$2\frac{1}{2}$	3	$\frac{5}{8}$	2.750	3.375

29 *Manometers Used.* As far as possible the simpler forms of manometers were used. The vertical U-tube water manometer, as in Fig. 5, No. 1; Fig. 4, *D;* Fig. 10, *S,* was used with the venturi meter, and part of the time with the orifice meter and flange nozzle meter, and also for the static line pressure. Where the readings had to be magnified, use was made of a 6-in. inclined one-leg reservoir oil manometer, 5 to 1 magnification (Fig. 5, No. 2; Fig. 4, *C*). For certain other readings an inclined U-tube oil manometer and a vertical U-tube two-liquid manometer were used (Fig. 5, Nos. 3 and 4).

30 A Foxboro differential mercury recording gage, Fig. 5. No. 5 and Fig. 4, *E,* was used to make comparisons with the water manometer used with the venturi meter. The flow conditions were maintained and checked at the orifice head by means of an Ellison inclined gage of 1 in. range and 10 to 1 magnification.

NATURE OF THE PULSATION

31 The first knowledge of the nature of the pulsation was gained through the use of an instrument which we have called the "photopulsometer." This instrument was made and loaned to us by Mr. H. N. Packard of the Cutler-Hammer Co., Milwaukee, Wis. Its principle is similar to that of the "phonedeik" designed by Prof. D. C. Miller[1] of Case School of Applied Science and used by him in photographic studies of musical sounds.

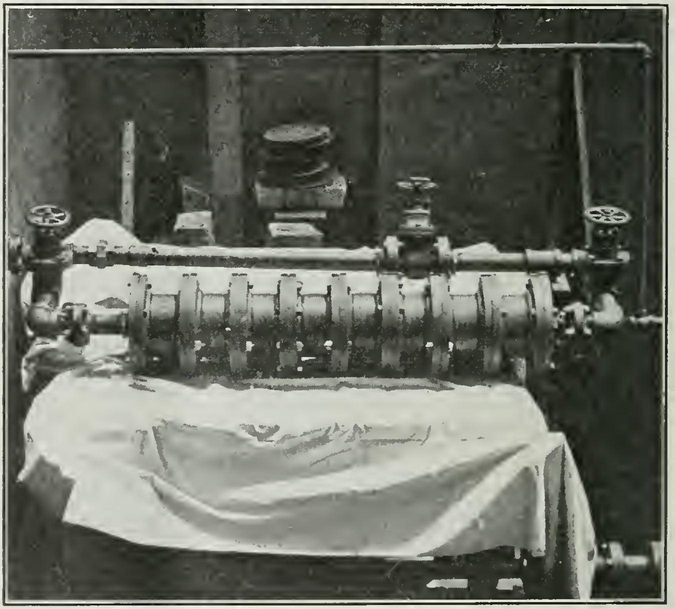

Fig. 7 Eight-Section Pipe-Flange Muffler

32 The photopulsometer is shown in the diagrammatic sketch of Fig. 11. A pitot tube with one leading and one trailing tip set with the opening in line on a vertical diameter was inserted in the center of the pipe. The leading or impact tip communicated with the under side of a diaphragm chamber. The trailing or static tip led to the upper side of the diaphragm chamber. The mica diaphragm, 0.011 in. thick, would therefore respond to changes in velocity of the air in the line as they occurred. These vibrations

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[1] The Science of Musical Sounds, p. 78, D. C. Miller.

were directly transmitted to a mirror hung in jeweled bearings and by means of a beam of light could be thrown on a photographic film giving a diagram proportional to the velocity. By means of a pendulum beating quarter- and half-seconds a chronographic record could also be made as shown on most of the films, for example in Fig. 14, by the breaks in the diagrams. A great many films were taken in this manner under a number of different running conditions and it proved to be a valuable method for providing a permanent record of the state of the flowing air in the line, either under violent pulsations due to various disturbing factors or for more steady flow due to the effect of certain quieting factors, as well as a record of the state of flow when under steady or pulsationless-flow conditions.

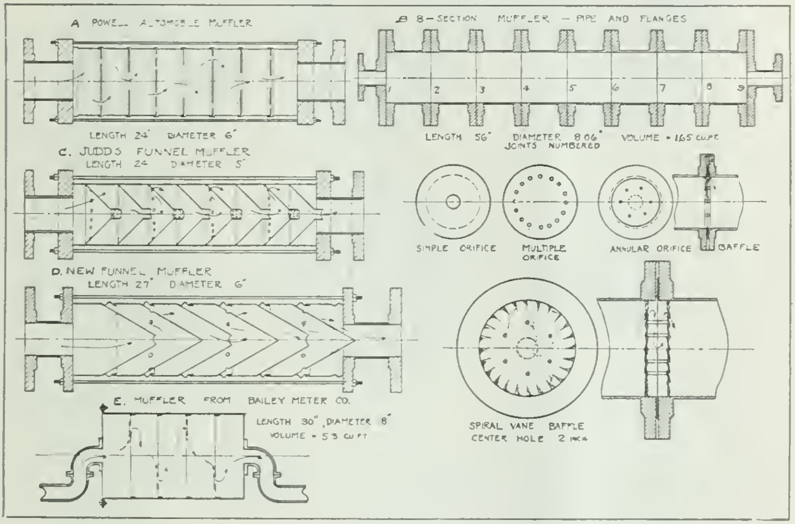

Fig. 8 Muffler Devices

33 *Velocity Diagrams.* Figs. 12 to 19, inclusive, give records of the velocity changes at the center of the pipe line at various points and under various flow conditions. The maximum effect of the pulsation in the open line direct from the compressor is shown in Fig. 12. There is little or no quieting effect due to a length of pipe equal to 183 diameters. As measured from the diagrams:

Average of maximum pulsation at 50 ft. from compressor = 1.05
Average of maximum pulsation at 111 ft. from compressor = 0.80
Square root of ratio of 80 to 105 = 0.875

This equals 12.5 per cent quieting effect due to length of pipe for a maximum error of 82 per cent due to pulsating flow.

34 Fig. 13 shows artificial pulsations as produced by the butterfly-valve interrupter and that they are much less violent than those shown in the previous figure where the valve to the compressor is an automatic valve with a light spring. Diagrams (*a*) and (*b*) approach more nearly the shape of a sine curve. It should be noted that whenever the interrupter was used the air supplied came from the tank *T*, Fig. 1, used as a storage tank. The air was therefore free from pulsations except those that were imparted by the interrupter itself.

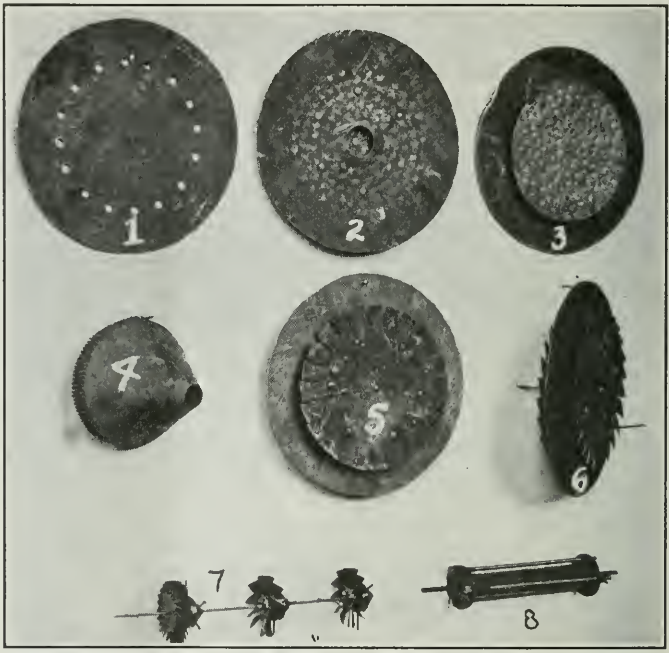

Fig. 9 Muffler Disks, Fan Units, and Small Quieting Devices

35 In Fig. 14 are shown artificial pulsations which are most violent near the interrupter. Those from a point 4 ft. [Diagram (*a*)] above the interrupter show only a slight disturbance indicating that the pulsation is retarded somewhat by the flowing current of air. Diagram (*d*), taken beyond the 24-in. volume,

shows its quieting effect and also shows the kind of diagram to be expected for pulsationless-flow conditions.

36 Diagram (*a*), Fig. 15, also shows pulsationless-flow conditions when the violent pulsations direct from the compressor have been eliminated by the use of the big tank *T* as a quieting device.

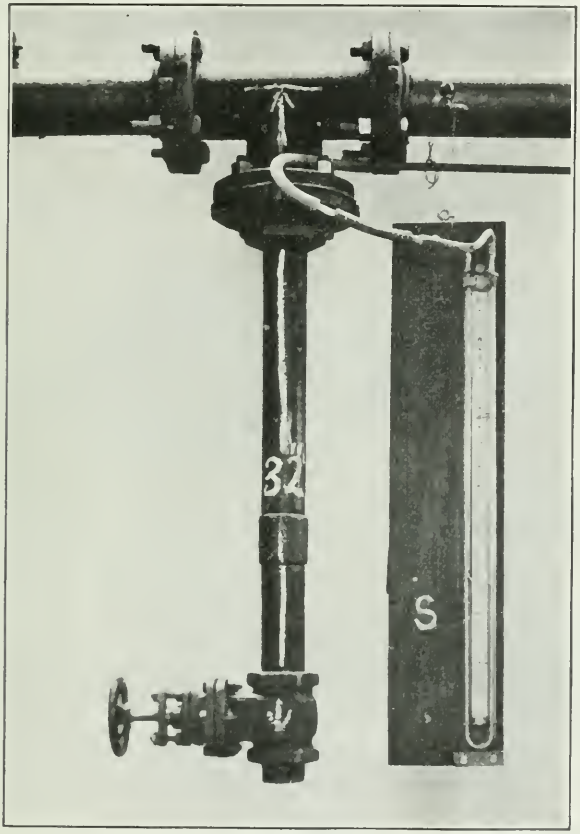

Fig. 10 Manometer for Pressure in Line

Diagram (*b*) shows that the diaphragm of the photopulsometer is not affected by the sound waves from a phonograph. Diagram (*c*) was taken to determine the natural period of vibration of the photopulsometer. This shows 6 vibrations in 0.103 sec., or 58 vibrations per second. This will explain the minute secondary pulsations occurring in Diagram (*a*) and in others under pulsationless flow. Most of the secondary pulsations seen in the other diagrams are likewise due to the same cause. Due to this cause also, some of the diagrams appear to go below the zero line when in reality they do not. Some other cases arise, however, where the pulsation appears to be negative, for which no good reason could be assigned.

37 The large tank *T* was tried as a quieting chamber, when connected similar to an air chamber to a pump. Fig. 16 shows that such an arrangement is worthless as a quieting device. This substantiates our later experience that, to be effective, tanks, or volumes, should be inserted in the line so that the air may pass through them axially.

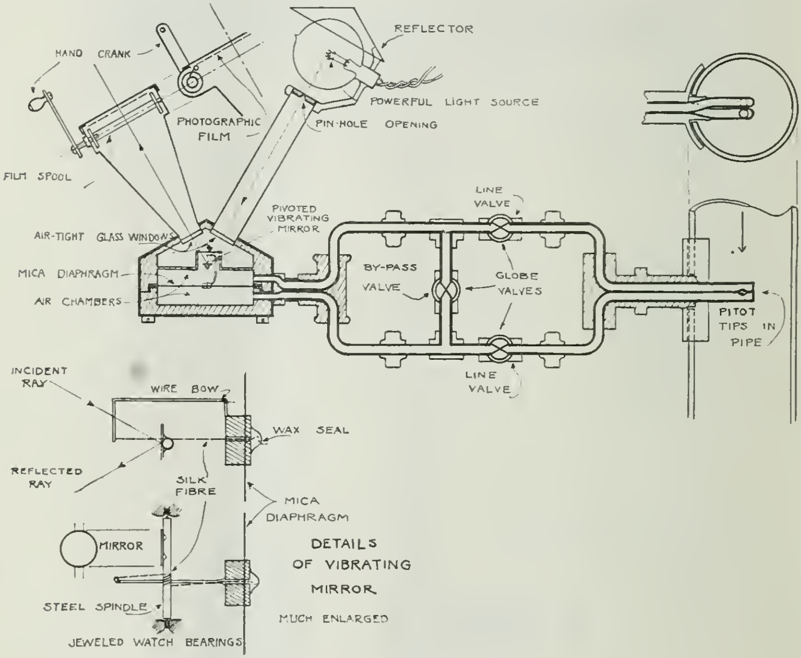

Fig. 11 Diagrammatic Sketch of Photopulsometer

38 One of the methods for eliminating pulsations is throttling by means of some kind of obstruction in the line such as a valve or an orifice. Diagram (*a*), Fig. 17, is taken for maximum pulsations direct from compressor. The diagrams appear to go below the zero pressure line, but when the secondary pulsations due to the natural period of vibration of the diaphragm are considered, it

will be seen that the true diagrams approach but do not go below the zero line. Diagram (*b*) shows the quieting effect of an orifice when the pulsation has been reduced to the condition of pulsationless flow.

39 *Effect of Pulsation of the Static Pressure in the Line.* For a compressor speed of 293 r.p.m., or nearly five revolutions per second, and for an air velocity in the pipe from 22 to 27 ft. per sec., the velocity wave length as shown by the diagrams would be from 4 to 6 ft. On starting the compressor it seemed frequently that the first impulse traveled much faster than the actual velocity of the air.

40 *Pipe-Line Pressure Diagrams.* To test this out and also to study the effect of pulsation on the static pressure in the line, two Crosby indicators were attached to the line, one on the compressor cylinder (Fig. 1, *C*), and the other 70 ft. distant (Fig. 1, *A*, No. 1 indicator) near the meter space. For details of arrangement of indicators for taking simultaneous pressure readings, see also Fig. 1, *B*. Diagrams from the compressor cylinder (Fig. 20, *A*, No. 1) had been taken several times before this as had also diagrams on indicator at point No. 1, Fig. 1, to show the rapid fluctuation in static pressure. When the indicator at point No. 1 was moved by hand a diagram was obtained closely resembling that shown on the films from the photopulsometer; but no diagrams had been taken simultaneously.

41 Since the sudden rise in the diagram taken by indicator No. 1 must mark the *beginning* of a pulsation, it was concluded that this initial point was the point at which the valve opened on the compressor, thus releasing a pulsation into the air line. This pulsation would continue with the upward stroke of the piston and at the instant that the valve on the compressor closed the pulsation on the record would cease.

42 The two indicator drums were connected by means of a light piano wire to the reducing motion on the engine. The two indicator pencil motions were connected by an electric circuit operating a detent motion. The pencil motion on the indicator at the compressor closed a switch and by means of a solenoid operated the pencil motion of the second indicator. Whenever simultaneous cards were taken, this method was used.

43 *Pressure Pulsation Greater Than Velocity Pulsation.* Three sets of such diagrams were taken and are shown at *A* and *B*, Fig. 20. Two important features were brought out: (1) The pulsa-

tion in the pipe produced a much greater pressure effect than that imparted to the velocity of flow. The diagram taken with the pitot tip (Fig. 20, *B*, No. 2) would be expected to give the combined effect due to pressure and velocity; and a larger and somewhat modified diagram might be looked for. On the contrary, the two diagrams taken with the static connection and with the pitot tip are strikingly similar in shape. (2) When simultaneous diagrams for a single stroke were recorded (Fig. 20, *A* and *B*, No. 3) it was found that the suction stroke of the compressor corresponded to the pressure stroke in the line. This seemed to indicate that the pulsation required about the time of a compressor revolution to travel a distance of 70 ft. in the pipe. For a speed of 291 r.p.m. this would mean about 0.1 sec. for $\frac{1}{2}$ revolution, or a pulsation velocity of 700 ft. per sec.

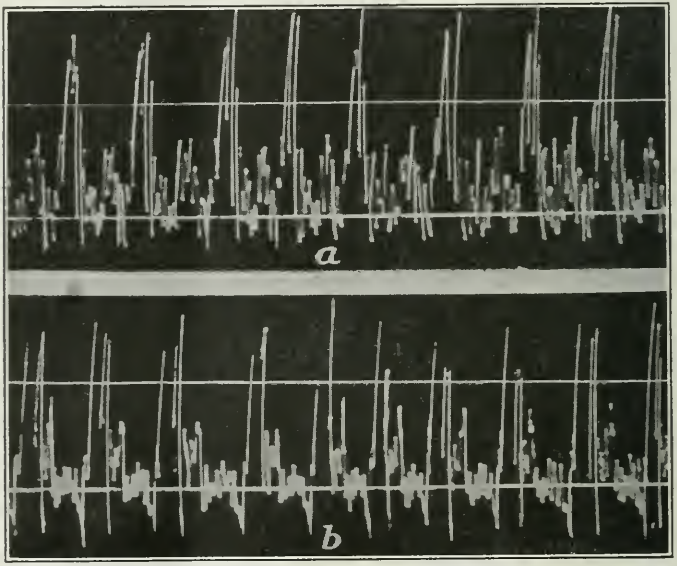

FIG. 12 SHOWING MAXIMUM EFFECT OF PULSATIONS AT DIFFERENT POINTS ON THE LINE — NO QUIETING EFFECT DUE TO INCREASE IN LENGTH OF LINE

(*a*) Maximum pulsation in line 50 ft. below compressor. (*b*) Do., 111 ft. below compressor.

44 To verify this assumption, two more indicators were added and their location rearranged as shown in Fig. 1, *A* and *B*. The one at the compressor remained unchanged, a second (at *O*, Fig. 1, *A*) was placed 18 in. distant in the discharge line to establish the

beginning of pulsation (see Cards 14*c*, 14*o*, Fig. 21); indicators I and II (Fig. 1, *B*) were placed in the 3-in. line, 54 ft. and 84 ft., respectively, from the compressor. They were connected to the detent motion through the electric circuit and were arranged so as to be operated either by hand or by the reducing motion. In addition, sparking points were attached to indicators I and II and to the secondary coil, grounded through the pipe line, so that, by means of a commutator, the cards on Nos. I and II would be perforated simultaneously by twelve sparks for each revolution of the drum while operated by hand. (See Fig. 21, Cards 11_{I}, 11_{II}.)

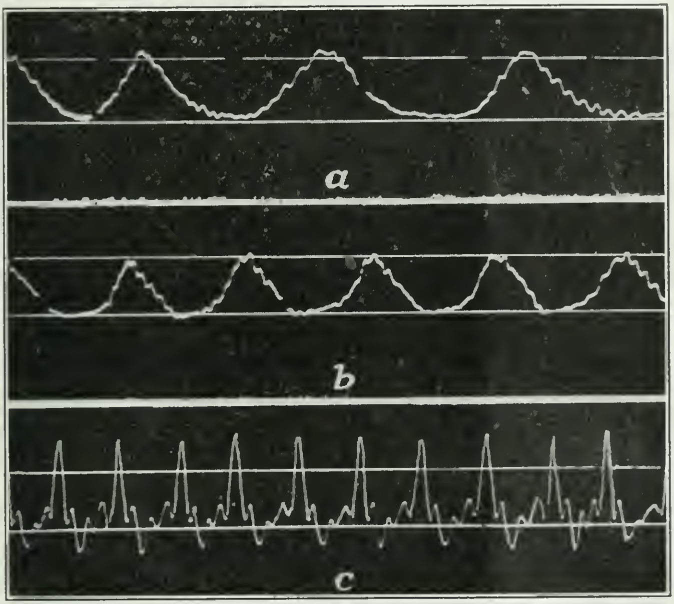

FIG. 13 PULSATIONS PRODUCED ARTIFICIALLY BY BUTTERFLY-VALVE INTERRUPTER. VELOCITY OF AIR IN LINE, 27 FT. PER SEC.

(*a*) 53 ft. below interrupter, 3 pulsations per sec. (*b*) 53 ft. below interrupter, 4 pulsations per sec. (*c*) 53 ft. below interrupter, 13.5 pulsations per sec.

45 *Analysis of Pressure Diagrams.* Fig. 21 comprises diagrams traced from cards taken on these indicators, including simultaneous cards at points *C* and *O*, *C* and *I*, *C* and II, *C*, I and II, and cards taken at I and II with coincident points shown by spark points. These diagrams include:

a Partial cards for establishing the fact that the initial point of pulsation is the initial point of pressure rise in the line which is seen to correspond to the point of valve opening in the compressor. (See diagrams to establish simultaneous pressure points, Cards 6*c*, 6*o*, 14*c*, 14*o*)

b Complete diagrams plotted in one direction, showing the use of the coincident lines. The drums were moved by hand and, at several points, simultaneous lines were drawn by means of the detent motion. This gives a method of accurate comparison of selected pulsations.

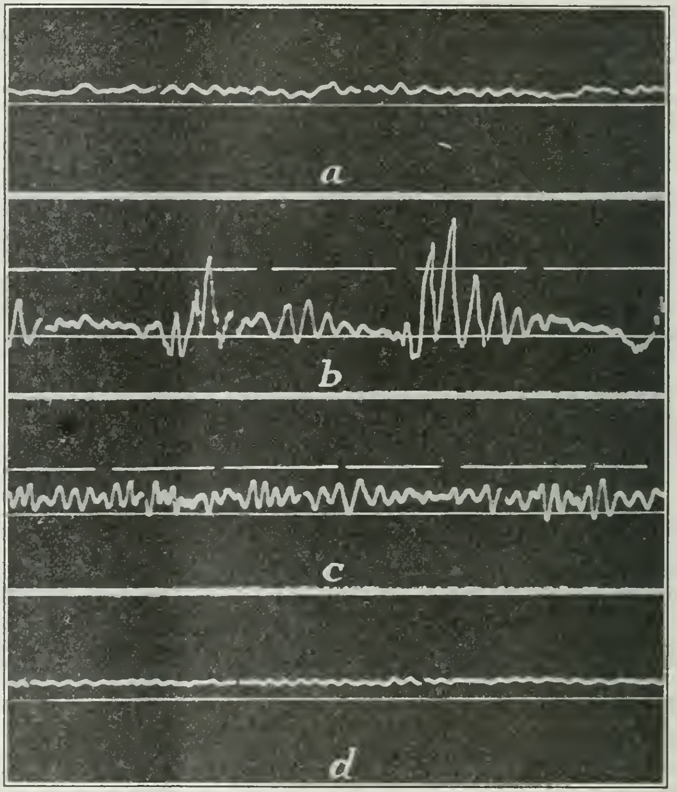

Fig. 14 Showing Artificial Pulsations at Different Points in the Line

Distances from butterfly-valve interrupter: (*a*) 4 ft. above; (*b*) 0.5 ft. above; (*c*) 2 ft. below (*d*) 60 ft. below, showing quieting effect of 24-in. volume.

c Also hand-operated diagrams to illustrate the initial pulsation points for repeated pulsations. The identity of the simultaneous strokes on the indicators is established and shown by means of the arrows of direction. (See Cards 3*c*, 3*o*.)

46 For the hand-operated cards the simultaneous lines are numbered alike and the initial points on the same pulsation at different points as it progresses are lettered *AAA*, *BBB*, *CCC*, etc. The progress of the pulsation down the pipe is shown clearly and the similarity of the shape as compared with the record from the photopulsometer is also apparent. (See Cards 3*c*; 3_I, 3_{II}; 11_I, 11_{II}.)

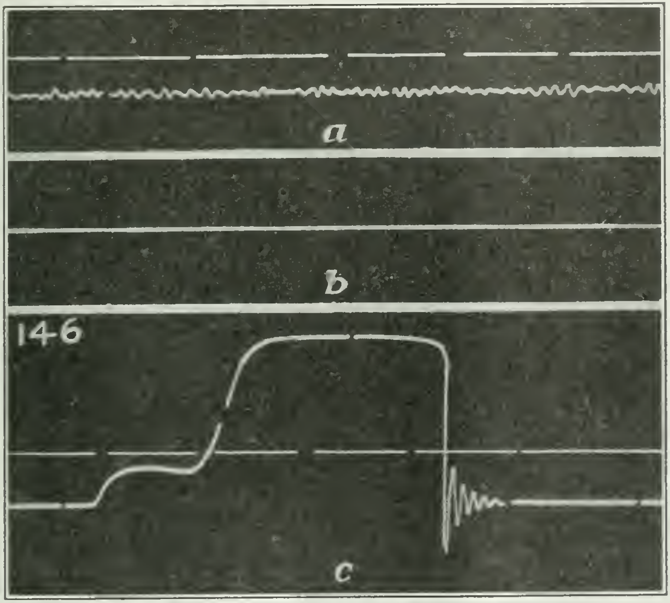

Fig. 15 Showing Velocity Conditions for Pulsationless Flow, and the Natural Period of Vibration of the Diaphragm

(*a*) Taken 111 ft. below compressor; air flowing from tank, no pulsations; velocity of air in line, 27 ft. per sec.

(*b*) Taken with tips placed in front of phonograph, sound concentrated in convergent nozzles.

(*c*) Showing the natural period of vibration of the photopulsometer.

47 *Rectification of Pressure Diagrams.* A few diagrams have been plotted to a uniform horizontal scale, or in other words, the diagrams have been converted into pressure-time diagrams. This method is clearly shown in Fig. 22. The motion of the piston has been considered as that of simple harmonic motion, which is not strictly true, owing to the finite length of the connecting rod. By plotting the complete pulsation as if the diagrams were taken in

TABLE 2 VELOCITY OF PULSATION IN PIPE

COMPUTATIONS OF VELOCITY OF PULSATION BETWEEN INDICATORS I & II

Card No.	Coincident line	Pulsation measured	Length of pulsation in 100ths of an inch	Value of 0.01 in. in seconds of time	Distance from beginning of pulsation to reference line		Lead I–II 100ths of an inch	Time taken by pulsation to travel 30 ft.	Velocity of pulsation, ft. per sec.	Number of holes at orifice head	Remarks
					I	II					
10	4	C–D	72	0.00285	38	28	10	0.0285	1052	4—3.4.6.7.	
10	6	D–E	73	0.00281	43	34	9	0.0253	1185	4–	
10	8	E–F	62	0.00330	48	40	8	0.0264	1135	4–	R.p.m. = 293
10	9	F–G	63	0.00325	26	17	9	0.0292	1028	4–	
11	3	A–B	74	0.00277	24	13	11	0.0305	985	3 4.6.7.	
11	5	B–C	70	0.00293	28	17	11	0.0323	930	3	
11	7	C–D	55	0.00373	34	28	6	0.0224	928	3	
11	10	E–F	53	0.00387	45	38	7	0.0271	1106	3	

COMPUTATIONS OF VELOCITY OF PULSATION BETWEEN INDICATORS C, I & II

Card No.	Coincident line	Pulsation measured	Length of pulsation in 100ths of an inch	Value of 0.01 in. in seconds of time	Distance from beginning puls. to line			Lead in 100ths of an inch			Time taken by pulsation to travel			Velocities, ft. per sec., based on distance		
					C	I	II	C–I	C–II	C–	54 ft.	84 ft.	30 ft.	54 ft.	84 ft.	30 ft.
5	2	A–B	145	0.00141	24	3	–10	21	34	13	0.0296	0.0480	0.0183	1830	1950	1640
5	6	B–C	122	0.00168	52	15	–4	37	56	19	0.0662	0.0941	0.0319	869	893	941
6	3	A–B	120	0.00171	42	21	5	21	37	16	0.0359	0.0632	0.0274	1510	1330	1095
6	6	B–C	110	0.00186	42	18	4	24	38	14	0.0446	0.0706	0.0261	1210	1190	1148
7	3	A–B	150	0.00136	32	–3	–18	35	50	15	0.0478	0.0683	0.0205	1130	1230	1465
7	7	B–C	105	0.00195	42	7	–12	35	54	19	0.0683	0.1052	0.0371	791	797	809

one direction only, the true characteristic shape of the pressure diagram is obtained. When the pulsation is foreshortened and repeated it bears a striking resemblance to those shown on the hand-operated diagrams in Fig. 22 and shown also on the film of the photopulsometer.

48 *The Velocity of the Pulsation.* The determination of the velocity of pulsation from the simultaneous sets of indicator diagrams for points I and II located 30 ft. apart was made as follows:

a Hand-operated diagrams were taken as previously explained.

b The diagrams were carefully compared, points of spark puncture located, coincident lines drawn thereform and numbered, and the initial points of the pulsations were identified and marked *AAA, BBB, CCC,* etc. The unavoidable irregularity of hand operation rendered the identical pulsations easily recognizable by means of their relative lengths.

c Measurements of the sets of diagrams were carried out, as shown in Table 2. It will be noticed that the computations shown include some of the diagrams illustrated in Fig. 21. All the diagrams shown are tracings of originals and are numbered and dated.

d Results were computed as shown on Table 2. Choosing Diagram 11 (see Fig. 21, Cards 11), coincident line No. 3, pulsation *A-B* is 0.74 in. long. For 293 r.p.m. one revolution = 0.205 sec. = time of one pulsation. If 0.74 in. or one pulsation = 0.205 sec., 0.01 in. = 0.00277 sec. Point *A* is 0.24 in. from line 3 for indicator I, and is 0.13 in. from line 3 for indicator II, a lead at point I over point II of 0.11 in., or the time taken to travel 30 ft. will be 0.00277 x 11 which is 0.0305 sec. Rate of travel per second = 30 ÷ 0.0305 = 985 ft., which is the velocity of pulsation. The velocity of sound in dry air at 0 deg. cent., 766 mm. pressure, is 1086 ft. per sec.

49 Table 3 gives the average results computed as in the example given and includes data taken for three points in the line, 30 ft., 54 ft. and 84 ft. from the compressor, and for six different velocities of air flow.

50 These results establish three significant facts:

a That, although our method shows results varying from the maximum to the minimum through a wide range, yet in no case does the velocity of pulsation so determined approach anywhere near the velocity of the flowing air.

b That, for a variation of velocity of flowing air ranging from zero to 27 ft. per sec., the velocity of pulsation was found to be independent of the velocity of the air.

c That the total average for 148 computations gave 1090 ft. per sec. as the velocity of pulsation. The velocity of sound in dry air at 32 deg. fahr. and 29.92 in. barometer is 1083 ft. per sec. The velocity of pulsation in all probability is equal to that of sound in air.

TABLE 3 AVERAGE RESULTS FOR VELOCITY OF PULSATION

Date	No.	Number of diagrams measured	Length of pipe used, ft.	Velocity of air per sec., ft.	Velocity of pulsation, ft. per sec.		
					Max.	Min.	Avg.
4-5-22	1	14	30	0	1462	640	1042
	2	16	30	5	1412	869	1088
	3	18	30	11	1440	805	1124
	4	20	30	16	1263	775	1019
	5	20	30	22	1290	815	1044
	6	13	30	27	1465	747	1100
	Total	101				Average...	1070
4-7-22	7	16	30	22	1640	809	1143
	8	15	54	22	1830	780	1107
	9	16	84	22	1750	797	1149
	Total	47				Average...	1133

51 For the average velocity of pulsation equal to 1090 ft. and for 4.88 pulsations per second, the pressure wave length as shown on the diagrams would be 223 ft.

52 Since the velocity of the pulsation is independent of the velocity of the flowing air and is evidently equal to the velocity of sound in air, it seems quite reasonable to conclude that the pulsation is a pressure change in the form of a wave front resembling a sound wave of low frequency. It seems also highly probable that these pulsations are similar in character to the pulsations set up by water hammer in a pipe line, since they also travel with the velocity of sound in water.

53 We gained some additional knowledge of the nature of the pulsation by noting its effect on manometers. Where a manometer was used to measure a differential head at a meter, the following effects were consistently present:

a For pulsationless flow the reading was very constant, the only variation being a slight long period surge due to appreciable variations in the compressor speed.

TABLE 4 OBSERVED DATA AT METER

No.	Setting of Line	Instrument Used	Readings on Manometer									Remarks
			1			2			3			
			Lt.	Rt.	Tot.	Lt.	Rt.	Tot.	Lt.	Rt.	Tot.	
1	9″×9″ vol. at com-	Vent. 28″ Man.	12.9	13.2	26.1	12.8	13.3	26.1	12.7	13.3	26.0	Temp 2
2	pressor	“ Foxboro	cha	rt	19.			19.			19.	= 86.5°
3		indicator card										R.p.m.
4		Film No. 255	at	Ia$_2$								=288
5	9″×9″ vol. at comp.	Vent. Foxboro			21.8			21.8			21.8	
7	9″×18″ vol. midway	“ 28 M.	13.5	13.8	27.3	13.3	13.8	27.1	13.2	13.5	26.7	
10	9″×9″ vol. at comp.	“ “	11.7	12.1	23.8	11.7	12.2	23.9	11.7	12.2	23.9	
11	9″×27″ vol. midway	1 “ Foxboro			20.6			20.6			20.6	Temp.
14	9″×9″ vol. at comp.	“ “			20.0			20.0			20.0	= 99°
15	9″×36″ vol. midway	“ 28″ M.	12.8	13.2	26.0	12.7	13.1	25.8	12.7	13.2	25.9	
18	9″×9″ vol. at comp.	“ “	12.9	12.2	25.1	12.6	13.3	25.9	12.1	12.8	24.9	(Runs
19	9″×45″ vol. midway	“ Foxboro			19.8			19.8			19.8	made
22	9″×9″ vol. at comp.	“ 28″ M.	11.9	12.6	24.5	11.9	12.4	24.3	12.0	12.6	24.6	7–20–21)
23	9″×54″ vol. midway	“ Foxboro			18.6			18.6			18.6	
26	9″×45″vol. at comp.	“ Foxboro			21.0			21.0			21.0	
27	9″×54″ vol. midway	“ 28″ Man.	12.0	12.6	24.6	12.0	12.7	24.7	12.0	12.7	24.7	
2	9″×45″ vol. at comp	Venturi meter with 28″ vertical water manometer	9.8	10.6	20.4	9.8	10.5	20.3	9.8	10.4	20.2	
5	9″×18″ vol. “ “		14.1	14.4	28.5	14.1	14.3	28.4	14.2	14.3	28.5	R.p.m.
8	9″×27″ vol. “ “		12.4	12.9	25.3	12.5	12.9	25.4	12.6	13.0	25.6	= 288
11	9″×36″ vol. “ “		11.3	11.9	23.2	11.2	11.7	22.9	11.2	11.8	23.0	(Runs
14	9″×54″ vol. “ “		8.8	9.7	18.5	9.1	9.6	18.7	9.0	9.7	18.7	made
17	Line Clear		15.8	17.0	32.8	15.9	17.0	32.9	15.7	17.0	32.7	7–21–21)
22	Pulsationless		4.1	5.1	9.2	4.1	5.1	9.2	4.1	5.1	9.2	
20	Pulsating — O.L.	1″ or 33% orifice meter with 28″ vertical water manometer	14.0	13.2	27.2	14.0	13.2	27.2	14.0	12.9	26.9	R.p.m.
22	Pulsationless		11.7	11.4	23.1	11.9	11.6	23.5	12.0	11.7	23.7	= 291
29	1–1″ Orifice		12.6	12.1	24.7	12.7	12.2	24.9	12.7	12.1	24.8	
30	2–1″ Orifices		11.8	11.4	23.2	12.5	11.7	24.2	11.5	12.5	24.0	T$_2$=95°
31	3–1″ “		12.2	11.5	23.7	12.2	11.5	23.7	12.2	11.7	23.9	(Runs
32	4–1″ “		12.3	11.6	23.9	12.1	11.5	23.6	11.6	11.6	23.2	made
33	5–1″ “		12.0	11.5	23.5	12.0	11.0	23.0	12.0	11.5	23.5	8–5–21)
2	1″ Orifice	33% orifice meter 28″ vertical manometer	13.9	13.3	27.2	14.0	13.4	27.4	13.8	13.3	27.1	R.p.m.
3	$\frac{28}{32}$″ “		12.3	12.0	24.3	12.5	12.2	24.7	12.4	12.1	24.5	= 291
4	$\frac{25}{32}$″ “		12.2	12.0	24.2	12.3	12.1	24.9	12.2	12.5	24.7	(Runs
5	$\frac{22}{32}$″ “		12.3	12.0	24.3	12.3	12.0	24.3	12.3	12.0	24.3	made
6	$\frac{19}{32}$″ “		12.4	12.1	25.5	12.7	12.3	25.0	12.7	12.3	25.0	8–6–22)

b For pulsating flow this surge was magnified greatly.

c For pulsating flow there is also a rapid vibration of the water column corresponding to the pulsations and depending in amplitude upon the local conditions at the manometer.

d The most significant characteristic of the readings for pulsating flow was the large increase over that for pulsationless flow. This increase was present for every type of manometer, meter and gage tried. It varied from a few per cent to several hundred per cent under extreme conditions.

THE ELIMINATION OF THE PULSATION

54 The problem of the elimination of the pulsation, or of the effects due to the pulsation, suggested two methods of attack: (1) modification of the existing metering devices so that the recorded flow would be unaffected whether the flow be steady or in pulsations; and (2) the use of devices which would correct or eliminate the pulsations before the flowing fluid reached the meter.

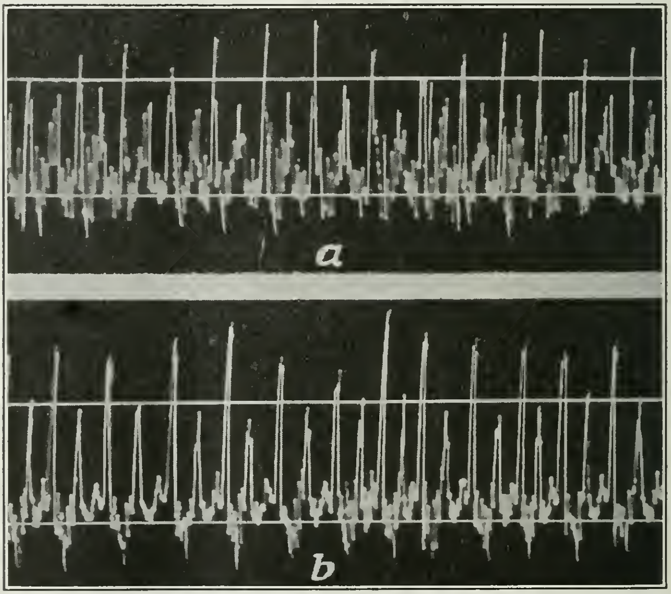

Fig. 16 Vertical Tank of 200 cu. ft. Capacity as Quieting Chamber—Similar to Air Chamber on Pump; 4.9 Pulsations per Sec.

Distances below compressor: (*a*) 50 ft.; (*b*) 117 ft.

55 The first of these suggested schemes was taken up to some extent in the study of the modification of manometer connection. The second suggestion, that of pulsation elimination, received the

major part of our attention. Of the five quieting devices used, the pulsating bag and the revolving fan operated by the air flow were studied by means of the photopulsometer. The use of throttling devices, the insertion of tanks, volumes or equalizing chambers in the line and a combination of the two devices, forming the so-called "muffler," comprise the remaining three quieting and eliminating devices. These five schemes, it is believed, cover nearly all, if not all, of the practical schemes which might be used for this purpose.

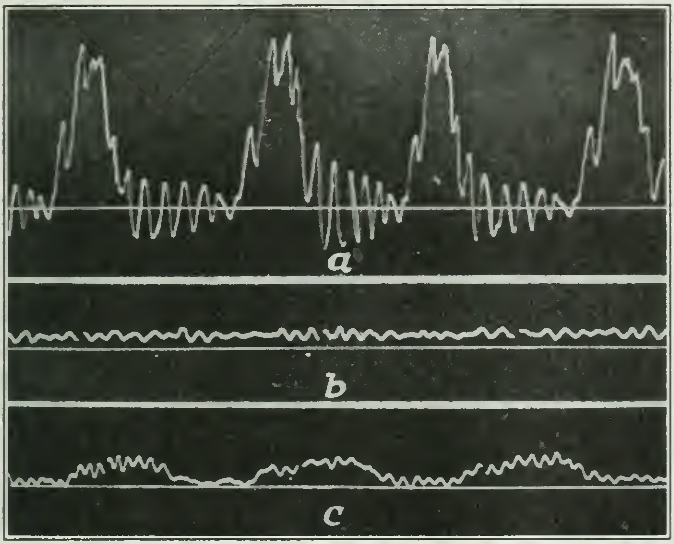

Fig. 17 Showing Quieting Effect of Throttling by Means of an Orifice

(*a*) 80 ft. below compressor, maximum pulsation in line. (*b*) 10 ft. below throttling orifice, pulsation destroyed. (*c*) 4 ft. above interrupter, 3.5 pulsations per sec.

56 The tabulated results which follow include the average observed data and the percentage of error as figured from these average data. The average observed data are the averages of three simultaneous readings taken for all manometer readings. Tables 4 and 5 are observed-data sheets. In most instances where any change in line setting was made a pulsation-flow run was followed immediately by a pulsationless-flow run so as to eliminate the error due to possible change in temperature. In the few cases during the first of the experimentation where this was not done, proper corrections were made later for the temperature effect.

The term " percentage of error " for any meter means the percentage by which the indicated velocity of flow with pulsations differs from the velocity of flow for quiet or pulsationless flow, both readings taken on the same manometer, or indicating device and with a constant quantity of air flowing under the same conditions. The condition for constant flow was assured by maintaining at all times

TABLE 5 OBSERVED DATA AT ORIFICE HEAD

Run No.	Orifice-head reading			Hydrometer		Instrument or line setting	Remarks
	1	2	3	Dry temp.	Wet temp.		
1	0.911	0.914	0.911	79.0	70.0	Pulsating flow. No. 1. Vol. 9″ × 9″ next to compressor, others from No. 2 to No. 6 at mid point. Venturi meter — alternating Foxboro gage and water manometer.	Bar. = 29.047 Zero of orifice-head gage = 0.003 in. (Runs made 7–20–21)
2	0.929	0.913	0.916	78.6	69.8		
3	0.909	0.917	0.916	78.9	70.0		
4	0.915	0.909	0.900	79.3	70.1		
5	0.911	0.903	0.905	80.5	70.6		
7	0.917	0.890	0.877	80.4	70.6		
10	0.920	0.916	0.914	81.1	70.0		
11	0.897	0.892	0.890	81.1	70.1		
14	0.915	0.910	0.891	81.2	70.0		
15	0.878	0.875	0.881	81.0	69.8		
18	0.905	0.903	0.887	82.1	67.7		
19	0.902	0.898	0.901	82.6	67.8		
22	0.895	0.887	0.896	82.8	67.7		
23	0.900	0.895	0.884	83.5	68.0		
26	0.915	0.907	0.894	83.3	67.6		
27	0.905	0.913	0.910	84.0	67.9		
2	0.905	0.906	0.908	74.3	66.5	Open line pulsationless	Bar = 29.195 Zero = 0.004 in. (Runs made 7–21–21)
5	0.895	0.904	0.898	76.3	66.5		
8	0.900	0.893	0.890	77.0	66.6		
11	0.881	0.890	0.903	77.1	66.6		
14	0.880	0.882	0.896	77.9	66.8		
17	0.911	0.910	0.910	78.6	66.7		
22	0.894	0.906	0.911	79.3	68.1		
20	0.898	0.902	0.901	79.3	70.1	Open line pulsationless	Bar. = 29.051 Zero = 0.006 in. (Runs made 8–5–21)
22	0.910	0.906	0.913	79.6	71.2		
29	0.913	0.914	0.913	80.0	70.0		
30	0.895	0.920	0.911	80.8	70.5		
31	0.915	0.910	0.910	80.5	69.1		
32	0.913	0.917	0.906	80.9	69.8		
33	0.886	0.890	0.882	81.2	71.0		
2	0.910	0.902	0.908	72.5	63.5	1 in.	Bar. = 28.98 (Runs made 8–6–21)
3	0.916	0.920	0.919	73.5	69.5	$\frac{28}{32}$ in.	
4	0.915	0.920	0.930	74.0	70.0	$\frac{25}{32}$ in.	
5	0.925	0.928	0.930	—	—	$\frac{22}{32}$ in.	
6	0.930	0.945	0.940	75.0	70.5	$\frac{19}{32}$ in.	

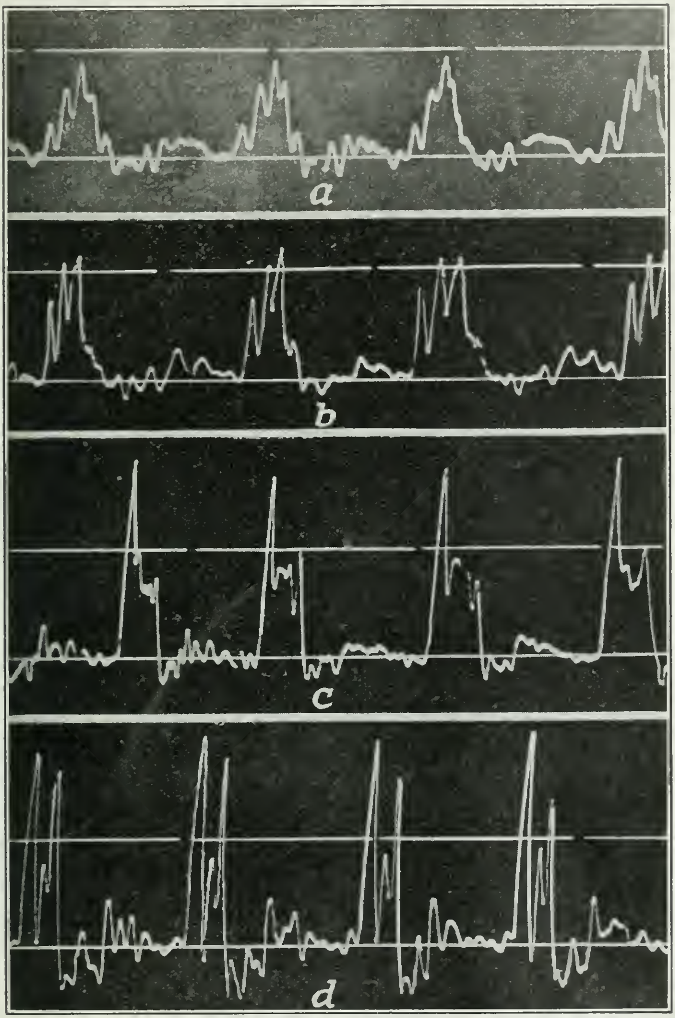

FIG. 18 SHOWING QUIETING EFFECT OF 3-IN. REVOLVING FANS DRIVEN BY CURRENT OF AIR AT VELOCITY OF 22 FT. PER SEC.

(*a*) 54 ft. below compressor, pulsation slightly throttled; maximum error due to pulsation, 68 per cent; 4.5 pulsations per sec.

(*b*) Same as (*a*) but at 80 ft. below compressor.

(*c*) 54 ft. below compressor, 6 ft. above fan section; maximum error due to pulsation, 68 per cent; 4.5 pulsations per sec.

(*d*) 20 ft. below fan section acting as quieting unit; error reduced from 68 per cent to 27 per cent; 4.5 pulsations per sec.

a static head of 0.9 in. of water at the orifice head, where special care was taken to secure pulsationless-flow conditions.

$$\text{Error in Per Cent} = (\sqrt{P_2/P_1} - 1)100$$

where P_2 = corrected manometer reading for pulsating flow
P_1 = corrected manometer reading for pulsationless flow.

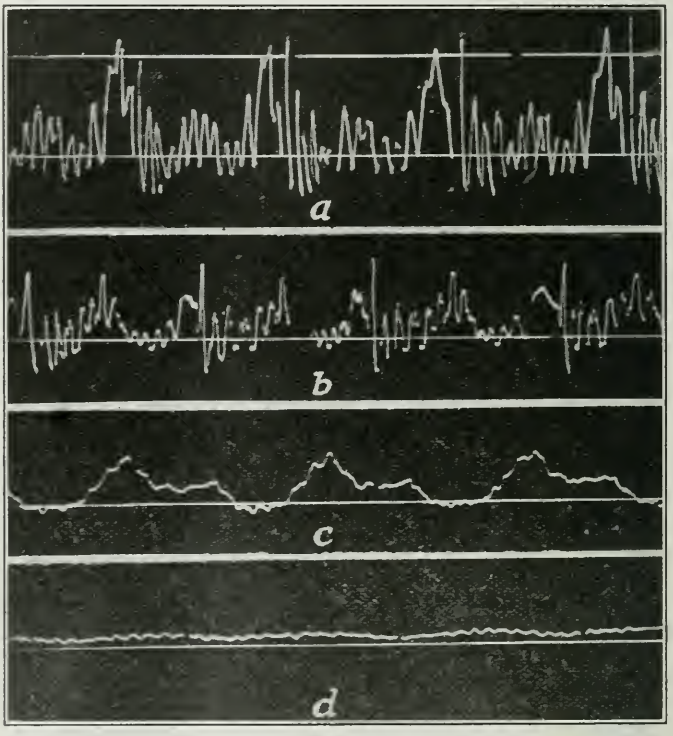

Fig. 19 Showing Quieting Effect of Pulsating Bag

(*a*) 54 ft. below compressor; maximum pulsation in line.
(*b*) 54 ft. below compressor; pulsating bag attached to quiet pulsations.
(*c*) 88 ft. below compressor; shows slight quieting effect of the 12-in. volume.
(*d*) 117 ft. below compressor; shows complete quieting effect of the 24-in. volume.

The error in velocity varies as the square root of the error in head reading. The manometer readings at the meter were corrected to a standard orifice head reading of 0.9 in., by direct proportion.

57 *Modification of Manometer Connections at the Meter.* The first attempt to reduce the error of pulsation by this means

was by throttling the manometer connections. These connections between the meter in the line and the manometer indicating the head readings were made of heavy rubber tubing having an inside diameter of $\frac{3}{16}$ in. It has long been the practice to quiet the swinging of a pressure-gage hand by throttling the gage connections. Following out this idea, a series of "orifice plugs," see Fig. 4, were made by drilling holes of known size in small pieces of brass rod.

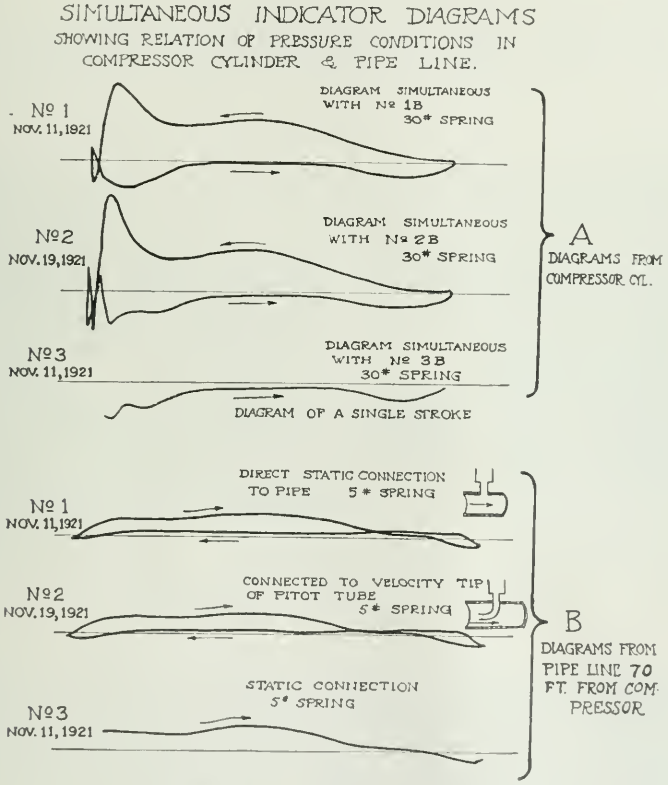
SIMULTANEOUS INDICATOR DIAGRAMS
SHOWING RELATION OF PRESSURE CONDITIONS IN COMPRESSOR CYLINDER & PIPE LINE.
Nº 1
NOV. 11, 1921
DIAGRAM SIMULTANEOUS WITH Nº 1B
30# SPRING
Nº 2
NOV. 19, 1921
DIAGRAM SIMULTANEOUS WITH Nº 2B
30# SPRING
A
DIAGRAMS FROM COMPRESSOR CYL.
Nº 3
NOV. 11, 1921
DIAGRAM SIMULTANEOUS WITH Nº 3B
30# SPRING
DIAGRAM OF A SINGLE STROKE
DIRECT STATIC CONNECTION TO PIPE
5# SPRING
Nº 1
NOV. 11, 1921
CONNECTED TO VELOCITY TIP OF PITOT TUBE
5# SPRING
Nº 2
NOV. 19, 1921
B
DIAGRAMS FROM PIPE LINE 70 FT. FROM COMPRESSOR
STATIC CONNECTION
5# SPRING
Nº 3
NOV. 11, 1921

Fig. 20 Simultaneous Indicator Diagrams

These plugs were inserted in the rubber manometer tubes both for the vertical U-tube water manometer and also for the Foxboro differential mercury gage.

58 Fig. 23 gives the average error for both these gages for maximum pulsation while using the venturi meter. The curve shows that throttling has no appreciable effect in reducing the error

until the opening has been reduced to less than 0.07 in. diameter, and that even for an obstruction so small as nearly to close the opening the percentage of error is not reduced to within practical limits. The Foxboro gage showed the same characteristic in regard to the effect of throttling, but gave an error about 60 per cent as large as that for the water manometer. In either case in order to reduce the error to 50 per cent of the maximum it would be necessary to use an orifice of 0.02 in. diameter, or the diameter of a No. 76 drill, clearly a ridiculous size. The surge, or pulsation, of the water column was completely destroyed, so that the effect is quite analogous to that of the steam gage when throttled. This indicates that while throttling a pressure gage does not affect its reading, it has no beneficial effect in reducing the error due to pulsation. That such an error might exist was pointed out by Mayo[1] in 1905, and the futility of throttling manometer connections to reduce the pulsation is also mentioned by Westcott[2] in 1922.

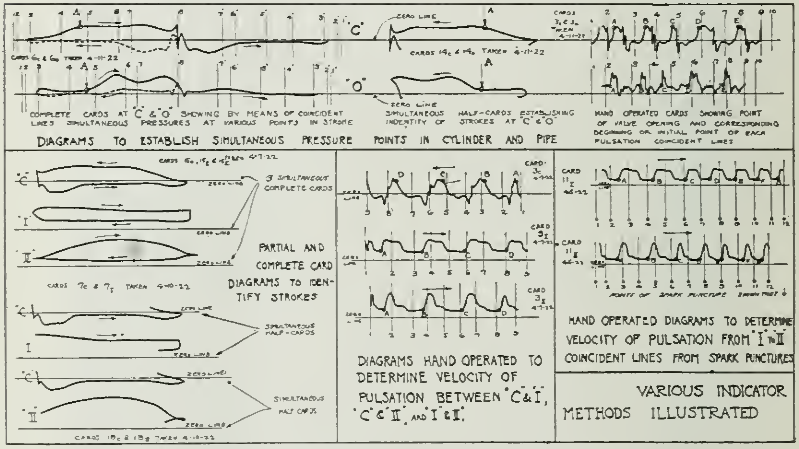

Fig. 21 Analysis of Indicator Diagrams

59 The efficient quieting effect of volumes when used in the test line suggested the possibility that small volumes inserted in the tubes leading to the manometers might serve to reduce the pulsation before the meter was reached.

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[1] Trans. Am.Soc.C.E., vol. 54, 1905, Part D, p. 502, Mayo.

[2] Measurement of Gas and Liquids by Orifice Meters, 1922, p. 143, H. P. Westcott.

60 Our first observations as to the possible effect of a volume in the manometer line were made by a comparison of the readings of the two manometers (the 6-in. inclined one-leg reservoir oil manometer and the 28-in. vertical U-tube water manometer) when used interchangeably. These two manometers when used to measure the pulsationless flow would agree exactly. When used for pulsating flow the two would not always agree, although the amount of difference between them was difficult to determine.

61 Several tests were made using the orifice meter and the 6-in. inclined gage with volumes of different sizes in one or both of the manometer connections. The results of these tests show that the use of volumes in the manometer connections does not give so favorable results as the method of throttling. There is only about 15 per cent reduction in the error for the 70 per cent orifice meter.

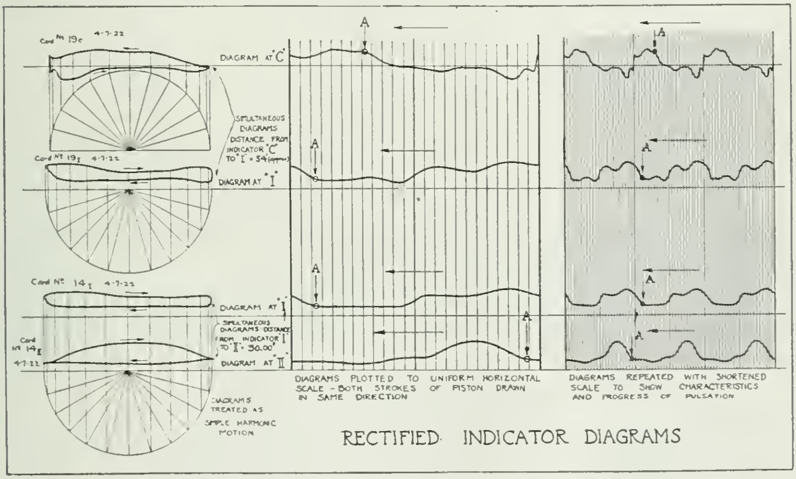
Card Nº 19c 4-7-22
DIAGRAM AT C
SIMULTANEOUS DIAGRAMS DISTANCE FROM INDICATOR C TO I = 54' (approx)
Card Nº 19I 4-7-22
DIAGRAM AT I
Card Nº 14I 4-7-22
DIAGRAM AT I
SIMULTANEOUS DIAGRAMS DISTANCE FROM INDICATOR I TO II = 30.00'
Card Nº 14II 4-7-22
DIAGRAM AT II
DIAGRAMS TREATED AS SIMPLE HARMONIC MOTION
A
A
A
A
A
A
A
A
DIAGRAMS PLOTTED TO UNIFORM HORIZONTAL SCALE – BOTH STROKES OF PISTON DRAWN IN SAME DIRECTION
DIAGRAMS REPEATED WITH SHORTENED SCALE TO SHOW CHARACTERISTICS AND PROGRESS OF PULSATION
RECTIFIED INDICATOR DIAGRAMS

Fig. 22 Analysis of Indicator Diagrams

62 It was thought, also, that the point of attachment of the manometer connection might have some influence on the error due to pulsation. With the orifice meter comparisons were made with the manometer connected (1) close to the orifice and (2) at a distance of one pipe diameter above the orifice and at points below the orifice ranging from ½ diameter to 19 diameters.

63 Tests made show that for all orifices including the 80 per cent orifice the error at the standard points of connection is greater than that for points near the orifice which averages 96.3

per cent of the error at the standard points of connection. For points farther distant from the orifice there is a tendency for the error first to increase and then to decrease as the 19 diameters point is approaching, in all covering a total range of 20 per cent error.

64 *Effect of Type of Manometer Used.* The error in measuring pulsating flow seemed to depend to some degree upon the type of manometer used to register the head, even if all other conditions of the line and meter were identical.

TABLE 6 MAXIMUM ERROR FOR VENTURI, FLANGE NOZZLE, AND PITOT METERS

Kind of meter	Error, per cent, average
Venturi meter	82.0
$1\frac{1}{8}$-in. flange nozzle meter	76.5
$1\frac{1}{2}$-in. flange nozzle meter	94.3
2-in. flange nozzle meter	142.0
$1\frac{1}{2}$-in. flange nozzle meter	199.0
No. 1 pitot-tube meter	137.0
No. 2 pitot-tube meter	103.0

65 It may be stated that all manometers properly graduated will give the same head reading for pulsationless flow. But when the flow is pulsating, the ratio of its reading to the true reading will differ somewhat according to the variations mentioned above. A mercury manometer will probably show an error less than that of a water manometer and the latter less than one using mineral oil. A manometer with small tubes is likely to read higher than one with a larger set of tubes, but this is merely a tendency. If the

TABLE 7 MAXIMUM ERROR FOR ORIFICE METER

Size of orifice, per cent	Error, per cent, average (from curve)
33	15.0
40	25.0
50	47.5
60	81.0
70	127.0
80	185.0
90	285.0

tubes are too small the capillary effect can be noted; and if they are too large, or if they end in a reservoir, the doubtful effect due to a " volume " will be introduced. The inclined leg of a manometer under some conditions may even cause less " surge " effect. The presence of a check value or damping device between the two manometer legs or chambers, or a float to actuate the recording arm may reduce the error, as is seen in the case of the Foxboro gage.

TABLE 8 COMPARISON OF METERS

Size, per cent	Error, per cent, average	
	Orifice meter	Flange nozzle meter
37.5	20.0	76.5
50.0	47.5	94.0
66.7	112.5	142.0
83.3	207.5	199.0
	Orifice meter	Venturi meter
33.0	15.0	82.0

66 *Maximum Percentage of Error Produced by Pulsation.* For our installation the maximum error for the meters was as given in Tables 6 to 9, inclusive. The results show that the less the obstruction to the flow of the air, the greater the percentage of error due to pulsation; also, the greater the restoration of pressure after passing the meter, the greater will be the error. The reason

TABLE 9 RESTORATION OF PRESSURE AFTER PASSING METER[1]

Type of meter	Maximum error, per cent	Restoration of pressure, per cent
Venturi meter	82.0	80.0
33% orifice meter	15.0	11.5
40% orifice meter	25.0	19.0
50% orifice meter	47.0	28.0
60% orifice meter	81.0	38.0
70% orifice meter	127.0	48.0
80% orifice meter	185.0	61.0
90% orifice meter	285.0	77.0

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[1] Trans. Am.Soc.M.E., vol. 38, 1916, p. 362, 264: Water Flow through Pipe Orifices, H. Judd.

for this relation appears to be that, since the pulsation is a form of pressure energy, that type of meter unit which in itself most completely dissipates the pulsation energy will show the least percentage of error.

67 *Distribution of Pulsation as Shown by Traverse.* The pipe was traversed by both types of pitot tubes for maximum pulsation conditions. The results are shown in Fig. 24. The curves of this figure show by both traverses that error due to the pulsating flow is least at the center of the pipe. There is a slight tendency, as shown by the curves, for the point of minimum error to be located a little to one side of the center of the pipe. It is not known just how much the pitot tubes themselves are influenced by their approach to the wall of the pipe.

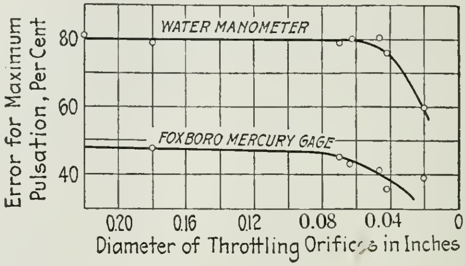

FIG. 23 EFFECT OF THROTTLING MANOMETER CONNECTIONS — FOR VENTURI METER

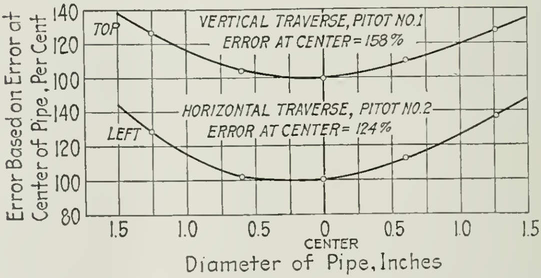

FIG. 24 DISTRIBUTION OF PULSATION ACROSS THE DIAMETER OF THE PIPE

68 *Quieting Effect of a Revolving Fan Section* (see Fig. 9, No. 7). The effect of a revolving fan section when placed in the pipe line is shown by Fig. 18. The revolving fan apparently has

some merit as a quieting device, but is of questionable practical value.

69 *Effect of the Pulsating Bag as a Quieting Device,* Fig. 6, is shown by Fig. 19. It is felt that the special design of such a device would be needed to cover the requirement of each individual installation in order to correct or eliminate the pulsating error.

70 *Elimination of Pulsation by Throttling.* The experiments carried on with the various devices for eliminating or modifying the pulsation led to the conclusion that the solution of the problem depended entirely on the absorption of the energy of the pulsation propagated as a pressure wave closely resembling a sound wave of low frequency. Whatever the device used, its value in killing the pulsation will be measured by its ability to absorb, or dissipate, this energy of pulsation.

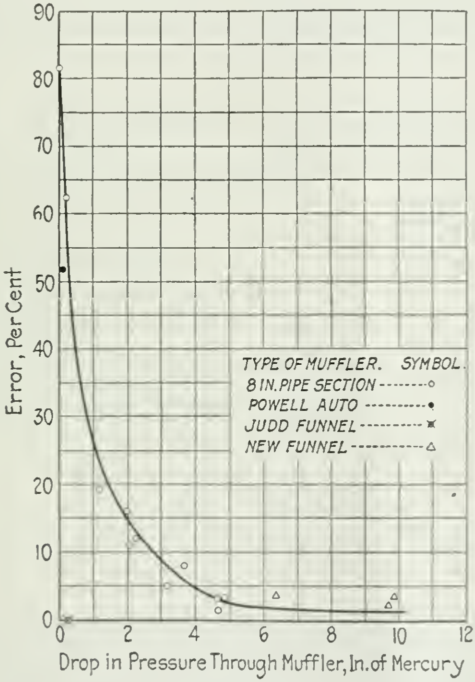

FIG. 26 EFFECT OF MUFFLERS FOR QUIETING PULSATION — USED WITH VENTURI METER

71 The first attempt made to quiet the pulsation was by means of throttling either by valve or by orifice. The loss of pres-

sure during passage through the valve or obstruction in the line was the eliminating factor.

72 The general effect of throttling is to reduce the error rapidly by means of a pressure drop up to 4 in. of mercury. The use of a greater pressure drop causes the error to be reduced more gradually. The manner of throttling is immaterial whether by gate or globe valve or by an orifice.

73 The general characteristics are summarized in Table 10. This table shows that a drop in pressure of 4 in. by throttling is needed to make an appreciable reduction in the error; and a drop of 6 in. or 3 lb. per sq. in. is necessary to bring the error within practical limits. In most cases the error is not reducible below 1 to 3 per cent, even with a sacrifice of a drop of 12 in. mercury.

TABLE 10 ELIMINATION OF THE PULSATION ERROR BY THROTTLING

Kind of meter used	Drop in pressure by throttling, inches of mercury				
	0	2	4	9	12
	Error, per cent				
Venturi	82.0	26.0	10.0	5.0	2.5
33% orifice	10.0	2.0	0.0	0.0	0.0
50% orifice	58.0	17.0	7.0	4.0	2.0
70% orifice	145.0	50.0	18.0	9.0	4.0
80% orifice	175.0	67.0	25.0	12.0	4.0
90% orifice	315.0	118.0	22.0	–5.0	18.0
1⅛-in. flange nozzle	44.0	18.0	8.0	3.5	1.0
1½-in. flange nozzle	83.0	28.0	10.0	5.0	1.0
2-in. flange nozzle	156.0	50.0	16.0	6.0	1.0
2½-in. flange nozzle	340.0	97.0	31.0	7.5	1.0
Pitot No. 1	160.0	50.0	20.0	–7.0	2.5
Pitot No. 2	140.0	81.0	5.0	0.0	3.5

74 *Elimination of Pulsation by the Use of Volumes.* Tanks, or volume capacities, or "volumes," as we have chosen to call them, were used in the line for the purpose of quieting the pulsation. These volumes were inserted in the line so that the direction of flow through them was along the axis of the volume. The connections were made by means of pipe flanges; and later orifices were inserted in these flanges at the entrance and exit of each volume. This produced an abrupt entrance into and an abrupt exit out of the volumes, which in itself would tend to cause a loss of energy; and hence would probably contribute toward the reduction of the energy imparted to the pulsation. These volumes were all cylin-

drical in shape and with the exception of the 8-in. and the 48-in. volumes were made of thin sheet metal, No. 24 gage.

75 The quieting effect due to the use of volumes is shown by the curve sheet in Fig. 25. The dotted curve is drawn in as a representative average curve for both venturi and orifice meters where volumes, alone, are used.

76 The 24-in. volume in our test line, situated below the meter, had a capacity of 29 cu. ft., and was plainly one of ample size to convert the pulsating flow into pulsationless flow when the air reached the line leading to the orifice head. It is evident from these results that a volume is also a practical means of eliminating the pulsation. The chief question is, whether in large installations volumes of sufficient size would be of practical use.

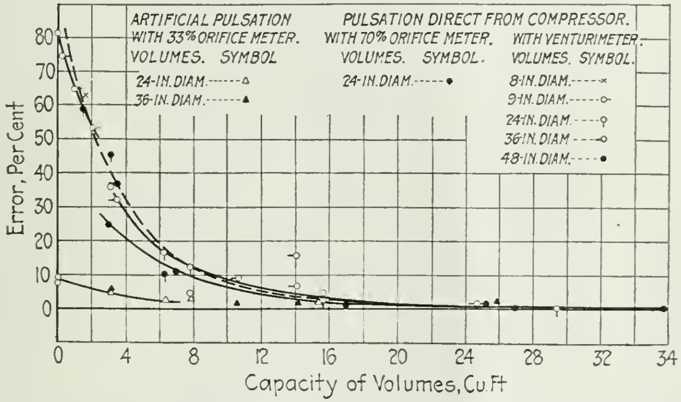

Fig. 25 Percentage of Error for Venturi and Orifice Meters — Pulsation Quieted by Volumes

77 *Effect of Varying the Shape of Volume* was also studied. In a general way a volume is probably more efficient when it is of relatively large diameter.

78 *Elimination of Pulsation by Combining Throttling with Volumes.* Since the pulsation could be nearly if not quite eliminated either by the use of throttling devices or by the use of volumes alone, the natural conclusion was that some combination of the two schemes might be discovered which would give satisfactory results without the objectionable large pressure drop or the excessive size of the volume. A series of runs was made, while the various volumes were in the line, where orifices of various sizes were placed

at the entrance and exit of the volumes. The venturi and the orifice meters were used in these tests. It was found that it was possible with a volume of several cubic feet, combined with a pressure drop of about two inches of mercury, to reduce the error to a small figure, even for a meter having a large maximum error.

79 *The Muffler as a Quieting Device.* Following the experiments with the volumes and orifices combined as a means of eliminating the pulsation, the idea was further developed by the combination of a volume with several orifices; or in other words, the adaptation of the principle of the automobile muffler to the problem of pulsating flow.

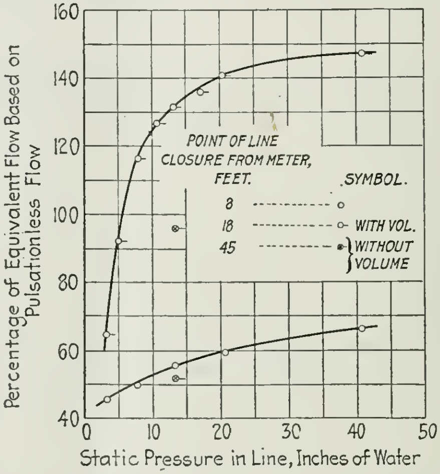

FIG. 27 EFFECT OF PULSATION ON VENTURI METER IN "DEAD-END" LINE

80 To study the effects of a muffler a device was constructed of 8-in. pipe-flange sections, as in *B*, Fig. 8. The curve given in Fig. 26 is based upon all the results obtained for every type of muffler tested. The results for the 8-section muffler were more complete and show at the upper limit an error of 81.5 per cent for pulsating flow for open pipe. The whole curve shows that the effectiveness of any single type of muffler, aside from its value as a volume alone, depends entirely upon the amount of throttling produced and very little upon the design and arrangement of its baffle work.

POSSIBILITY OF ADJUSTMENT OF ERRORS

81 There is a possibility that the error shown by a meter when measuring pulsating flow may be so adjusted or so compensated for, that the true quantity passing through the meter may be known. This is mentioned as a possibility only, and forms a basis for comments on some of the circumstances which would attend such an attempt.

82 There is the first possibility of the integration of the curve, obtained by means of the photopulsometer, which is a series of velocity-time diagrams. The second possibility would be the use of a correction factor for the manometer itself. A third possibility would be the determination of a pulsating factor by calibration under actual running conditions.

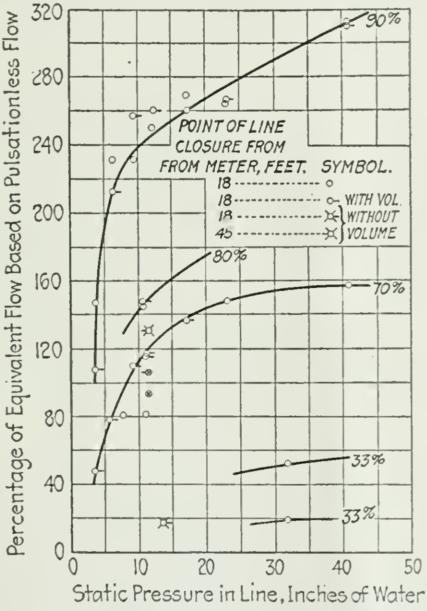

Fig. 28 Effect of Pulsation on Orifice Meter in "Dead-End" Line

83 *Effect Produced by "Dead-End" Pulsation.* During our experiments in connection with the effect of pulsations on the static pressure we had noticed indications of a reading on the meter for zero velocity in the line. Later a series of tests was made with each of the four meters in while the test line was closed at different points

beyond the meter station. The static pressure, at a point in the line 6 ft. above the meter, was maintained the same as during the regular line of tests for 0.9 in. static head at the orifice head. This static pressure was secured by regulating a by-pass valve in the line 18 ft. above the meter (see Fig. 10). The line could also be closed at flanges located at 8 ft., 18 ft., 40 ft., and at the orifice head, 45 ft. below the meter. For the " dead-end " tests most of the runs were made with the large tank, *T*, in the line, and a few runs were made with the large tank out of the line.

84 The effect produced by the " dead-end " pulsation is represented by the flow equivalent to the reading on the meter as compared with the flow under pulsationless-flow conditions. The curves shown in Figs. 27–31 show this relation for the four types of meters. The standard flow conditions as represented, during the regular runs, by the 0.9 in. static head at the orifice head, the static pressure in the line above the meter would read about 9.5 in. of water for pulsationless flow and the corresponding static head under pulsating flow would range from 9.5 in. to 16 in. of water, according to the type of meter used. Generally speaking, as would be expected, the static pressure in the line read a little higher under pulsating flow than under pulsationless flow, in some cases showing an increase of 3.5 in. in 10 in. This would be equal to an error of 16 per cent due to pulsation.

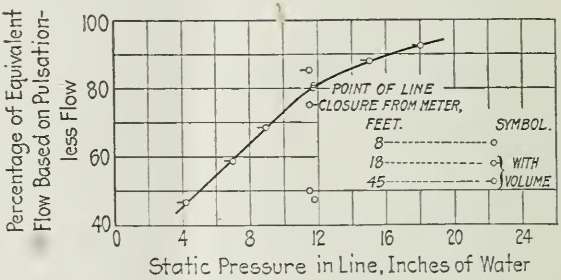

Fig. 29 Effect of Pulsation on 1½-in. Flange-Nozzle Meter in " Dead-End " Line

85 The curves, Figs. 27–31, give, especially, the effect due to change in static pressure in the line while the meters are in a " dead-end " line. For all meters there is an increase in pulsating effect as the static end is increased up to 20 in., after which they increase more gradually up to 40 in. static pressure, the limit of the tests.

86 In most cases there was a tendency toward an increase in the false flow reading with increase of distance to point of line closure from the meter. In two instances the meter showed a negative reading. The rapid change in this effect on the meter with variation in static pressure, together with the marked variation in apparent flow with change in point of line closure from the meter makes it exceedingly doubtful whether any reliability could be placed on this method of obtaining the pulsation factor for any given installation.

87 *Application of Proposed Formula to Pulsating Flow.* An attempt has been made to test the relation expressed by the following formula by means of the "dead-end" flow data. This formula[1] is: $P_2/(\sqrt{P_1} \pm \sqrt{D})^2 \times 100 = 100$ per cent, where P_1, P_2 and D are respectively the corrected manometer readings for pulsationless flow, pulsating flow, and "dead-end" flow. The positive value of D has been used since it was considered to be better adapted to the data taken.

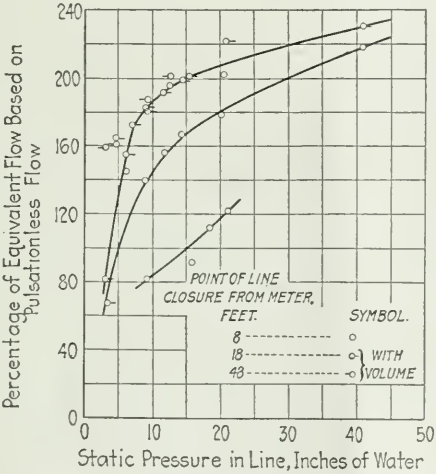

FIG. 30 EFFECT OF PULSATION ON PITOT TUBE No. 1 IN "DEAD-END" LINE

88 Fig. 32 shows the relation between the ratio P_2 to $(\sqrt{P_1} \pm \sqrt{D})^2$ and the point of line closure. The curve in broken lines is the average for the total number of results; the full line is the average for the few points for the line without the volume, Q. For the 8-ft. distance the results are widely scattered and indicate

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[1] Measurement of Gas and Liquids by Orifice Meter, H. P. Westcott, 1922.

that this distance is too close to the meter. The 18-ft. distance gives results showing more uniformity with the average close to 100 per cent. This would indicate the formula would apply better for line closure at 18 ft. The 45-ft. point, though representing the end of the duplicate line, with an average ratio falling below 100 per cent, would indicate less agreement with the formula. The volume Q does not make any marked effect, or as much effect as is shown by difference in length of line closure from the meter.

89 The uncertainty as to where the point of closure should be made or as to whether a length of line duplicating any given line would give the true pulsating-head factor make it doubtful whether the proposed formula could be applied with any assurance of a reasonable degree of accuracy.

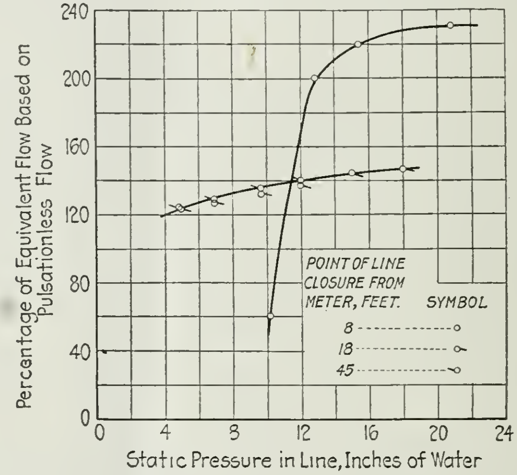

FIG. 31 EFFECT OF PULSATION ON PITOT TUBE NO. 2 IN "DEAD-END" LINE

CONCLUSIONS

90 This investigation relating to our special installation is summarized as follows:

A Nature of Pulsations:

a Pulsations in a pipe line, originating from a reciprocating piston, or a similarly disturbing system, consist of sudden changes both in the velocity and in the pressure of the fluid.

b The pressure change is the most apparent and is probably the greatest factor in producing errors in metering devices.

c The pressure change is in the form of a wave front resembling a traveling sound wave of low frequency.

d The pressure wave travels in the pipe with the velocity of sound.

e The velocity of the pulsation is independent of the velocity, or quantity, of fluid flowing.

f Pulsations in air flow are similar to the compression waves set up by water hammer. Both travel at the velocity of sound in the fluid and are independent of the velocity of flow.

g The effect of this pulsation on a flow meter is to increase its reading, often causing an error of great magnitude. The magnitude of this error depends upon the frequency of pulsation, nominal static pressure of the fluid, type of meter used and adjacent fixtures in the pipe line.

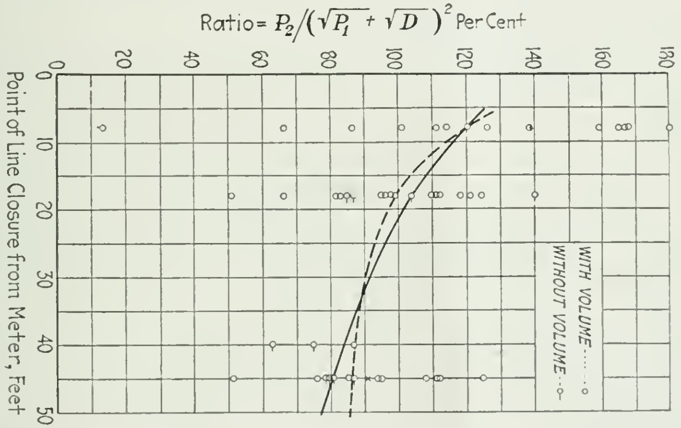

FIG. 32 EFFECT OF LINE CLOSURE ON PROPOSED FORMULA — WITH "DEAD-END" PULSATIONS

h With orifice meters and flange nozzle meters the pulsating error increases as the diameter of the orifice, or nozzle, approaches the diameter of the pipe.

i The throttling or modification of the manometer connections to the meter does not appreciably reduce the error.

j The point of attachment of manometer connection has no great effect on the error due to pulsating flow.

k The pulsation error at the center of the pipe is 35 per cent less than that at the wall of the pipe.

l A meter on a "dead-end" connection will usually show a positive error of considerable magnitude.

m The pulsation must be eliminated or greatly reduced in order to have the meter read without objectionable error.

B Practical Elimination of Pulsations:

n Because of the high velocity of the pulsation, an excessive length of pipe line would be necessary to destroy the pulsation.

o Throttling is effective but requires a pressure drop of 6 in. of mercury to reduce the error to 5 per cent.

p Abrupt volume enlargements in the pipe line will eliminate the error, if of sufficient capacity. A volume capacity of 20 cu. ft. is required for an error within 2 per cent.

q Generally speaking, for the same capacity, a volume of relatively large diameter is more effective than one of small diameter.

r No relation was found between the compressor displacement and the capacity of the volume chambers.

s The combination of throttling with volumes forming the "muffler" device probably is the most effective device for the mechanical elimination of pulsations.

t The pulsating bag, or diaphragm, and the fan, or revolving baffles, are partially successful in eliminating the pulsations, but their installation is thought to offer serious practical objections.

u The effectiveness of any of these quieting devices seems to depend upon their ability to dissipate or change the energy of pulsation which is effected chiefly through a drop in pressure.

v The device which will destroy the pulsating energy with the least obstruction to the flow of the fluid is the most desirable.

w The effectiveness of the meter element itself in quieting the pulsation depends upon the degree of restoration of the pressure beyond the meter. The greater the percentage of restoration, the higher the percentage of error shown for any given type of meter.

C Adjustment of Error of Pulsation:

x It is probably not feasible to correct any meter by means of a correction factor owing to the disturbing effects which may arise from slight changes in the installation and running conditions.

y The experimental establishment of a pulsating correction factor and its relation as shown in the proposed formula is not considered feasible with our present experimental knowledge of the laws of pulsating flow.

z It seems probable that each installation where pulsating flow is present would present its own peculiar problem for which an individual study and consideration of the existing conditions would be necessary for a satisfactory solution.

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DISCUSSION

H. N. Packard. We do not agree that the "pressure change . . . is the greatest factor in producing errors in metering devices." For instance, imagine a compressor cylinder discharging through a short length of pipe, in which is mounted a pitot tube, into a large volume such as a gasometer. In the pipe section no measurable static pressure change during the cylinder discharge can be detected, but a very appreciable variation in rate of flow must occur with the consequent error of meter reading. This is readily confirmed by actual test with the pitot tube. As a further proof of this point, in the Thomas meter, which is entirely independent of pressure conditions and indicates only the standard units flowing through it, we have errors with heavily pulsating flows. Assuming a pulsating flow of sine wave form, the meter error is negligible up to the point where the flow at the peak of the wave does not vary more than 30 per cent from the mean rate. When the wave amplitude is as great as the mean flow, giving instantaneous stoppage of flow in each cycle, the meter error reaches a maximum of nearly 8 per cent. As practically all meter installations are fairly close to the pulsation producing piston, we believe the errors are mostly due to actual instantaneous flow variations through the metering device.

We believe that the velocity of transmission of the wave is that of sound plus that of the gas velocity, slightly modified by the size and shape of the pipe line. We have found a number of references in standard works on physics that the velocity of sound in air is dependent on the wind velocity as well as the density of the air. Do the authors feel that their data can support their statement? Their maximum test velocity was 27 ft. per sec. compared with about 1100 ft. per sec. for sound, and it seemed to us difficult to be sure that the experimental error was not as great as this.

The curves on Fig. 23 would indicate some reduction in manometer error with throttling of connections. Have the authors any explanation of this other than possible leaks? We can see no other reason for the gain in accuracy.

The statement is made that at least a 6-in. mercury pressure drop is required to reduce pulsations to a practical limit. Do the authors consider this a general statement or applicable only to their test conditions? It would appear to me to be a function of the density of the fluid, its velocity and the pulsation wave form (magnitude of pulsation) if made as a general statement.

We are still of the opinion that there is some relation between the piston displacement and volume of a quieting receiver which will give good results. Taking the two absurd extremes of a volume equal to piston displacement and an infinite volume, in one case we know that no effect will be produced and in the other perfect quieting of pulsations will occur. We believe that the quantity of fluid discharged per stroke, the number of strokes per minute and the volume and diameter between the source of pulsations and the meter determine the pulsation effect at the meter, at least with elastic media such as gases.

On dead-end error tests we believe that there is an actual displacement of gas back and forth in the meter, this flow effect being due to the elasticity of the gas which is alternately compressed and expanded in the dead-end volume. In our own meter the dead-end effect does not occur at all except with such large dead-end volume that the alternate gas flows through it, traveling at least seven inches in a reverse direction. This condition has been met but once in our commercial experience. Our meters are normally set vertically with the normal flow vertically upward. At zero load a very small amount of heat is still left in the heater to maintain control and due to convection current the heated gas rises through the exit thermometer and maintains its temperature higher than the entrance thermometer sufficiently to give a constant tendency to shut down the meter. With large dead-end volume and severe pulsations there is sufficient back flow to overcome the convection currents and carry heated gas from the heater into the entrance thermometer, increasing its temperature to approximately the same as the exit thermometer. Immediately the control for the meter goes to full load under such conditions.

J. M. Spitzglass. Prior to the advance of Professor Judd's experimental work on pulsating flow there was an idea prevalent that the error in the measurement was due mainly to the magnifying effect of the differential column, reading the average height and the corresponding square root of this average instead of the average of the instantaneous square roots which are the equivalent of the varying flow.

With the development of the flow meter, we sought to eliminate this error by making the meter respond electrically to the instantaneous instead of the average height of the differential column. This provision was thought to eliminate the part of the error which the authors of the paper designate as the "effect of the type of manometer used." We soon discovered that there was a much larger error due to the "harmonic" effect of the pulsations in the flow. Still, in all our observations with reciprocating flow this error seldom exceeded 25 per cent under any circumstances. Furthermore, this error could be easily eliminated by moderate restrictions in the form of additional orifice plates on either side of the differential medium.

The writer was greatly surprised when he first glanced at the tables in the paper to note that the errors in some of the meters were as high as 200 per cent and over. The writer does not for a moment cast any doubt on the data of the given observations, but he has felt from the beginning that there must be something misleading in the algebraic presentation of the results. After reading the paper a second and third time, the solution of the riddle presented itself very clearly.

In summarizing the tabular results of the investigation, the authors adopted the velocity pressure of the flow as *the* basis for comparing the effect of the pressure pulsations, which, according to their own explanations and results, was not in the least a function of that velocity pressure. What they actually did was to compare a variable quantity, the pressure pulsations, on the basis of another and more variable quantity, the velocity pressure of the flow in the given meter. It will be observed, therefore, that whenever the assumed basis, the velocity pressure of the meter, decreased, the apparent percentage of error increased in a corresponding ratio.

The writer believes that this is the real reason why the percentage of error increased when the size of the orifice or the flange nozzle was increased. The same reasoning applies and is a sufficient explanation for the fact that the percentage error increased in the case of the pitot tube at the wall of the pipe, where the velocity pressure (the assumed basis) is the lowest. Furthermore, the dead-end phenomenon shows conclusively that the effect of pulsation is not a factor of the velocity pressure of the air.

As the data and the results of the investigation are exceedingly important for the users of the meter on pulsating flow, the writer would like to ask the authors to include in the paper a summary table, similar to Table 4, giving the actual values of the static pressure in the line, the actual velocity pressure, and the difference in static pressure between the pulsating and pulsationless flow for all meters tested. It can be readily seen that if the difference has a value of, say, 5-in. of water, it may form a square-root error of 145 per cent on a meter whose velocity pressure is only one inch of water, while the same 5-in. difference will form an error of only 5 per cent on a meter whose velocity pressure amounts to 50-in. of water; and what is more important, if by means of moderate restriction or increased volume the difference is reduced from 5 to 0.5 in., it will still give an error of 22 per cent in the first case and will be entirely insignificant in the second case. To state this in another way: The effect of pulsation, according to the writer's understanding of the investigation, is shown to be rather in the nature of an additional term than a factor in the algebraic expression of the flow for a given meter.

R. J. S. Pigott. The paper on pulsating flow reports the results of research work undertaken for the Special Committee on Fluid Meters, of the Society's Research Committee.

One point about pulsating flow seems to be coming more strongly to the fore; that is, the problem is largely an acoustic one. All the data go to show that the variability of the conditions is due to the fact that the acoustic conditions in the pipe differ with every installation, and it is hard to see how pulsating flow can be stopped in every case until a study is made of the phenomena from an acoustic standpoint.

Up to the present time we have not developed devices for detecting the acoustic variations. We have been working along lines of mechanical devices almost wholly, and have never given enough attention, as yet, to demonstrating clearly what are the acoustic conditions in the pipes.

One of the earliest problems in the opinion of the members of the Fluid Meters Committee was that of either providing correction for the effect of pulsating flow upon the indications yielded by the flow meter mechanism, or to so reduce the pulsations as to render their effect insignificant. The net result of research has definitely confirmed the belief that any type of meter operating on a difference of head which is proportioned to the square of velocity will register high on pulsating flow. The other belief, that it is very difficult, if not impossible, to provide suitable correction factors for the readings of the meter, is also very largely confirmed. The problem, therefore, is mainly reduced to providing commercially practicable means for suppressing pulsations to a point where they do not have a marked effect in the registration of the meter.

The experimental work so far carried out has indicated that it is feasible to accomplish this end by means of a combination of throttling with enlargement of volume. The scope of the experimental work has not been large enough as yet to definitely establish the amount of throttling and the amount of volume enlargement required for any particular case and it is probable that the variations in velocity and size of lines will render an exact solution for any specific case difficult. However, there is a good deal of encouragement to be drawn from the evidence that in all likelihood proper relations of the two requirements can be determined to cover a considerable variation in pulsation condition. The situation on pulsating flow is only one factor of several which have tended to reduce the reliance placed upon flow meters for accurate measurements, and it is not generally recognized that the inaccuracies found in the commercial operation of flow meters are due almost entirely to the conditions under which the flow meter is installed. Under proper installation conditions, it can be demonstrated that any well designed flow meter can give results with perfectly satisfactory precision. However, in the great majority of installations insufficient attention is given to the effect of pulsating flow, eddy currents in the lines due to valves, elbows or similar obstructions and leakage in the lines, transmitting a pressure difference to the meter indicating and recording mechanism.

The first report of the Fluid Meters Committee was to have been produced for the Annual Meeting but the amount of work to be done both in editing the report and preparing for printing was too much to permit publication at this time.

This report will cover the matter of installation very fully as well as the theory and accuracy of the devices employed. It is to be hoped that this report will provide for the designers and users of flow meters a complete summary of information available on the subject. Hitherto there has been no single source from which this information could be obtained and it has been scattered through three or four hundred different publications.

John L. Hodgson.[1] In the opinion of the writer, the results obtained from the elaborate researches described in the paper might have been very much greater had a careful analysis been made beforehand of the ways in which pulsating flows cause errors in meters which are based upon the measurement of a differential pressure.

By making such an analysis the writer has found it possible to reach wider and more general conclusions than the authors of the present paper at the expense of far less experimental work.

Some of the most important of these conclusions are summarized below.

A pulsating air flow may be considered to consist of:

a A "pressure variation" which is transmitted with the velocity of the fluid in the pipe, plus the velocity of the sound in the fluid proper to the particular size and roughness of pipe used, and the nearness to the source of pulsation of the point where the velocity is measured.

b A "velocity variation" during which the whole of the air in the pipe is accelerated or retarded.

The fluid at a point distant from the source of pulsation does not however change its velocity until the impulse, transmitted with the velocity stated under *a*, reaches it.[2]

Both these pressure and velocity variations cause errors[3] in the meter; but in quite different ways.

The error due to the pressure variation occurs when the pressure pipes leading to the meter have different coefficients of discharge for inflows and outflows, and when the capacity in the meter on the two sides of the water or mercury column are different.[4] It is then possible to obtain an actual difference of pressure on the two sides of the meter by the pressure variation alone and when there is no velocity variation at all in the pipe.[1]

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[1] Eggington House, Beds, England.

[2] The authors state that the velocity of propagation of the impulse is that of sound. That may *apparently* be so in the case of their particular experiments; as the effect of the pipe walls is to retard the speed, and of the moving air to increase it.

[3] The above sources of error, and also the effects of "square root" and "viscous" damping of the manometer or meter were pointed out in a paper by the writer in 1916; see *Proc.* Inst. C. E., vol. CCIV, p. 134 to 137.

[4] In the case of the photo-pulsometer, shown in Fig. 11, the upstream

The error due to the pressure variation[2] may easily be brought down to a very small amount by using pressure connections which have equal coefficients of discharge in both directions, and by keeping the capacities in the meter about equal.

There remains the error due to velocity variation, which is the real source of trouble.

This causes error because the flow depends upon the mean of the square root of the pressure differences across the measuring device; whereas the meter reading[3] depends (approximately)[4] upon the mean of the pressure differences.

It can be shown by calculation that for certain wave forms this "velocity variation" may produce errors of several hundred per cent.

The error due to this cause can be calculated or determined by calibration for any particular conditions; but as it varies with the rate of flow, and the wave form, and the product of the specific volume and the absolute pressure of the fluid, and the loss of pressure in, and the capacity of, the pipe line, it is best reduced to a small amount rather than allowed for.

The only way to reduce it is to smooth out the wave form of the "velocity variation" at the metering point.

This can be done in many ways, the simplest of which (not mentioned by the authors) is to insert a capacity *and* a throttling

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cavity is larger than the downstream cavity, with the result that when a change of pressure occurs the pressure will rise and fall most quickly in the downstream cavity; thus exaggerating all the readings. Many of the diagrams taken with this instrument indicate negative flows (instead of the positive flows which must actually have existed) because of this defect.

[1] The "dead end" condition referred to by the authors.

[2] The "pressure change" is wrongly stated by the authors to be "probably the greatest factor in producing errors." The "pressure change" is more rightly considered as a "symptom" than a "cause."

[3] That is, assuming that the meter is adjusted so as to show no appreciable error due to the "pressure variation."

[4] This is only true when the "damping" in the pressure pipes and the meter follows the "viscous" law. If it follows the "square-root" law the mean meter reading is not the true mean of the pressure differences. This may explain the change of manometer error as the pressure pipes are throttled shown in the authors' Fig. 23. The data given in the paper are, however, insufficient to enable the point to be settled.

device between the source of pulsation and the metering point. If the meter itself offers sufficient resistance it may form the throttling device; if it does not, an additional throttling device may be added. The capacity should be placed between the source of pulsation and the meter, and the additional throttling device, if any, should be placed on the downstream side of the meter. (See Fig. 33.).

In a paper read before the Midland Institute of Mining, Civil and Mechanical Engineers in January 1921, and again in a paper read before the Institute of Naval Architects in April 1922, the writer showed that, if certain assumptions were made in order to simplify the reasoning[1], it could be proved[2] that the percentage error of a meter for any particular wave form depended upon the value of the ratio:

$$FCL/ZQ$$

where F is the frequency with which the wave form repeats itself

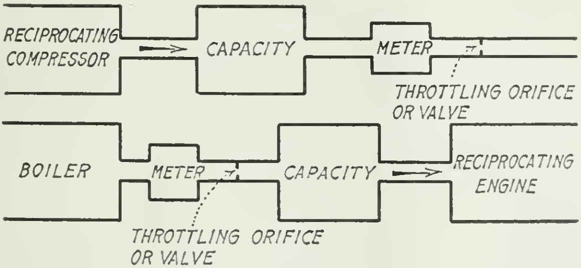

Fig. 33 Location of Capacity with Respect to Source of Pulsation and Meter

C is the capacity, and

L is the loss of pressure between the entrance to the capacity C and the side of the metering device which is furthest from the source of pulsation

Z is the product of the specific volume of the fluid and its absolute pressure

Q is the rate of weight flow.

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[1] Such as that the meter is adjusted so that the "pressure variation" causes no error; that the only error is caused by the meter taking the mean of the pressure differences across the measuring device, instead of the mean of the square root of the pressure differences; that L varies in Q^2, etc.

[2] For an elementary proof, see the writer's paper published in the *Proc.*, Inst. Naval Architects, April 1922.

The writer has developed methods which enable curves connecting the values of FCL/ZQ and the per cent error of the meter due to the "velocity variation" to be calculated and drawn out, so that, given the wave form, and the values of FZQ, the value of CL which will reduce the pulsation error to small limits can be immediately read off.

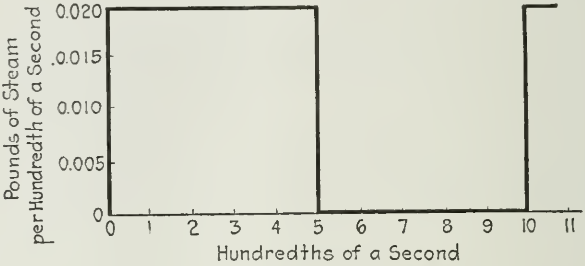

FIG. 34 SQUARE WAVE FORM

Such a curve for a square wave form, Fig. 34, is shown in Fig. 35.[1]

Many interesting results follow from the FCL/ZQ relation, among which are:

1 A large meter error may often be reduced to a negligible amount by trebling or quadrupling the value of CL by providing for additional throttling and by putting the meter further from the source of pulsation so as to secure additional capacity. It will be seen that in order to reduce the pulsation error, it is equally efficacious to

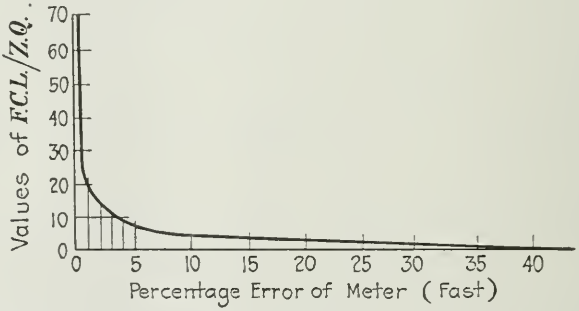

FIG. 35 RELATION OF VALUES OF FCL/ZQ AND PER CENT OF ERROR OF METER

increase C or L. The energy lost is, however, least when the pulsation is reduced by increasing C.

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[1] Compare the authors' Fig. 26, which is plotted against L only.

2 If the rate of flow is reduced on account of the compressor slowing down (the wave form remaining the same) the meter error will be increased, as F and L are reduced, and L falls off more rapidly than Q.

3 Similarly when a steam engine governs, the meter error increases as the flow is reduced. The increase in the error is greatest when the engine governs on the "cut off" (instead of "on the throttle"), as the wave form is then altered for the worse. If there is an appreciable steam chest capacity on the engine side of the throttle, the pulsation error may actually be reduced when the engine governs "on the throttle."

4 If the throttling orifice shown in Fig. 33 consists of a valve (instead of a fixed orifice) and this valve is shut down (either automatically or by hand) as the flow is reduced so that the original L is maintained, the pulsation error will remain constant at all flows, if F and Q fall off in the same ratio.

5 It will be seen on reference to Fig. 35 that if the pulsation error is large, any small change in the value of FCL/ZQ will cause a large change in the pulsation error, whereas if the pulsation error is reduced to 1 or 2 per cent there may be large variations in the value of FCL/ZQ without causing any appreciable change in the overall error of the meter. It is therefore far better to reduce the pulsation error to a small amount by increasing FCL/ZQ than it is to "rate" the meter by actual calibration for a large pulsation error.[1]

6 The FCL/ZQ relation explains the authors' conclusions $A(h)$ and $B(w)$, since it shows that the pulsation error is large when L is small.

7 It can be shown that for given values of FLZ and Q the amount that the meter reads fast is (roughly) inversely proportional to C^2, or for given values of CLZ and Q to F^2 and so on.

It should be understood that the FCL/ZQ relation, being deduced from premises which simplify the actual conditions, does not hold with absolute rigidity in practice, and also that it only holds over a limited range of conditions.[1] At the same time it serves as a most valuable key to the meaning of what is otherwise a mass of unrelated data.

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[1] Compare the authors' conclusions $A(m)$ and $C(x)$.

The writer would say that he disagrees with the authors' conclusions $A(b)(d)(e)(f)(i)$, and $B(n)$.

It will be seen that he is generally in agreement with conclusions $C(x)$ and $C(z)$. With regard to the latter he would say that curves connecting the values of CFL/ZQ and the percentage error of the meter which he has had calculated out enable his firm (Messrs. Geo. Kent of London and Luton, England) to decide on the proper value of CL for any particular case at the expenditure of a few minutes' work only.

In conclusion he would like to congratulate the authors upon the scope of their work, and upon the clear way in which they have set forth their results.

The Authors. In reply to Mr. Packard, we can readily see that his type of meter would not be greatly affected, if any, by the pressure changes, even though the static pressure gage might read higher due to the pulsation. We would also conclude from our investigation that his meter would be less affected by the pulsation because the effect on the velocity head seems to show much less error than that produced in meters depending on the pressure drop readings.

In regard to the effect produced by the static pulsation we have failed to convey the proper meaning. The change or effect on static pressure produced by the pulsation is much greater than the effect produced on the velocity head. This pulsation, like sound, seems to be propagated as a pressure wave and the effect produced on any measuring device, especially where difference in pressure head is used, is much greater than the effect recorded on the velocity diagrams from the photo-pulsometer. Hence, the conclusion that the "pressure change" was the greatest disturbing factor was drawn. This we believe to be borne out in our work. By the photo-pulsometer the diagrams with air flow through the venturi showed a maximum velocity change of $1\frac{1}{2}$ in. = 4.5 in. water as taken with the pitot tips which from their construction would produce a magnified reading while for the same flow conditions, the indicator diagrams giving static pressure fluctuations showed a change about 1.2 lb. or say 35-in. water or about eight times as great as was shown in the velocity head reading neglecting the fact that the instrument gave a magnified reading.

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[1] For instance, it does not hold when the rate of flow is so great that the loss of pressure, L, no longer follows the square root law.

Furthermore in Fig. 20, the diagrams 1 and 2, section B, show a pressure card and a pressure and velocity head card respectively. A comparison of the two cards show little or no difference and at least indicate no great change, if any, due to the velocity head when added to the static head, as taken by a pitot tip attached to the indicator cock.

It seems apparent, therefore, that the pulsation, (assuming its propagation as a pressure wave) is transmitted in the pipe by means of the air (either flowing or quiet) as a medium; and that with the dead-end meter connection the pulsation is surging back and forth independent of the air which itself may also have some slight movement back and forth. This transmission of pulsation in the dead-end line would seem to be similar in this respect to the surge of pressure in a water line due to water hammer which is very much greater in magnitude as compared with the effect due to velocity.

In regard to the velocity of the pulsations, the statement should be modified to show that the velocity of the pulsation included the velocity of the wave and the velocity of the air. For the maximum velocity of 27 ft. per sec. the error would be less than 3 per cent if we neglect the velocity of the air which of course is much less than the actual experimental error.

The conclusions given in the summary are made with reference to the installation which we tested; also reference to the throttling effects of a six-inch mercury pressure drop considers our test conditions only, and should be modified much according to Mr. Packard's suggestion.

In regard to error due to throttling manometer connections, we do not believe that there were any leaks, since these manometer throttling plugs were made up in sets all of the same material and in the same way, and furthermore the data seemed to be consistent.

Referring to the relation to piston displacement of the volume of a quieting cylinder, it is probably true that *some relation* exists, but it seemed to us that it would take such an extended investigation to establish anything approaching a law, as to render the solution impracticable.

In the dead-end meter installation, we agree with Mr. Packard that there is an actual forward and back flow of the fluid due to the elasticity of the gas; but it is also true, we think, that the pulsation in the form of the compression wave travels forward in undiminished amplitude and returns in more or less diminished amplitude depending on the length, shape, and volume of the dead-end connection.

Mr. Spitzglass states in his discussion that the authors in finding the per cent of error due to pulsating flow have compared "a variable quantity, the pressure pulsations, on the basis of another and more variable quantity, the velocity pressure of the meter."

The error due to pulsating flow was based on the velocity, or quantity of flow, or its proportional equivalent the square root of the pressure difference through the meter element for pulsationless flow. For the four types of meters used the velocity head is equal to or proportional to the drop, or pressure difference, through the meter element. From whatever cause the pulsating flow may have been produced it is quite evident that its effect would have to be determined from the reading on the meter manometer.

It appears to us, therefore, that while the pressure pulsation seems to be the greatest factor in the error due to pulsating flow it is the velocity head reading that is observed on the meter. In our opinion it is the velocity-head readings as shown by the meter for both conditions of flow that should be compared. In fact we are at a loss to know of any other way of establishing the per cent of error.

Mr. Spitzglass also calls attention to the fact that for the orifice meter and flange nozzle meter the percentages of error increase approximately as the inverse ratio of the velocity heads. The reason he assigns is that as the velocity through the meter element decreases the effect due to the static pressure pulsations increases; and likewise by the same reasoning the apparent increase in percentage of error, as the walls of the pipe are reached, can be explained. This relation for the orifice and flange-nozzle meters can be noted in the following table but it is felt that the data is not sufficient in relation to the behavior of pressure pulsation at different points of the line to warrant more than a mention of this apparent relation.

Owing to the limited capacity of the air compressor it was not possible to experiment with pulsating air flow under varying static pressure conditions. For the dead-end meter connection static pressure changes could be noted and it was observed that a rapid increase in percentage of error occurred with increase of static pressure. Hence in the discussion, where it is stated that a 5-in.

differential would give 145 per cent error for a flow condition of one inch true differential and the same differential of 5 in. would give only 5 per cent error for a true differential of 50 in. of water, it would be unsafe to predict that the differential would still be 5 inches for a 50-in. true differential. A flow under 50-in. head would necessarily require a much higher static pressure which is likely to result in a differential between pulsationless and pulsating flow much greater than the assumed 5 inches and hence the percentage of error would be correspondingly increased.

TABLE 11 FLOW CONDITIONS IN THE TEST LINE

Meter used	Static pressure in line above meter, in. water			Velocity head by meter man., in. water	Maximum error, Per cent
	Pulseless	Pulsating	Difference		
1	2	3	4	5	6
Venturi	10			9.29	
		13.5	3.5	30.15	81.0
Orifice	27.7			22.00	
33%		32.0	4.3	26.35	9.5
Orifice	11.8			4.10	
50%		13.4	1.6	10.48	60.0
Orifice	9.8			0.94	
70%		11.4	1.6	4.83	128.0
Orifice	9.4			0.41	
80%		10.8	1.4	2.93	167.0
Orifice	8.9			0.0865	
90%		9.6	0.7	1.083	254.0
Flange-nozzle	13.0			5.59	
1⅛ in.		16.3	3.3	11.73	45.0
Flange-nozzle	10.0			1.64	
1½ in.		11.4	1.4	6.16	94.0
Flange-nozzle	9.3			0.47	
2 in.		9.9	0.6	2.74	143.0
Flange-nozzle	8.7			0.121	
2½ in.		9.4	0.7	1.090	200.0
Pitot	8.8			0.163	
No. 1		9.2	0.4	1.110	162.0
Pitot	8.8			0.224	
No. 2		9.6	0.8	1.000	115.0

The authors are glad to comply with Mr. Spitzglass' request to include a table giving in summary a statement of the flow conditions in the test line:

Columns 2 and 3 give the static pressure in the line for pulseless and pulsating flow respectively. Column 4 gives the difference in static pressure for pulsating flow over pulseless flow for a velocity of flow in the three-inch line of twenty-two feet per second.

It will be noticed that although the flow conditions were uniform for the different meter elements the pulsating effect on the static manometer varied, ranging from 0.4 to 4.3 in. of water. This variation depended upon the kind of meter element used. Those elements which obstructed the pipe most, namely: the venturi, the 33-per cent orifice and the 1⅛-in. flange-nozzle, showed the greatest effect due to the static pulsation. Where the pipe was least obstructed as with the orifice, flange-nozzle, and pitot meters the static manometer showed the least effect. Also the maximum percentage of error is seen to vary approximately inversely as the error shown by the static manometer.

As pointed out in the paper and as further emphasized by Mr. Pigott, the authors believe that very little can be done to establish suitable correction factors for meters operating under pulsating flow and that the solution of the problem is reached when some suitable means is provided which will reduce the pulsations to a negligible point. The adaptation of the "muffler" device is apparently the most effective mechanical device for reducing the pulsations. However, further study and experimentation is necessary to establish the proper combination of throttling and volume space for static pressures and pulsating conditions approaching those in general practice.

The authors feel greatly honored in having their paper reviewed by Mr. John L. Hodgson of England.

Mr. Hodgson takes exception to certain conclusions in the paper, in some cases justly so, and in others due apparently to a wrong interpretation of the paper. He points out the importance of having equal spaces in the manometer connections of the meter and in the case of the photo-pulsometer equal spaces above and below the diaphragm. The authors also recognized the importance of this and so far as possible all manometer connections were made of equal length, although this relation could not be maintained while the manometers were in use. Our experiments showed (see Fig. 23) that, when the manometer tubes were throttled, a throttle-plug, with a diameter of 0.02 in. was necessary to reduce the error from 80 to 60 per cent for the venturi meter, with throttling plug coefficients the same for flow in either direction and other conditions of flow remaining constant. The use of volumes in the manometer connections in various combinations and with uniform and similar connecting tubes, $\frac{1}{8}$ in. diameter opening (see "M," Fig. 4), gave no more favorable results than did the throttle plugs.

With the photo-pulsometer the space below the diaphragm was made equal to the space above as were also the connections to the searching tubes as shown in the corrected drawing of Fig. 11. A partition with a glass shutter extended across the diaphragm chamber so that the diaphragm was equidistant from each wall. The diagrams showing negative pressure could not therefore be caused by unequal capacities. The photo-pulsometer was used chiefly as an indicator for illustrating and confirming pulsationless flow conditions.

It is conceivable that the pressure pulsations might be lessened, perhaps nearly eliminated, by the use of the proper amount of throttling, observing at the same time that equal spaces were provided at the manometer connections for varying quantities of flow. We concluded that it was an extremely doubtful and uncertain method, if not wholly dangerous, to rely too fully on such expedients for reducing the pulsating error.

The question raised in regard to the relative effect of the pulsation on the static pressure is answered, we believe, in the reply to Mr. Packard. The authors have designated as static pulsation changes all pulsating changes which act in a direction at right angles to the stream flow. For the venturi meter, with the usual static pressure connections, acting under pulsating flow, the liquid in the manometer tubes would steadily rise until a differential head was reached equal to more than three times the differential head due to pulsationless flow. The column would vibrate through a range of $\frac{1}{2}$ to $\frac{3}{4}$ of an inch corresponding in frequency to that of the flowing air. This vibration is due, in our opinion, to the in and out flow through the manometer tubes, and represents the only effect on the manometer tubes due to the velocity pulsation, the greater part of the manometer reading being due to the static pulsations.

We agree with Mr. Hodgson and Mr. Packard that the velocity of propagation of the pulsating wave approaches that of sound in the flowing fluid plus the velocity of the flowing air. However, in our opinion, as based on our experiments, we are not willing to concede that the pressure pulsation has a less effect than the velocity pulsation in meter installations where these pulsations act on manometers with static connections at right angles and even with the inside of the pipe. From the "dead-end" flow experiments it would also seem evident that the pressure pulsation effect is many times greater than that due to the velocity pulsation.

The statement was made by Mr. Hodgson that the simplest way of reducing the pulsation by the use of a capacity combined with throttling was not mentioned by the authors. In conclusion *B* (*s*) we have stated that the "muffler" device, a combination of capacity, or volume, with throttling is probably "the most effective device for the mechanical elimination of pulsations." The capacity or "muffler" was always inserted between the disturbing element and the meter. It would not seem advisable in our opinion, to insert the throttling device in the line below the meter. It would seem better to eliminate the pulsations as far as possible before the meter was reached.

The authors regret that Mr. Hodgson's paper containing his latest investigations involving the development of a formula for pulsating flow was not available for examination and study until after their paper was written. The grouping of the factors involved in pulsating flow in order shown in the formula seems feasible. Mr. Hodgson is to be congratulated in being able to reduce the results of his researches to a working formula which shows the factors involved in their proper relation. We are in full accord with the opinion of Mr. Hodgson that the only sure way to meter pulsating flow is to reduce the pulsations by means of suitable capacity and throttling, and we likewise believe that even the formula proposed, or any similar formula, while serving in a general way cannot be too rigidly applied in practice. Each installation will present its own peculiar problem.

APPENDIX A — HISTORICAL

For at least twenty years it has been noticed that pulsations would complicate and render inaccurate any attempt to measure the flow of fluids by means of a meter. In 1902, Williams, Hubbell and Fenkell (*1) investigated the effect of turbulent flow produced by flow of water in pipe bends. This was followed by further study along the same line in 1916 by Moreel (*2). In 1905 Mayo (*3) noticed that the pulsations caused by the impeller of a centrifugal pump raised the delivery gage reading 25 per cent higher than the design and running conditions of the pump would warrant.

In 1916, Rogers and Smith (*4) and in 1917 F. B. Seely (*5) mentioned the action of turbulence connected with tests on water-discharge through submerged mouthpieces and orifices. E. G. Bailey (*6) also pointed out the serious effect of "swirling" in fluid flow.

F. P. Fisher (*7) found erratic disturbances in gas mains due to pulsations and tried to eliminate them, with partial success, by using a "pulsation equalizer" made up of a manifold of small parallel pipes of different lengths.

L. A. Wilson, (*8) while using a Venturimeter for air flow from a compressor, found that the meter would register a flow reading for a closed pipe indicating that the pulsations were transmitted to the meter resulting in a reading even with no air flow.

H. P. Westcott (*9) following this lead suggested a formula for correcting the effect due to pulsation by making use of the "dead-end" reading on a flowmeter.

From 1908 to 1918, studies (*10) of the effect of water hammer in pipe lines have been carried on. This is mentioned because of the close relation which, it is believed, can be shown to exist between water hammer and pulsations in the air line, the difference being that of degree only.

During the past two years, more definite steps have been taken in an attempt toward eliminating, or compensating for, the error due to pulsating flow: 1. As reduced by throttling (*11); 2. by the use of tanks (*12); 3. by the use of tanks combined with vibrating diaphragms as used by Professor Trinks (*13) for measuring the output of air compressors; 4. by the recent introduction of a type of silencer (*14) to be applied to exhaust lines in Power Stations.

*1. Trans. A. S. C. E. Vol. 47, 1902, p. 1–369. "Experiments of Effect of Curvature upon the Flow of Water in Pipes." Williams, Fenkell, and Hubbell.

*2. Engineering News, Vol. 75, p. 302–304. "Lost Head Diagrams for Bends in Water Pipe." Ben Moreel.

*3. Trans. A. S. C. E. Vol. 54, 1905, part D, p. 502. "Pulsation Effect on Static Gage Reading." Mayo.

*4. Engineering News, Vol. 76, 1916, p. 825–827. "Experiments with Submerged Orifices and Tubes." T. C. Rogers and T. L. Smith.

*5. Engineering Experiment Station, U. of Ill., Bull. 96, 1917, p. 48.

"The Effect of Mouthpieces on Flow of Water through a Submerged Short Pipe." F. B. Seely.

*6. Power, June 13, 1916, p. 854. "Measurement of Steam Flow." E. G. Bailey.

*7. Trans. A. S. M. E. Vol. 38, 1916, p. 278. "Establishing a Standard of Measurement for Natural Gas in Large Quantities." Fisher.

*8. Power Vol. 44, 1916, p. 425. "Action of Venturimeter with Pulsating Flow." L. A. Wilson.

*9. "Measurement of Gas and Liquids, by Orifice meter," p. 143. H. P. Westcott.

*10. Trans. A. S. C. E. Vol. 82, 1918, p. 185–249. "Pulsations in Pipe-lines as Shown by Some Recent Tests." H. C. Vensano.

*11. Report No. 49. National Advisory Committee for Aeronautics. 1920, p. 30, "Metering Characteristics of Carburetors."

*12. Technical Notes No. 40. National Advisory Committee for Aeronautics, 1921, p. 3. "Effect of Reversal of Air Flow Upon the Discharge Coefficient of Durley Orifices." Marsden Ware.

*13. Power Plant Engineering. Nov. 1, 1921, p. 1044. "Use of Nozzles for Measuring Flow." W. Trinks.

*14. Power. Jan. 17, 1922, p. 91.

Power. June 27, 1922, p. 1024. The Maxim Industrial Silencer in the Power Plant.

APPENDIX B — EXPLANATORY NOTES

(Page and paragraph numbers refer to corresponding page and paragraph numbers in the paper)

Page 8, Paragraph 11. When the project was first undertaken, the only reliable source of power, owing to fuel shortage, was that of natural gas. We, therefore, were obliged to limit ourselves at the time to a small air compressor driven by a gas engine to furnish the air supply. It was then the intention to investigate, also, the effect of pulsations produced in steam and water, but more than enough work appeared in connection with air flow to occupy all of the available time.

Page 13, Paragraph 22. This check orifice was placed in the flange near the tapered end of the orifice head with a manometer arranged to read the drop across it simultaneously with the manometer at the orifice head. All the single orifices were checked against the check orifice by comparing the square roots of the ratios of the manometer readings.

Page 14, Paragraph 24. Of the four meter elements used in the study of the effects of pulsation, the venturi meter, the orifice meter, and the flange nozzle meter depend upon the same fundamental principle, that of pressure drop due to change in velocity; the pitot tube depends upon the "impact" principle or the conversion of the velocity of flow into its equivalent static head. These four types of meters were chosen as being representative of the majority of the general class of recording meters of the inferential type.

Page 21, Paragraph 38. The photo-pulsometer was not available for use during the latter half of the investigation. This was regretted in some instances especially when studies were being made of the static pressure changes in the line and of the probable velocity of the pulsation. However, by means of such velocity diagrams as were taken we were able to verify the different velocity changes produced by the various modifying causes and especially to check up our equipment and to establish the conditions for pulsationless flow at the standardized orifice head.

Page 37, Paragraph 60. The "surge" of the vertical manometer often made it necessary to have two observers to get accurate readings, and also a more accurate reading could be made on the inclined manometer than on the vertical manometer. One reason assigned for this difference was the presence of considerable volume of air in the reservoir at the low pressure end of the inclined manometer.

Page 38, Paragraph 64. The variations possible in manometers in common use are as follows: Variation in (1) density of the liquid used, (2) size, shape, and position of the manometer tubes, (3) connection between manometer legs, (4) recording mechanism. Our experimental knowledge of these different variations is quite fragmentary but a few tendencies were observed. As previously mentioned, the recording head reading on the Foxboro differential recording mercury gage was consistently only 60% of the head reading on the water manometer. See also Tables 11, 12, 13, 14, 30 Appendix C.

Page 39, Paragraph 65. It is our opinion, also, that if the connections from the pipe to the manometer were short, direct, and as large, or nearly so, as the manometer tubes and if the frequency of the pulsation was comparatively low the pulsation would be unusually violent and would cause the liquid in the manometer to assume "a wholly indeterminate condition of churned foam" as one investigator (*7) describes it. The liquid would attempt to "recover" or return to a lower different head between pulsations and the violent fluctuation of the column would make accurate reading impossible. In most cases, however, the manometer connections consist of a series of enlargements and contractions, hence the further addition of special volume units would only tend to accentuate this condition.

If the pulsation is allowed to propagate itself through still air instead of through a flowing stream of air, the effect is quite different on the various types of manometers. This may be seen by comparing the results shown on Table 27. Here four types of manometers were attached to a meter under "no flow" conditions. The pipe line could be closed at different points and the same static pressure was maintained while the compressor was running under the standard conditions except that air was by-passed off from the test line 18 feet above the meter.

For the Flange Nozzle (Table 30) with pulsationless flow, the reading checks closely for the water manometer, the inclined U-tube manometer, and for the one-leg inclined manometer. When the pulsations were admitted to the "dead-end" line (for 8 ft. closure) for the same static pressure, 11.8″ water (increased 2″ due to effect of pulsation on the static manometer) the water manometer gave a negative reading of .25″ (for manometer connections both direct and reversed) but there was no agreement among any of the other manometers for the same pulsation conditions and all manometers except the water manometer showed a wide variation when the manometer connections were reversed.

Similarly for the 70 % orifice meter under the same pulsating conditions, while the pulsationless flow readings showed reasonably consistent agreement, the action of all of the manometers under the effect of pulsation was very erratic with no agreement either individually when the connections were reversed or when compared with one another. The reason for this disagreement seemed to be due entirely to the differences in the size, shape, and position of the manometer tubes when in use. The "dead-end" pulsation acted very differently and was much more erratic in its effect than the pulsation occurring while the air was flowing. The presence of an enlarged reservoir bulb on the low pressure leg of the manometer seemed to have less disturbing effect than one connected to the high pressure leg. The effect was more pronounced when the bulb was only partly filled with liquid owing probably to the increased hydrostatic effect due to the increased surface exposed to the pulsation.

Page 40, Paragraph 67. The increase in per cent of error at the walls of the pipe is 30 % for Pitot No. 1 and 31 % for Pitot No. 2. No reason can be assigned either for this manner of distribution or for the manner of variation of these curves in direct opposition to the usual velocity curves as shown by a traverse. If Bernouilli's law holds in this case, for a reduction in velocity near the walls of the pipe there would be the corresponding increase in pressure. Increase in pressure is shown to result in increase in pulsation error but it is thought that the pressure increase would not be sufficient to account for the full amount of increase in error indicated by the traverses. See also Table 21.

Page 41, Paragraph 68. Diagrams *a*, *b*, *c*, Fig. 18, are taken with the fan section removed and the line slightly throttled thus reducing the error due to pulsation for open pipe from 80 % to 68 %. The fan-section consisting of three closely fitting but freely revolving fans was then placed in the line. The slight amount of throttling in addition to the effect of the high speed of the fans reduced the error from 80 % to 27 %. The fans seem to modify the character of the pulsation somewhat as Diagram *d* shows. There is a distinct dip below the zero pressure line. The maximum pulsations range as high if not higher than those in Diagram *c*. No diagrams were available for the point beyond the 24″ quieting volume (Fig. 1, Part *Q*). In all probability at that point a diagram would have shown pulsationless flow. The per cent of error quoted above was computed from other data taken.

Page 41, Paragraph 69. Comparing the maximum pulsations in the line for diagram *a*, Fig. 19, with the maximum pulsations for diagram *b* after the pulsating bag had been attached shows:

Average maximum pulsation for diagram $a = 0.63$
Average maximum pulsation for diagram $b = 0.25$
Square root of ratio 25 to 63 = 0.63
Quieting effect due to pulsating bag = 37 %

Under certain conditions a tank or volume with vibrating sides would, no doubt, be of service in reducing the effect of the pulsations. It is felt that special design of such a device would be needed to cover the requirement of each individual installation in order to correct or eliminate the pulsating error.

Page 42, Paragraph 72. This rate of change was not so rapid when the pulsation was artificially produced. See Curves 2 and 3 Fig. 34. See also Tables 17, 18, 19, 20, 21, Figs. 34, 35, 36, 37.

For the 90% orifice meter, conditions were such as to produce a negative error after the drop in pressure exceeded 5 inches. A similar case was observed for the pitotmeter No. 1 and the same phenomenon was also noted in a test with the same pitotmeter made earlier but not included in these data. As mentioned in the discussion under *Effect of Type of Manometer Used* for the action of the meters under "dead-end" flow, it is believed that certain conditions may arise whereby the effect of the pulsation is as if there were an instantaneously negative flow of air which might cause an apparent negative reading on the meter. The occurrence although not easily accounted for cannot be accidental for the instances referred to occurred during tests taken at intervals several months apart. It is possible that the partial restoration of pressure after leaving the orifice meter with the large orifice may be a contributing cause for the occurrence of the negative error and the subsequent increase in error with increase of drop in pressure.

Page 43, Paragraph 74.

DIMENSIONS OF VOLUMES USED TO QUIET PULSATIONS

Diameter, Inches	Length, Inches	Capacity, Cu. Ft.	Diameter, Inches	Length, Inches	Capacity, Cu. Ft.
8	56	1.65	24	30	7.86
9	9	0.33	24	60	15.73
9	18	0.67	36	6	3.54
9	27	1.00	36	18	10.62
9	36	1.33	36	24	14.14
9	45	1.67	36	42	24.75
9	54	2.00	48	3	3.00
18	30	4.43	48	7	7.00
24	12	3.14	48	16	17.00
24	6	1.57	48	25	27.00
24	24	6.28	48	34	36.00

Page 43, Paragraph 75. Where the combined errors of the venturi and orifice meters are plotted, the general effect is seen to be the same regardless of the kind of meter used. While for the smaller capacities there is a marked advantage shown by the volumes of larger diameter over those of smaller diameter (for equal capacities) yet the curves indicate that about 20 cu. ft. capacity is sufficient to bring the error due to pulsation below 2%, for all diameters. See also Tables 22, 23.

Page 43, Paragraph 76. An attempt was made to establish a relation between the capacity of the minimum size of volume and the piston displacement of the compressor. Such a relationship might be useful if there were any close relation between the velocity pulsation and the pressure pulsation; but we have not been able to find any reason for assuming that any relation exists between piston displacement and volume capacity.

Page 43, Paragraph 77. See curves in Fig. 38.

Page 44, Paragraph 78.

Data and Results for Pulsation Reduced by Orifices Combined with Volumes:

Meter Used	*Data and Results*	*Curves*
Venturi	Table 24	Fig. 39, 40.
33% & 70% Orifices	Table 25	Fig. 41.

The curves in Fig. 39 show for the venturi meter the variation of error with drop in pressure for an orifice placed at the entrance to volumes of constant capacity ranging from 1/3 cu. ft. to 200 cu. ft. Points are also plotted for orifices at both entrance and exit to the volumes. It will be noted that two orifices, for the same constant volumes, gave a slightly greater reduction in the error. It was also observed while running that a volume fitted with two orifices instead of one, quite noticeably reduced the vibration and the sound effect which accompanies pulsating flow.

The curves in Fig. 40 show for the venturi meter the effect of throttling by means of two orifices combined with different volumes for the complete range of volumes of constant capacity. The curves when read horizontally give the combinations of volume and drop in pressure for any given per cent of error. Likewise readings taken vertically give for a certain definite drop in pressure, the per cent error to be expected for any given volume. For example: For 5% error and a drop in pressure of 2 inches of mercury, a volume of 3.54 cu. ft. would be required; and for a drop of 3 inches and volume of 2 cu. ft. the error would be 6 per cent. The curves in Fig. 41 show in a similar way the possible combinations for the 70% orifice meter when using the set of 24-inch volumes.

Page 44, Paragraph 79. Some preliminary experiments were made with several mufflers (See Fig. 8). First a Powell auto muffler was used and directly following that a muffler was hastily constructed using tin funnels for the baffling units. This gave unusually good results, reducing the error to almost zero with a drop of a few inches of water. It was thought at the time that the problem of pulsating flow was solved. However, when a new muffler was made of funnel sections more carefully constructed and a trial was made the previous results could not be duplicated. It required a drop in pressure of 6 inches of mercury with the new funnel muffler to reduce the error to 3.5%.

A fourth muffler (Type E, Fig. 8) larger in initial volume and containing four baffles with small holes was tried. This muffler as a simple volume tank gave an error of 4.3%, (which checked previous tests for the same conditions); and with the baffles in place gave an error of 2.5% for a pressure drop of 3 or 4 inches of water.

Page 44, Paragraph 80. We were again convinced that the error of pulsation can be reduced only by a loss in energy. Since the pulsating flow is an energetic fluctuation of velocity and pressure; in a large tank, or volume, these fluctuations become mutually eliminated, or dissipated. When throttling is used to quiet pulsations it is effective in so far as it absorbs the energy of the pulsations. The 33% orifice meter gives less error than a larger orifice meter or the venturi meter because there is a greater unrestored pressure loss through it. When the excess energy of the pulsating flow is absorbed, the pulsations are eliminated. See also Table 26.

Page 45, Paragraph 82. Velocity diagrams could be treated in a manner similar to an indicator card and integrated to determine the average pipe velocity. It would require the conversion of the velocity head diagram into a new diagram involving the square root of the velocity head before integrating, or the employment of a special form of rectifying planimeter. Such a method has been tried with partial success by Mr. H. N. Packard in his study of pulsating flow.

Use of correction factor: If the meter could be calibrated for pulsationless flow and the flow conditions under which it was to be used were to remain constant, then such a correction might be applied. The difficulties arising are as follows:

1. It would be difficult to produce pulsationless flow in most commercial installations where pulsating flow is the source of the difficulty.

2. It is doubtful whether or not true pulsationless flow could be assured.

3. It would be difficult to establish the same quantity of flow for both pulsating and pulsationless flow. The exception might be in the case of a steam engine, where the steam used might be condensed and weighed as a check.

4. Any number of slight variations are apt to arise in the flow conditions which would affect the amount of the error. A correction factor would have to be re-determined frequently.

Use of a pulsating factor has been suggested by H. P. Westcott (*9) as applied to an orifice meter. A second line with a closed end is connected with the main line, parallel to it, ahead of the main orifice meter. It is subjected to the same static pressure and to the effect of pulsating flow under "no flow" conditions. The main meter reading will be influenced by two factors, the pulsating effect and the velocity of flow; the meter in the "dead-end" line is assumed to be influenced by the pulsating effect alone. The meter reading for pulsationless flow should also be found. A formula is proposed to express the relation between these three quantities:

$$P_2 = (\sqrt{P_1} \pm \sqrt{D})^2$$

P_2 = corrected manometer reading for pulsating flow
P_1 = corrected manometer reading for pulsationless flow
D = corrected manometer reading for "dead-end" flow

If this relation holds true, then:

$$\frac{P_2}{(\sqrt{P_1} \pm \sqrt{D})^2} = 1$$

Page 46, Paragraph 85. From Tables 27, 28, 29, 30 it can be seen that a meter when out of service in a "dead-end" line but otherwise subjected to the effect of pulsations may register a false flow reading ranging from 19% to 239% according to the type of meter. (See, also, Table 33.)

Page 47, Paragraph 87. Table 31 contains the values for this ratio as computed from the data for the four points of line closure.

APPENDIX C, TABLES.

TABLE 11 EFFECT OF THROTTLING MANOMETER CONNECTIONS FOR VENTURI METER (SEE P. 34, PAR. 57)

Date	Run, No.	Throttling Plugs		Kind of Flow	Manometer Reading, Inches of Water			Ratio	$\sqrt{\text{Ratio}}$	Error Per Cent
		Diam. Orifice, Inches	Sizes, Inches		At Orifice Head	At Venturi Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
28-Inch Vertical Water Manometer, Fig. 5										
7–27–21	1	.....	....	puls'less	0.900	9.68	9.68	....	.....	....
	2	0.1800	$\frac{1}{4}''$ diam.	pulsating	.902	31.10	31.05	3.21	1.790	79.0
	3	0.0700	$\frac{3}{8}''$ length	pulsating	.905	31.17	31.00	3.20	1.788	78.8
	4	0.0635	....	pulsating	.900	31.33	31.33	3.24	1.800	80.0
	5	0.0465	....	pulsating	.899	31.33	31.34	3.25	1.803	80.3
	6	0.0420	....	pulsating	.904	30.33	30.20	3.12	1.766	76.6
	7	0.0200	....	pulsating	.920	25.10	24.65	2.53	1.593	59.3
Foxboro Differential Mercury Gage, Fig. 5										
7–27–21	8	.....	....	puls'less	0.900	9.66	9.66	....	.....	....
	9	0.1800	$\frac{1}{4}''$ diam.	pulsating	.893	20.90	21.08	2.18	1.479	47.9
	10	0.0700	$\frac{3}{8}''$ length	pulsating	.900	20.50	20.50	2.12	1.456	45.6
	11	0.0635	....	pulsating	.902	20.00	19.95	2.07	1.437	43.7
	12	0.0465	....	pulsating	.906	19.50	19.39	2.01	1.415	41.5
	13	0.0420	....	pulsating	.902	17.90	17.87	1.85	1.360	36.0
	14	0.0200	....	pulsating	.902	18.60	18.58	1.94	1.394	39.4

TABLE 12 PER CENT ERROR FOR ORIFICE METERS, EFFECT DUE TO POINT OF MANOMETER CONNECTION (SEE P. 37, PAR. 62)

Date	Run, No.	Manometer Connection		Kind of Flow	Manometer Reading, Inches of Water			Ratio	$\sqrt{\text{Ratio}}$	Maximum Error, Per Cent	Error Compared with Standard Connection Per Cent
		Above Orifice, Diams.	Below Orifice, Diams.		At Orifice Head	At Orifice Meter	Corrected to 0.9				
		1	2	3	4	5	6	7	8	9	10

33% Orifice Meter

1-3-22	1	$2\frac{2}{5}$	$2\frac{2}{5}$	puls'less	0.930	22.88	22.15				...
	2	$2\frac{2}{5}$	$2\frac{2}{5}$	pulsating	.931	27.30	26.40	1.191	1.092	9.2	93.9
	3	1	$\frac{1}{2}$	puls'less	.935	23.13	22.25	..	.....		..
	4	1	$\frac{1}{2}$	pulsating	.929	27.60	26.76	1.203	1.098	9.8	100.0
	5	1	2	puls'less	.934	21.40	20.63	...	......	..	.
	6	1	2	pulsating	.935	27.03	26.05	1.261	1.123	12.3	125.5
	7	1	4	puls'less	.924	20.08	19.58	....	......	...	......
	8	1	4	pulsating	.929	24.47	24.73	1.262	1.123	12.3	125.5
	9	1	8	puls'less	.932	20.28	19.60	.	......	....	...
	10	1	8	pulsating	.930	24.53	23.75	1.210	1.100	10.0	102.0
	11	1	19	puls'less	.935	20.43	19.70	.	.....	....	.....
	12	1	19	pulsating	.931	24.13	23.33	1.185	1.090	9.0	91.8

40% Orifice Meter

1-3-22	13	$2\frac{2}{5}$	$2\frac{2}{5}$	puls'less	0.937	10.90	10.47	....	......	..	...
	14	$2\frac{2}{5}$	$2\frac{2}{5}$	pulsating	.944	16.77	16.03	1.531	1.237	23.7	100.0
	15	1	$\frac{1}{2}$	puls'less	.948	11.03	10.48	....	......	...	..
	16	1	$\frac{1}{2}$	pulsating	.943	16.97	16.01	1.530	1.237	23.7	100.0

50% Orifice Meter

1-3-22	17	$2\frac{2}{5}$	$2\frac{2}{5}$	puls'less	0.936	4.35	4.18	....	......	...	.. ..
	18	$2\frac{2}{5}$	$2\frac{2}{5}$	pulsating	.936	10.27	9.89	2.362	1.536	53.6	91.0
	19	1	$\frac{1}{2}$	puls'less	.940	4.28	4.11	....	.....	....	.. ..
	20	1	$\frac{1}{2}$	pulsating	.934	10.87	10.47	2.552	1.599	59.9	100.0

60% Orifice Meter

1-3-22	21	$2\frac{2}{5}$	$2\frac{2}{5}$	puls'less	0.937	1.92	1.846	....	.....	...	.. ...
	22	$2\frac{2}{5}$	$2\frac{2}{5}$	pulsating	.937	6.90	6.630	3.590	1.898	89.8	98.2
	23	1	$\frac{1}{2}$	puls'less	.944	1.90	1.810	....	.....	.	...
	24	1	$\frac{1}{2}$	pulsating	.935	7.03	6.640	3.670	1.915	91.5	100.0

70% Orifice Meter

1-3-22	25	$2\frac{2}{5}$	$2\frac{2}{5}$	puls'less	0.977	1.010	0.930	. .	......	.	. .
	26	$2\frac{2}{5}$	$2\frac{2}{5}$	pulsating	.987	5.306	4.840	5.210	2.283	128.3	97.6
	27	1	$\frac{1}{2}$	puls'less	.982	1.010	0.926	..	..	...	....
	28	1	$\frac{1}{2}$	pulsating	.982	5.100	4.950	5.340	2.315	131.5	100.0
	29	1	2	puls'less	.985	0.643	0.588	..	....	...	....
	30	1	2	pulsating	.985	4.113	3.755	6.380	2.525	152.5	117.0
	31	1	4	puls'less	.976	0.510	0.471	.	. ..	....	. .
	32	1	4	pulsating	.988	3.130	3.120	6.630	2.575	157.5	120.0
	33	1	8	puls'less	.975	0.510	0.471	. .	... .	....	. ....
	34	1	8	pulsating	.983	3.320	3.040	6.460	2.540	154.0	117.2
	35	1	19	puls'less	.975	0.543	0.502		.. ..	...	. ...
	36	1	19	pulsating	.983	3.220	2.940	5.870	2.120	112.0	108.0

80% Orifice Meter

1-3-22	37	$2\frac{2}{5}$	$2\frac{2}{5}$	puls'less	0.980	0.460	0.422				.
	38	$2\frac{2}{5}$	$2\frac{2}{5}$	pulsating	.970	3.300	3.065	7.260	2.695	169.5	93.0
	39	1	$\frac{1}{2}$	puls'less	.980	0.432	0.397	.			
	40	1	$\frac{1}{2}$	pulsating	.969	3.400	3.160	7.960	2.820	182.0	100.0

TABLE 12 — *Continued*

Date	Run, No.	Manometer Connection		Kind of Flow	Manometer Reading, Inches of Water			Ratio	√ Ratio	Maximum Error, Per Cent	Error Compared with Standard Connection Per Cent
		Above Orifice, Diams.	Below Orifice, Diams.		At Orifice Head	At Orifice Meter	Corrected to 0 9				
		1	2	3	4	5	6	7	8	9	10
				90% Orifice Meter							
1–3–22	41	$\frac{2}{25}$	$\frac{2}{25}$	puls'less	0.970	0.175	0.161	....	......	....	....
	42	$\frac{2}{25}$	$\frac{2}{25}$	pulsating	.989	2.080	1.892	11.750	3.430	243.0	69.0
	43	1	½	puls'less	.969	0.090	0.084	....	......	....	....
	44	1	½	pulsating	.985	1.880	1.720	20.450	4.520	352.0	100.0
				95% Orifice Meter							
1–3–22	45	$\frac{2}{25}$	$\frac{2}{25}$	puls'less	0.945	0.055	0.0524	....	......	....	....
	46	$\frac{2}{25}$	$\frac{2}{25}$	pulsating	.990	1.520	1.3820	26.40	5.140	414.0	45.6
	47	1	½	puls'less	.950	0.010	0.0095	....	......	....	....
	48	1	½	pulsating	.990	1.060	0.9640	101.80	10.080	908.0	100.0
				100% Orifice Meter							
1–3–22	49	$\frac{2}{25}$	$\frac{2}{25}$	puls'less	0.900	0.007	0.0070	....	......	....	....
	50	$\frac{2}{25}$	$\frac{2}{25}$	pulsating	.930	1.032	1.0000	143.0	11.96	10.96	....
	51	1	½	puls'less	.940	0.007	0.0067	....	......	....	....
	52	1	½	pulsating	.940	0.992	0.9950	141.5	11.90	10.90	100.0
	53	1	2	puls'less	.922	0.007	0 0068	....	......	....	....
	54	1	2	pulsating	.922	1.292	1.2620	186.0	13.65	12.65	....
	55	1	4	puls'less	.931	0.007	0.0068	....	......	....	....
	56	1	4	pulsating	.931	1.232	1.1900	175.0	13.23	12.23	....
	57	1	8	puls'less	.932	0.017	0.0160	....	......	....	....
	58	1	8	pulsating	.937	0.952	0.9150	57.2	7.56	6.56	....
	59	1	19	puls'less	.929	0.038	0.0370	....	......	....	....
	60	1	19	pulsating	.935	0.802	0.7720	20.8	4.67	3.67	....

TABLE 13 ERROR COMPARED WITH THAT FOR STANDARD CONNECTIONS (1-DIAMETER ABOVE, ½-DIAMETER BELOW ORIFICE; (SEE P. 37, PAR. 62)

No.	Size of Orifice, Per Cent	Point of Attachment, (A Above, B Below Orifice) Pipe Diams.					
		$\frac{2}{25}$ A, $\frac{2}{25}$ B	1 A, ½ B	1 A, 2 B	1 A, 4 B	1 A, 8 B	1 A, 19 B
				Error, Per Cent			
1	33	93.0	100	125.5	125.5	102.0	91.8
2	40	100.0	100	....	....	....	....
3	50	91.0	100	....	....	....	....
4	60	98.2	100	....	....	....	....
5	70	97.6	100	117.0	120.0	117.2	108.0
6	80	93.0	100	....	....	....	....
7	90	69.0	100	....	....	....	....
8	95	45.6	100	....	....	....	....
9	100	100.6	100	116.0	112.0	60.0	34.0
Average (omitting 7, 8, 9)		96.3	100	121.2	122.8	109.6	....

TABLE 14 EFFECT OF VOLUMES INSERTED IN MANOMETER CONNECTIONS (SEE P. 37, PAR. 61)

Date	Run, No.	Pipettes, 1¼″ Diam., 9″ Long			Kind of Flow	Manometer Reading, Inches of Water			Ratio	$\sqrt{\text{Ratio}}$	Error Per Cent
		Total Number Used	Position at Orificeer Meter			At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9	10
40% Orifice Meter Using 28-Inch Vertical Manometer With Glass Pipettes											
1-20-22	1	0	.....	.....	puls'less	0.918	10.53	10.33	....	.....	....
	2	2	1 above	1 below	puls'less	.918	10.53	10.33	1.000	1.000	0.0
	3	0	.....	.....	pulsating	.918	17.07	16.72	1.620	1.273	27.3
	4	2	1 above	1 below	pulsating	.921	15.17	14.81	1.433	1.197	19.7
	5	1	none	1 below	pulsating	.916	16.93	16.65	1.602	1.267	26.7
	6	1	1 above	none	pulsating	.915	15.40	15.15	1.466	1.210	21.0
	7	2	2 above	none	pulsating	.913	13.27	13.09	1.266	1.125	12.5
70% Orifice Meter, Using 6-Inch Inclined Manometer With Glass Pipettes											
1-20-22	8	0	.....	.....	puls'less	.914	0.955	0.940	....	.....	...
	9	2	1 above	1 below	puls'less	.916	0.960	0.943	1.002	1.001	0.1
	10	0	.....	.....	pulsating	.904	5.127	5.110	5.430	2.333	133.3
	11	2	1 above	1 below	pulsating	.908	4.427	4.380	4.660	2.160	116.0
	12	1	none	1 below	pulsating	.905	4.800	4.780	5.090	2.255	125.5
	13	1	1 above	none	pulsating	.906	4.687	4.660	4.950	2.225	122.5
	14	2	2 above	none	pulsating	.908	4.180	4.140	4.410	2.100	110.0

TABLE 15 EFFECT OF VOLUMES INSERTED IN MANOMETER CONNECTIONS (SEE P. 37, PAR. 61)

Date	Run No.	Volume Units, 1⅛″ Diam., 1″ Long			Kind of Flow	Manometer Reading, Inches of Water			Ratio	$\sqrt{\text{Ratio}}$	Error Per Cent
		Total Number Used	Position at Orifice Meter			At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9	10
70% Orifice Meter, Using 6-inch Inclined Manometer with Volume Units											
2-14-21	1	0	......	......	puls'less	0.901	1.12	1.12	....	.....	...
	2	0	......	......	pulsating	.902	5.60	5.58	4.99	2.235	123.5
	3	1-N	1 above	none	pulsating	.900	5.10	5.40	4.83	2.200	120.0
	4	2-N	1 above	1 below	pulsating	.902	5.13	5.41	4.83	2.200	120.0
	5	2-N	2 above	none	pulsating	.902	5.27	5.26	4.70	2.167	116.7
	6	4-N	2 above	2 below	pulsating	.900	5.10	5.40	4.83	2.209	120.0
	7	3-N	3 above	none	pulsating	.900	5.10	5.10	4.56	2.135	113.5
	8	6-N	3 above	3 below	pulsating	.901	5.30	5.30	4.74	2.179	117.9
	9	4-N	4 above	none	pulsating	.902	5.00	4.98	4.46	2.113	111.3
	10	8-N	4 above	4 below	pulsating	.901	5.16	5.15	4.61	2.145	114.5
	11	5-N	5 above	none	pulsating	.902	4.83	4.81	4.30	2.075	107.5
	12	10-N	5 above	5 below	pulsating	.902	4.84	4.82	4.31	2.076	107.6
	13	6-N	6 above	none	pulsating	.903	4.86	4.84	4.33	2.080	108.0
	14	12-N	12 above	none	pulsating	.903	4.92	4.90	4.38	2.093	109.3
	15	6-N	......	......	......	....	....	..	....	...	..
		12-0	6 above	12 below	pulsating	.902	4.49	4.48	4.02	2.005	100.5
	16	6-0	6 above	none	pulsating	.903	4.71	4.69	4.20	2.047	104.7
	17	12-0	6 above	6 below	pulsating	.901	4.70	4.70	4.20	2.047	104.7
	18	12-0	12 above	none	pulsating	.902	4.61	4.59	4.11	2.025	102.5

TABLE 16 MAXIMUM PER CENT ERROR FOR ORIFICE METER

(SEE P. 39, PAR. 66)

Date	Run No.	Orifice Used		Kind of Flow	Manometer Reading, Inches of Water			Ratio	√Ratio	Error Per Cent
		Size, Per Cent	Diam., Inches		At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
8-13-21	1	33	1.098	puls'less	0.902	22.070	22.000	......	......	.....
	2	33	1.098	pulsating	.905	26.470	26.350	1.196	1.092	9.2
	3	50	1.530	puls'less	.914	3.930	3.780	......	......	.....
	4	50	1.530	pulsating	.900	10.000	10.000	2.642	1.623	62.3
	5	60	1.836	puls'less	.914	1.685	1.660	......	......	.....
	6	60	1.836	pulsating	.894	6.420	6.460	3.950	1.986	98.5
	7	70	2.142	puls'less	.914	0.800	0.788	......	......	.....
	8	70	2.142	pulsating	.910	3.800	3.760	4.770	2.185	118.5
	9	80	2.448	puls'less	.898	0.348	0.349	......	......	.....
	10	80	2.448	pulsating	.904	2.720	2.713	7.780	2.790	179.0
	11	90	2.754	puls'less	.918	0.079	0.077	......	......	.....
	12	90	2.754	pulsating	.878	0.807	0.828	10.240	3.200	220.0
1-3-22	13	33	1.098	puls'less	.935	23.130	22.250	......	......	.....
	14	33	1.098	pulsating	.929	27.600	26.750	1.203	1.098	9.8
	15	40	1.224	puls'less	.948	11.030	10.490	......	......	.....
	16	40	1.224	pulsating	.943	16.970	16.000	1.525	1.235	23.5
	17	50	1.530	puls'less	.940	4.280	4.100	......	......	.....
	18	50	1.530	pulsating	.934	10.970	10.480	2.560	1.600	60.0
	19	60	1.836	puls'less	.944	1.900	1.810	......	......	.....
	20	60	1.836	pulsating	.935	7.030	6.640	3.670	1.915	9.15
	21	70	2.142	puls'less	.982	1.010	0.927	......	......	.....
	22	70	2.142	pulsating	.982	5.400	4.950	5.340	2.315	131 5
	23	80	2.488	puls'less	.980	0.432	0.397	......	......	.....
	24	80	2.488	pulsating	.969	3.400	3.160	7.960	2.820	182.0
	25	90	2.754	puls'less	.969	0.090	0.084	......	......	.....
	26	90	2.754	pulsating	.985	1.880	1.720	20.450	4.520	352.0
	27	95	2.907	puls'less	.950	0.010	0.0095	......	......	.....
	28	95	2.907	pulsating	.990	1.060	0.964	101.800	10.080	908.0
	29	100	3.060	puls'less	.940	0.007	0.0067	......	......	.....
	30	100	3.060	pulsating	.940	0.992	0.950	142.000	11.900	1090.0
1-19-22	31	40	1.224	puls'less	.932	10.830	10.470	......	......	.....
	32	40	1.224	pulsating	.905	17.030	16.950	1.620	1.272	27.2
	33	50	1.530	puls'less	.907	4.100	4.070	......	......	.....
	34	50	1.530	pulsating	.920	10.800	10.600	2.610	1.615	61.5
	35	60	1.836	puls'less	.917	1.850	1.816	......	......	.....
	36	60	1.836	pulsating	.920	7.030	6.780	3.730	1.932	93.2
	37	70	2.142	puls'less	.909	0.670	0.664	......	......	.....
	38	70	2.142	pulsating	.919	3.880	3.800	5.730	2.395	139.5
	39	80	2.488	puls'less	.903	0.410	0.408	......	.....	.....
	40	80	2.488	pulsating	.905	0.287	0.286	7.000	2.647	164.7
	41	90	2.754	puls'less	.914	0.080	0.079	......	......	.....
	42	90	2.754	pulsating	.924	1.170	1.140	14.800	3.845	284.5
	43	50	1.530	puls'less	.901	4.000	3.990	......	......	.....
6-28-22	44	50	1.530	pulsating	.905	8.270	8.220	2.060	1.435	43.5
	45	70	2.142	puls'less	.906	4.975	4.950	......	......	.....
	46	70	2.142	pulsating	.912	25.800	25.750	5.200	2.380	138.0
	47	80	2.488	puls'less	.902	2.195	2.190	......	......	.....
	48	80	2.488	pulsating	.900	15.540	15.540	7.190	2.655	165.5
	49	90	2.754	puls'less	.903	0.460	0.458	......	......	.....
	50	90	2.754	pulsating	.910	5.800	5.740	12.530	3.543	254.3
	51	40	1.224	puls'less	.918	10.530	10.330	......	......	.....
1-20-22	52	40	1.224	pulsating	.918	17.070	16.720	1.620	1.273	27.3
	53	70	2.142	puls'less	.914	0.955	0.940	......	......	.....
	54	70	2.142	pulsating	.904	5.127	5.110	5.430	2.333	133.3
	55	70	2.142	puls'less	.901	1.120	1.118	......	......	.....
	56	70	2.142	pulsating	.902	5.600	5.580	4.990	2.235	123.5

TABLE 17 PER CENT ERROR FOR VENTURI METER, PULSATIONS QUIETED BY THROTTLING (SEE P. 42, PAR. 73)

Date	Run No.	Drop Through Valve, In. Mercury	Manometer Readings, Inches of Water			Ratio	$\sqrt{\text{Ratio}}$	Error Per Cent
			At Orifice Head	At Venturi Meter	Corrected to 0.9			
		1	2	3	4	5	6	7
2–15–21	1	puls'less	0.901	9.55	9.54	.....	.....	....
	2	0.03	.900	11.37	11.37	1.193	1.093	9.3
	3	0.93	.900	11.24	11.24	1.182	1.087	8.7
	4	1.93	.899	10.75	10.74	1.129	1.063	6.3
	5	4.59	.900	10.14	10.14	1.064	1.032	3.2
	6	6.54	.899	9.99	9.98	1.047	1.023	2.3
	7	7.01	.902	9.86	9.84	1.031	1.015	1.5
	8	8.16	.903	9.71	9.68	1.013	1.007	0.7
	9	9.82	.901	9.74	9.73	1.021	1.011	1.1
	10	12.06	.908	9.79	9.70	1.017	1.009	0.9
	11	puls'less	.900	9.58	9.58	.....	.....	....
	12	0.06	.899	11.69	11.72	1.222	1.105	10.5
	13	0.45	.903	11.57	11.53	1.204	1.097	9.7
	14	1.93	.902	10.79	10.76	1.120	1.058	5.8
	15	4.18	.912	10.33	10.20	1.063	1.031	3.1
	16	5.90	.895	9.97	10.05	1.050	1.025	2.5
	17	5.82	.893	9.91	10.09	1.052	1.026	2.6
	18	7.36	.898	9.88	9.90	1.033	1.016	1.6
	19	9.64	.901	9.81	9.80	1.022	1.011	1.1
	20	12.42	.914	9.89	9.72	1.015	1.008	0.8
2–28–21	21	puls'less	.910	9.85	9.74	.....	.....	....
	22	0.24	.922	12.33	12.05	1.238	1.113	11.3
	23	0.48	.919	12.08	11.82	1.215	1.102	10.2
	24	0.48	.919	12.12	11.86	1.218	1.104	10.4
	25	0.75	.902	11.69	11.67	1.198	1.095	9.5
	26	2.60	.915	10.97	10.79	1.108	1.053	5.3
	27	3.86	.933	10.87	10.48	1.076	1.037	3.7
	28	6.40	.930	10.33	10.01	1.028	1.014	1.4
	29	8.35	.928	10.23	9.91	1.018	1.009	0.9
	30	12.85	.944	10.23	9.77	1.001	1.002	0.2
	31	puls'less	.931	9.63	9.32	.....	....	....
	32	0.00	.907	11.91	11.83	1.272	1.129	12.9
4–22–21	33	1.00	.924	11.51	11.20	1.202	1.096	9.6
	34	3.20	.964	10.95	10.23	1.100	1.048	4.8
	35	4.90	.965	10.59	9.86	1.058	1.028	2.8
	36	8.90	.902	9.59	9.57	1.028	1.014	1.4
	37	18.40	.955	10.02	9.42	1.010	1.005	0.5
	38	puls'less	.898	9.40	9.42	.....	.....	....
5–6–21	39	0.20	.891	30.90	31.20	3.320	1.820	82.0
	40	2.00	.939	15.52	14.89	1.580	1.258	25.8
	41	4.50	.912	11.20	11.00	1.170	1.082	8.2
	42	7.90	.935	10.50	10.10	1.072	1.035	3.6
	43	12.00	.941	10.40	9.95	1.057	1.027	2.7
	44	18.50	.895	9.70	9.76	1.036	1.018	1.8
	45	0.00	.937	11.75	11.30	1.200	1.095	9.5
	46	0.20	.945	11.43	10.90	1.159	1.077	7.7
	47	1.10	.945	10.17	9.95	1.057	1.027	2.7
	48	2.80	.931	9.85	9.53	1.012	1.007	−0.7
	49	4.60	.934	9.85	9.50	1.009	1.005	−0.5
	50	6.80	.911	9.51	9.40	0.997	0.999	−0.1
	51	12.00	.937	9.76	9.37	0.996	0.998	−0.2
	52	puls'less	.900	9.25	9.25	.....	.....	....
	53	0.00	.900	31.50	31.50	3.410	1.845	81.5
	54	0.00	.911	32.80	32.60	3.350	1.830	83.0
	55	0.00	.905	28.82	28.70	3.120	1.768	76.8

TABLE 18 PER CENT ERROR FOR ORIFICE METER, PULSATIONS QUIETED BY THROTTLING (SEE P. 42, PAR. 73)

Date	Run, No.	Orifice Used		Drop Thru Valve, Inches, Mercury	Manometer Readings, Inches of Water			Ratio	√Ratio	Error Per Cent
		Size Per Cent	Diam., Inches		At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
5-4-21	1	33	1.098	puls'less	0.900	23.17	23.17	.....	......	...
	2	...	.....	0.50	.870	25.90	26.80	1.156	1.075	7.5
	3	...	.....	2.40	.938	25.13	24.10	1.035	1.018	1.8
	4	...	.....	5.10	.900	23.90	23.90	1.030	1.015	1.5
	5	...	.....	5.90	.918	24.43	24.00	1.035	1.016	1.6
	6	...	.....	6.40	.905	23.77	23.61	1.020	1.010	1.0
	7	...	.....	11.20	.905	23.77	23.61	1.020	1.010	1.0
	8	...	.....	17.00	.888	23.83	24.15	1.040	1.020	2.0
	9	...	.....	puls'less	.935	23.13	22.25	.....	......	...
	10	...	.....	0.0	.929	27.60	26.75	1.200	1.095	9.5
1-13-22	11	...	.....	puls'less	.927	23.20	22.50	.....	......	...
	12	...	.....	0.25	.922	27.60	26.95	1.198	1.093	9.3
	13	...	.....	0.95	.928	25.10	24.30	1.080	1.040	4.0
	14	...	.....	1.85	.921	23.50	22.95	1.020	1.010	1.0
	15	...	.....	3.10	.931	23.20	22.50	1.000	1.000	0.0
	16	...	.....	4.45	.933	23.50	22.65	1.008	1.004	0.4
	17	...	.....	5.90	.935	23.60	22.75	1.012	1.006	0.6
	18	...	.....	8.40	.938	23.70	22.75	1.012	1.006	0.6
	19	...	.....	12.30	.931	23.40	22.63	1.008	1.004	0.4
	20	...	.....	18.00	.938	23.70	22.75	1.010	1.005	0.5
	21	...	.....	puls'less	.902	22.07	22.00	.....	......	...
6-28-22	22	...	.....	0.0	.905	26.47	26.35	1.196	1.092	9.2
	23	...	.....	0.65	.915	25.40	25.00	1.138	1.066	6.6
	24	...	.....	1.05	.906	23.93	23.76	1.079	1.038	3.8
	25	...	.....	2.40	.912	23.30	23.00	1.057	1.026	2.6
	26	...	.....	4.40	.910	22.83	22.60	1.027	1.013	1.3
6-28-22	27	50	1.530	puls'less	.902	4.000	3.990	.....	......	...
	28	...	.....	0.0	.905	8.270	8.220	2.060	1.435	43.5
	29	...	.....	0.65	.909	8.367	8.280	2.075	1.440	44.0
	30	...	.....	1.00	.906	6.803	6.760	1.693	1.300	30.0
	31	...	.....	2.20	.908	5.263	5.220	1.308	1.143	14.3
	32	...	.....	4.30	.901	4.500	4.500	1.128	1.062	6.2
	33	...	.....	6.40	.903	4.327	4.315	1.080	1.040	4.0
	34	...	.....	8.40	.900	4.237	4.237	1.062	1.030	3.0
	35	70	.....	puls'less	.982	1.01	0.926	.....	......	...
12-23-21	36	...	.....	0.00	.982	5.40	4.950	5.350	2.275	127.5
	37	...	.....	puls'less	.938	0.67	0.635	.....	......	...
1-13-22	38	...	.....	0.25	.944	4.04	3.855	6.071	2.460	146.0
	39	...	.....	1.00	.934	2.63	2.535	4.000	2.000	100.0
	40	...	.....	2.00	.934	1.62	1.560	2.460	1.570	57.0
	41	...	.....	4.20	.939	0.89	0.854	1.394	1.160	16.0
	42	...	.....	8.30	.946	0.73	0.695	1.094	1.045	4.5
	43	...	.....	12.30	.945	0.73	0.695	1.094	1.045	4.5
	44	...	.....	16.90	.938	0.72	0.692	1.090	1.043	4.3
	45	...	.....	19.20	.944	0.72	0.677	1.067	1.033	3.3
6-28-22	46	...	.....	puls'less	.910	5.00	4.95	.....	......	...
	47	...	.....	0.10	.912	25.80	25.75	5.200	2.380	138.0
	48	...	.....	0.60	.919	21.57	21.15	4.270	2.066	106.6
	49	...	.....	1.30	.915	15.19	14.93	3.020	1.747	74.7
	50	...	.....	2.20	.900	9.47	9.47	1.911	1.382	38.2
	51	...	.....	4.30	.901	6.31	6.30	1.272	1.129	12.9

TABLE 18 PER CENT ERROR FOR ORIFICE METER, PULSATIONS QUIETED BY THROTTLING (SEE P. 42, PAR. 73) — *Continued*

Date	Run, No.	Orifice Used		Drop Through Valve, Inches, Mercury	Manometer Readings, Inches of Water			Ratio	√ Ratio	Error Per Cent
		Size Per Cent	Diam., Inches		At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
	52	...	.....	6.10	0.906	5.72	5.68	1.147	1.071	7.1
	53	...	.....	8.40	.930	5.59	5.40	1.090	1.043	4.3
	54	...	.....	9.60	.912	5.43	5.36	1.082	1.041	4.1
6–29–22	55	80	2.448	puls'less	.902	2.195	2.190	....	......	...
	56	...	.....	0.20	.900	15.540	15.540	7.010	2.655	165.5
	57	...	.....	0.65	.910	12.800	12.650	5.750	2.395	139.5
	58	...	.....	1.20	.914	10.740	10.590	4.810	2.195	119.5
	59	...	.....	2.30	.907	4.940	4.900	2.230	1.490	49.0
	60	...	.....	4.50	.910	2.720	2.690	1.220	1.104	10.4
	61	...	.....	6.40	.916	2.400	2.360	1.072	1.034	3.4
	62	...	.....	8.40	.904	2.340	2.330	1.060	1.030	3.0
	63	...	.....	11.10	.884	2.320	2.345	1.068	1.033	3.3
	64	90	2.754	puls'less	.903	0.460	0.458	....	.....	.....
	65	...	.....	0.17	.910	5.800	5.740	12.530	3.543	254.3
	66	...	.....	0.70	.910	5.360	5.310	11.600	3.404	240.4
	67	...	.....	1.35	.905	4.610	4.580	10.000	3.160	216.0
	68	...	.....	2.65	.907	1.250	1.240	2.710	1.645	64.5
	69	...	.....	4.30	.902	0.470	0.469	1.025	1.012	1.2
	70	...	.....	6.20	.902	0.430	0.430	0.938	0.967	−3.3
	71	...	.....	7.40	.916	0.420	0.413	0.902	0.950	−5.0
	72	...	.....	8.60	.895	0.440	0.443	0.970	0.985	−1.5
	73	...	.....	11.15	.886	0.550	0.559	1.220	1.105	10.5

NOTE: — Inclined manometer was used for Runs 46 to 73. To reduce Column 6 to inches of water, multiply by 0.17.

TABLE 19 PER CENT ERROR FOR FLANGE NOZZLE–METER, PULSATIONS QUIETED BY THROTTLING (SEE P. 42, PAR. 73)

Date	Run No.	Nozzle Used		Drop Through Valve, Inches, Mercury	Manometer Readings, Inches of Water			Ratio	√Ratio	Error Per Cent
		Diam., Inches	Length, Inches		At Orifice Head	At Nozzle Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
6–15–22	1	1⅕	3⅗	puls'less	0.908	3.23	3.210	.....	.....	.....
	2	....	.....	0.0	.907	10.10	10.020	3.120	1.765	76.5
	3	....	.....	1.3	.922	5.50	5.370	1.672	1.293	29.3
	4	....	.....	2.4	.900	4.05	4.050	1.260	1.122	12.2
	5	....	.....	3.9	.900	3.63	3.630	1.130	1.063	6.3
	6	....	.....	6.3	.900	3.49	3.490	1.120	1.060	6.0
	7	....	.....	8.7	.905	3.38	3.365	1.080	1.040	4.0
	8	....	.....	10.0	.900	3.29	3.290	1.026	1.013	1.3
	9	....	.....	11.4	.895	3.36	3.380	1.050	1.025	2.5
	10	....	.....	puls'less	.910	5.72	5.660	.....	.....	.....
	11	....	.....	0.0	.902	11.60	11.570	2.025	1.422	42.2
	12	....	.....	puls'less	.900	5.57	5.570	.....	.....	.....
	13	....	.....	0.0	.902	11.77	11.730	2.100	1.450	45.0
6–16–22	14	1½	3⅜	puls'less	.899	1.663	1.664	.....	.....	.....
	15	....	.....	0.00	.903	6.300	6.280	3.775	1.943	94.3
	16	....	.....	0.65	.913	4.830	4.760	2.860	1 690	69.0
	17	....	.....	1.10	.907	4.530	4.500	2.700	1.642	64.2
	18	....	.....	1.15	.911	3.750	3.730	2.240	1.497	49.7
	19	....	.....	1.30	.895	4.250	4.270	2.665	1.632	63.2
	20	....	.....	2.40	.899	2.470	2.480	1.490	1.218	21.8
	21	....	.....	3.10	.893	2.140	2.155	1 294	1.138	13.8
	22	....	.....	4.50	.897	1.953	1.960	1.177	1.083	8.3
	23	....	.....	6.20	.914	1.900	1.870	1.122	1.061	6.1
	24	....	.....	puls'less	.900	1.636	1.636	.....	.....	.....
	25	....	.....	0.00	.901	6.170	6.160	3.770	1.942	94.2
6–26–22	26	2	3⅜	puls'less	.901	2.883	2.880	.....	.....	.....
	27	....	.....	0.0	.910	17.270	16.880	5.860	2.420	142.0
	28	....	.....	0.8	.901	14.120	14.110	4.900	2.215	121.5
	29	....	.....	1.5	.895	9.907	9.960	3.460	1.860	86.0
	30	....	.....	2.7	.901	5.160	5.160	1.790	1.338	33.8
	31	....	.....	3.4	.905	4.160	4.140	1.436	1.197	19.7
	32	....	.....	4.7	.916	3.593	3.535	1.228	1.109	10.9
	33	....	.....	puls'less	.917	2.757	2.710	.....	.....	.....
	34	....	.....	puls'less	.912	2.777	2.720	.....	.....	.....
	35	....	.....	0.0	.905	16.390	16.290	6.000	2.450	145.0
	36	....	.....	0.0	.910	16.720	16.540	6.090	2.467	146.7
6–27–22	37	2½	3⅜	puls'less	.904	0.716	0.713	.....	.....	.....
	38	....	.....	0.0	.900	6.400	6.400	8.960	2.990	199.0
	39	....	.....	1.0	.905	6.050	6.020	8.440	2.905	190.5
	40	....	.....	1.4	.904	4.963	4.940	6.920	2.630	163.0
	41	....	.....	2.3	.908	2.197	2.180	3.060	1.750	75.0
	42	....	.....	4.2	.913	0.917	0.904	1.263	1.125	12.5
	43	....	.....	6.0	.910	0.827	0.818	1.145	1.070	7.0
	44	....	.....	8.1	.912	0.747	0.738	1.030	1.014	1.4
	45	....	....	9.7	.912	0.743	0.733	1.027	1.013	1.3

NOTE: — Puls'less = pulsationless.

TABLE 20 PER CENT ERROR FOR PITOT-METER, PULSATIONS QUIETED BY THROTTLING (SEE P. 42, PAR. 73)

Date	Run No.	Pitot Used		Drop Through Valve, Inches, Mercury	Manometer Readings, Inches			Ratio	√Ratio	Error Per Cent
		Name	Position of Tip		At Orifice Head	At Pitot-Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
2-15-21	1	No. 1	At	puls'less	0.901	.1463	.1462	.....	.....	....
	2	Pitot	Center	0.025	.900	.1667	.1667	1.140	1.068	6.8
	3	....	.....	0.930	.900	.1557	.1557	1.072	1.036	3.6
	4	....	.....	1.930	.899	.1460	.1470	1.005	1.003	0.3
	5	....	.....	4.590	.900	.1535	.1535	1.050	1.025	2.5
	6	....	.....	6.540	.899	.1500	.1502	1.028	1.014	1.4
	7	....	.....	7.010	.902	.1360	.1357	0.925	0.962	−3.8
	8	....	.....	8.160	.903	.1425	.1421	0.971	0.985	−1.5
	9	....	.....	9.820	.901	.1385	.1384	0.945	0.972	−2.8
	10	....	.....	12.060	.908	.1403	.1392	0.952	0.975	−2.5
	11	....	.....	puls'less	.900	.1340	.1340	.....	.....	....
	12	....	.....	0.058	.899	.1540	.1550	1.120	1.058	5.8
	13	....	.....	0.450	.903	.1505	.1500	1.082	1.041	4.1
	14	....	.....	0.850	.900	.1550	.1550	1.120	1.058	5.8
	15	....	.....	1.930	.902	.1405	.1403	1.046	1.023	2.3
	16	....	.....	4.180	.912	.1295	.1280	0.965	0.983	−1.7
	17	....	.....	5.900	.895	.1360	.1370	1.022	1.011	1.1
	18	....	.....	5.820	.893	.1540	.1550	1.120	1.058	5.8
	19	....	.....	7.360	.898	.1425	.1430	1.069	1.034	3.4
	20	....	.....	9.640	.901	.1385	.1384	1.033	1.017	1.7
	21	....	.....	12.420	.914	.1285	.1266	0.948	0.974	−2.6
2-28-21	22	No. 1	At	puls'less	.911	2.10	2.08	.....	.....	.....
	23	Pitot	Center	0.24	.922	2.42	2.37	1.140	1.067	6.7
	24	....	.....	0.48	.919	2.25	2.21	1.063	1.031	3.1
	25	....	.....	0.48	.919	2.20	2.16	1.038	1.019	1.9
	26	....	.....	0.75	.902	2.33	2.32	1.117	1.057	5.7
	27	....	.....	2.60	.915	2.05	2.02	0.972	0.986	−1.4
	28	....	.....	3.86	.933	2.32	2.24	1.078	1.038	3.8
	29	....	.....	6.40	.930	1.87	1.81	0.870	0.933	−6.7
	30	....	.....	8.35	.928	2.02	1.96	0.942	0.971	−2.9
	31	....	.....	12.85	.944	2.00	1.91	0.918	0.958	−4.2
6-30-22	32	No. 1	At	puls'less	.900	0.982	0.982	.....	.....	....
	33	Pitot	Center	0.55	.915	5.60	5.510	5.610	2.370	137.0
	34	....	....	1.05	.913	4.85	4.780	4.870	2.204	120.4
	35	....	....	1.75	.911	3.52	3.480	3.545	1.880	88.0
	36	....	.....	3.30	.912	1.13	1.115	1.137	1.065	6.5
	37	....	.....	5.40	.908	0.85	0.842	0.856	0.925	−7.5
	38	....	.....	7.30	.908	0.89	0.882	0.897	0.946	−5.4
	39	....	....	9.30	.901	0.96	0.960	0.976	0.988	−1.2
	40	....	....	11.50	.915	1.03	1.012	1.030	1.011	1.4
	41	No. 2	At	puls'less	.904	1.41	1.406	.....	.....	...
	42	Pitot	Center	0.68	.905	5.80	5.770	4.120	2.030	103.0
	43	....	.....	1.50	.904	4.50	4.480	3.200	1.788	78.8
	44	....	.....	2.30	.901	2.73	2.730	1.950	1.395	39.5
	45	....	.....	4.30	.912	1.46	1.440	1.030	1.015	1.5
	46	....	.....	6.20	.908	1.41	1.400	1.000	1.000	0.0
	47	....	.....	8.20	.912	1.44	1.420	1.014	1.007	0.7
	48	....	.....	10.30	.899	1.48	1.480	1.057	1.028	2.8
	49	....	.....	12.30	.891	1.49	1.520	1.082	1.040	4.0

NOTE: — Inclined manometer was used. To reduce Column 6 to inches of water, multiply by 0.17.

TABLE 21 DISTRIBUTION OF PULSATION AS SHOWN BY PITOT TRAVERSES
(SEE P. 40, PAR. 67)

Date	Run No.	Pitot Used		Kind of Flow	Manometer Readings, Inches			Ratio	$\sqrt{\text{Ratio}}$	Error Per Cent
		Name	Position of Tip		At Orifice Head	At Pitot Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
6–30–22	1	No. 1	.837r	puls'less	0.906	0.660	0.655	....	......	.....
	2	Pitot	.837r	pulsating	.912	6.060	5.980	9.13	3.025	202.5
	3	Vertical	.410r	puls'less	.908	0.900	0.892	....	......	.....
	4	Traverse	.410r	pulsating	.915	6.380	6.280	7.04	2.655	165.5
	5	......	center	puls'less	.912	0.970	0.968	....	......	.....
	6	......	center	pulsating	.906	6.490	6.450	6.66	2.580	158.0
	7	......	.410r	puls'less	.901	0.840	0.839	....	......	.....
	8	......	.410r	pulsating	.912	6.440	6.360	7.58	2.753	175.3
	9	......	.837r	puls'less	.897	0.610	0.612	....	......	.....
	10	......	.837r bottom	pulsating	.908	5.630	5.580	9.12	3.020	202.0
	11	......	right .837r	puls'less	.905	0.630	0.625	....	......	.....
	12	No. 2	.837r	pulsating	.907	4.630	4.590	7.34	2.710	171.0
	13	Pitot	.410r	puls'less	.900	1.060	1.060	....	......	.....
	14	Hori-	.410r	pulsating	.910	6.180	6.120	5.77	2.400	140.0
	15	zontal	center	puls'less	.904	1.410	1.406	....	......	.....
	16	Traverse	center	pulsating	.908	7.170	7.110	5.06	2.247	124.7
	17	......	.410r	puls'less	.903	1.470	1.465	....	......	.....
	18	......	.410r	pulsating	.909	7.770	7.590	5.17	2.275	127.5
	19	......	.837r	puls'less	.904	1.070	1.066	....	......	.....
	20	......	.837r left	pulsating	.912	7.280	7.180	6.74	2.600	160.0

NOTE: — To reduce Column 6 to inches of water, multiply by 0.17.
r = radius of pipe.

TABLE 22 PER CENT ERROR FOR VENTURI METER, PULSATIONS QUIETED BY VOLUMES (SEE P. 43, PAR. 75)

Date	Run No.	Volume Used		Drop Through Volume, Inches, Water	Manometer Readings, Inches of Water			Ratio	√Ratio	Error Per Cent
		Diam., Inches	Volume, Cu. Ft.		At Orifice Head	At Venturi Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
	1	8.06	.....	puls'less	0.900	9.10	9.10	....	......	...
	2	8.06	1.65	3.0	.900	24.20	24.20	2.660	1.630	63.0
	3	9.00	.....	puls'less	.900	9.68	9.68	.....	.....	....
	4	9.00	0.33	0.9	.890	29.36	29.66	3.175	1.780	78.0
	5	9.00	0.67	0.9	.899	28.47	28.48	2.950	1.717	71.7
	6	9.00	1.00	0.9	.894	25.43	25.63	2.660	1.630	63.0
	7	9.00	1.33	0.9	.891	23.03	23.25	2.400	1.550	55.0
	8	9.00	1.66	0.9	.906	20.30	20.18	2.090	1.445	44.5
	9	9.00	2.00	0.9	.886	18.63	18.94	1.900	1.378	37.8
	10	9.00	0.33	0.9	.890	29.33	29.62	3.060	1.750	75.0
	11	9.00	1.00	0.9	.909	26.53	26.27	2.720	1.650	65.0
	12	9.00	2.00	0.9	.901	22.70	22.68	2.350	1.535	53.5
	13	24.00	.....	puls'less	.900	9.20	9.20	.....	.....	....
	14	24.00	3.14	0.0	.909	17.13	16.97	1.845	1.360	36.0
	15	24.00	6.28	0.0	.903	12.60	12.56	1.365	1.167	16.7
	16	24.00	7.86	0.0	.899	11.63	11.64	1.266	1.125	12.5
	17	24.00	15.73	0.0	.893	9.75	9.83	1.062	1.031	3.1
	18	36.00	3.54	0.0	.909	16.16	16.00	1.740	1.320	32.0
	19	36.00	10.62	0.0	.876	10.70	11.00	1.196	1.093	9.3
	20	36.00	14.14	0.0	.900	10.50	10.50	1.140	1.068	6.8
	21	36.00	24.75	0.0	.897	9.57	9.60	1.043	1.022	2.2
	22	48.00	.....	puls'less	.900	9.68	9.68	.....	.....	....
	23	48.00	3.00	0.0	.901	15.13	15.12	1.563	1.250	25.0
	24	48.00	7.00	0.0	.895	11.90	11.97	1.236	1.110	11.0
	25	48.00	17.00	0.0	.900	9.97	9.97	1.030	1.015	1.4
	26	48.00	27.00	0.0	.880	9.63	9.85	1.015	1.007	0.7
	27	48.00	36.00	0.0	.898	9.83	9.85	1.0150	1.007	0.7

NOTE: — Puls'less = pulsationless.

TABLE 23 PER CENT ERROR FOR ORIFICE METER (SEE P. 43, PAR. 75)
With 33% Orifice, Pulsations Quieted by Volumes

Date	Run No.	Volume Used		Drop Through Volume	Manometer Readings, Inches of Water			Ratio	√Ratio	Error Per Cent
		Diam., Inches	Volume, Cu. Ft.		At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
8-18-21	1	....	.....	puls'less	0.900	22.00	22.00	....	......	...
	2	24	3.14	0.0	.898	23.90	23.94	1.088	1.043	4.3
	3	24	6.28	0.0	.885	22.60	23.00	1.045	1.022	2.2
	4	24	7.86	0.0	.874	22.53	23.20	1.054	1.027	2.7
	5	24	15.73	0.0	.895	22.80	22.90	1.040	1.020	2.0
	6	36	3.54	0.0	.894	24.10	24.23	1.100	1.049	4.9
	7	36	10.62	0.0	.898	22.80	22.84	1.040	1.020	2.0
	8	36	14.14	0.0	.882	22.40	22.86	1.040	1.020	2.0
	9	36	24.75	0.0	.893	23.00	23.19	1.053	1.026	2.6

With 70% Orifice, Pulsations Quieted by Volumes

Date	Run No.	Diam., Inches	Volume, Cu. Ft.	Drop Through Volume	At Orifice Head	At Orifice Meter	Corrected to 0.9	Ratio	√Ratio	Error Per Cent
8-18-21	1	24	1.53	puls'less	0.900	4.330	4.330	....	.....	....
	2	24	1.53	0.0	.915	11.130	10.930	2.530	1.590	59.0
	3	24	3.14	puls'less	.900	4.250	4.250	....	.....	....
	4	24	3.14	0.0	.899	9.010	9.020	2.125	1.458	45.8
	5	24	6.28	puls'less	.900	4.310	4.310	....	.....	....
	6	24	6.28	0.0	.897	5.220	5.230	1.214	1.100	10.0
	7	24	7.86	puls'less	.900	4.290	4.290	....	.....	....
	8	24	7.86	0.0	.887	4.590	4.760	1.086	1.043	4.3
	9	24	15.73	0.0	.913	4.780	4.710	1.096	1.048	4.8
	10	36	3.54	puls'less	.900	4.330	4.330	....	.....	....
	11	36	3.54	0.0	.887	8.716	8.840	1.880	1.372	37.2
	12	36	10.62	puls'less	.900	4.290	4.290	....	.....	....
	13	36	10.62	0.0	.878	5.020	5.150	1.200	1.096	9.6
	14	36	14.14	puls'less	.900	4.330	4.330	....	.....	....
	15	36	14.14	0.0	.896	5.780	5.800	1.340	1.160	16.0
	16	36	24.75	0.0	.914	4.570	4.500	1.040	1.020	2.0

TABLE 24 PER CENT ERROR FOR VENTURI METER, PULSATIONS QUIETED BY VOLUMES WITH ORIFICES (SEE P. 43, PAR. 78)

Date	Run No.	Volume Used		Drop Through Volume, Inches, Mercury	Manometer Readings, Inches of Water			Ratio	√Ratio	Error Per Cent
		Diam., Inches	Volume, Cu. Ft.		At Orifice Head	At Venturi Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
7-30-21	1	9.0	.....	puls'less	0.900	9.68	9.68	....	......	...
	2*	9.0	0.33	1.70	.887	16.75	17.00	1.757	1.322	32.2
	3*	9.0	0.33	2.80	.886	14.20	14.59	1.508	1.228	22.8
	4	9.0	0.33	3.30	.907	11.70	11.62	1.201	1.097	9.7
	5*	9.0	0.33	4.00	.892	12.56	12.68	1.310	1.141	14.1
	6	9.0	0.33	5.40	.893	10.30	10.39	1.074	1.035	3.5
	7*	9.0	0.33	6.60	.907	11.20	11.12	1.150	1.071	7.1
	8	9.0	0.33	8.50	.911	10.03	9.95	1.025	1.012	1.2
	9*	9.0	0.33	11.50	.910	10.63	10.53	1.089	1.045	4.5
	10	9.0	0.33	12.60	.888	9.60	9.73	1.005	1.003	0.3
	11*	9.0	1.00	1.90	.875	13.33	13.70	1.417	1.189	18.9
	12*	9.0	1.00	3.10	.913	12.40	12.23	1.264	1.124	12.4
	13	9.0	1.00	3.70	.915	10.50	10.33	1.068	1.032	3.2
	14*	9.0	1.00	4.50	.910	11.03	11.90	1.127	1.061	6.1
	15	9.0	1.00	5.80	.888	9.60	9.72	1.004	1.002	0.2
	16*	9.0	1.00	7.10	.960	10.90	10.22	1.057	1.027	2.7
	17	9.0	1.00	8.40	.893	9.57	9.64	0.996	0.998	−0.2
	18*	9.0	1.00	11.20	.917	10.40	10.21	1.055	1.026	2.6
	19	9.0	1.00	12.90	.856	9.20	9.60	1.000	1.000	0.0
	20*	9.0	2.00	2.10	.921	12.00	11.73	1.212	1.100	10.0
	21*	9.0	2.00	3.00	.894	10.53	10.60	1.096	1.045	4.5
	22	9.0	2.00	3.70	.930	10.40	10.07	1.041	1.020	2.0
	23*	9.0	2.00	4.60	.911	10.20	10.08	1.042	1.020	2.0
	24	9.0	2.00	6.00	.940	10.20	9.77	1.010	1.004	0.4
	25*	9.0	2.00	7.20	.959	10.57	9.92	1.024	1.011	1.1
	26	9.0	2.00	8.20	.911	9.80	9.69	1.001	1.001	0.1
	27*	9.0	2.00	11.40	.911	9.90	9.79	1.011	1.005	0.5
	28	9.0	2.00	12.60	.895	9.50	9.56	0.988	0.994	−0.6
	29	24.0	.....	puls'less	.900	9.20	9.20	....	......	...
	30	24.0	1.57	3.50	.882	9.55	9.74	1.058	1.028	2.8
	31	24.0	1.57	5.50	.881	9.30	9.49	1.032	1.016	1.6
8-15-21	32	24.0	1.57	7.60	.881	9.37	9.57	1.040	1.020	2.0
	33	24.0	3.14	3.63	.884	9.27	9.13	1.025	1.012	1.2
	34	24.0	3.14	5.52	.912	9.34	9.23	1.003	1.002	0.2
	35*	24.0	6.28	2.59	.905	9.87	9.82	1.072	1.035	3.5
	36	24.0	6.28	3.75	.889	9.03	9.14	0.993	0.997	−0.3
	37	24.0	6.28	5.65	.891	9.15	9.25	1.005	1.003	0.3
	38	24.0	7.86	2.00	.903	9.50	9.47	1.030	1.015	1.5
	39	24.0	7.86	2.60	.886	9.20	9.35	1.015	1.007	0.7
	40	24.0	15.73	2.07	.894	9.23	9.29	1.010	1.005	0.5
	41	36.0	3.54	2.50	.908	9.70	9.62	1.046	1.022	2.2
	42	36.0	3.54	2.93	.916	9.57	9.41	1.026	1.013	1.3
	43	36.0	3.54	3.67	.912	9.20	9.08	0.988	0.993	−0.7
	44	36.0	3.54	5.34	.916	9.37	9.21	1.000	1.000	0.0
	45	36.0	3.54	5.94	.879	8.98	9.19	1.000	1.000	0.0
	46	36.0	10.62	2.53	.884	9.17	9.33	1.014	1.007	0.7
	47	36.0	10.62	2.94	.901	9.24	9.20	1.000	1.000	0.0
	48	36.0	10.62	3.79	.899	9.13	9.11	0.994	0.997	−0.3
8-20-21	49	36.0	10.62	5.18	.884	9.03	9.19	1.000	1.000	0.0
	50	36.0	10.62	5.59	.885	9.08	9.23	1.003	1.002	0.2
	51	36.0	14.14	2.48	.913	9.20	9.07	0.987	0.992	−0.8
	52	36.0	14.14	3.69	.890	8.90	9.00	0.978	0.988	−1.2
	53	36.0	24.75	1.87	.900	9.30	9.30	1.011	1.005	0.5
	54	36.0	24.75	2.54	.900	9.37	9.37	1.018	1.009	0.9
	55	36.0	24.75	3.62	.884	9.10	9.27	1.007	1.003	0.3
2-8-22	56	8.06	.....	puls'less	.900	9.10	9.10	....	......	...
	57	8.06	1.65	1.30	.900	12.83	12.83	1.432	1.204	20.4
	58	8.06	1.65	1.20	.900	13.00	12.90	1.480	1.217	21.7

* Throttled by one orifice at entrance to volume.

TABLE 25 PER CENT ERROR FOR ORIFICE METER (SEE P. 43, PAR. 78)
With 33% Orifice, Pulsations Quieted by Volumes With Orifices

Date	Run No.	Volume Used		Drop Through Volume, Inches, Mercury	Manometer Readings, Inches of Water			Ratio	√Ratio	Error Per Cent
		Diam., Inches	Volume, Cu. Ft.		At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
8–19–21	1	....	.....	puls'less	0.900	22.00	22.00	....	......	...
	2	24	1.53	3.31	.891	22.50	22.70	1.033	1.016	1.6
	3	24	1.53	5.74	.892	22.30	22.50	1.023	1.011	1.1
	4	24	1.53	7.60	.909	23.15	22.90	1.040	1.020	2.0
	5	24	3.14	3.49	.902	21.93	21.90	0.994	0.997	−0.3
	6	24	3.14	5.52	.910	23.02	22.80	1.037	1.018	1.8
	7	24	3.14	5.62	.908	23.00	22.81	1.037	1.018	1.8
	8	24	3.14	7.48	.898	22.50	22.53	1.023	1.011	1.1
	9*	24	6.28	2.63	.904	22.50	22.40	1.018	1.009	0.9
	10	24	6.28	3.68	.882	21.60	22.10	1.004	1.002	0.2
	11	24	6.28	5.65	.894	21.97	22.10	1.004	1.002	0.2
	12	24	7 86	1.97	.904	22.72	22.63	1.030	1.015	1.5
	13	24	7.86	2.53	.891	22.55	22.79	1.035	1.017	1.7
	14	24	15.73	2.07	.892	22.38	22.60	1.027	1.013	1.3
	15	36	3.54	2.50	.907	22.30	22.08	1.003	1.002	0.2
	16	36	3.54	2.93	.880	21.17	21.63	0.983	0.992	−0.8
	17	36	3.54	3.67	.887	22.06	22.41	1.020	1.010	1.0
	18	36	3.54	5.34	.915	22.28	21.92	0.996	0.998	−0.2
	19	36	3.54	5.94	.902	21.90	21.86	0.994	0.996	−0.4
	20	36	10.62	2.53	.893	21.60	21.75	0.988	0.994	−0.6
	21	36	10.62	2.94	.896	21.95	22.06	1.002	1.001	0.1
	22	36	10.62	3.79	.896	22.25	22.36	1.015	1.007	0.7
	23	36	10.62	5.18	.885	21.50	21.86	0.994	0.996	−0.4
	24	36	10.62	5.59	.914	22.15	21.95	0.998	0.999	−0.1
	25	36	14.14	2.48	.900	22.40	22.40	1.018	1.009	0.9
	26	36	14.14	3.69	.896	22.20	22.30	1.013	1.006	0.6
	27	36	24.75	1.87	.905	22.83	22.75	1.033	1.016	1.6
	28	36	24.75	2.54	.887	22.35	22.68	1.031	1.015	1.5
	29	36	24.75	3.62	.888	22.30	22.64	1.030	1.015	1.5

Note: — Puls'less = pulsationless.

* Throttled by one orifice at entrance to volume.

WITH 70% ORIFICE, PULSATIONS QUIETED BY VOLUMES WITH ORIFICES

TABLE 25 PER CENT ERROR FOR ORIFICE METER — *Continued*

Date	Run No.	Volume Used		Drop Through Volume, Inches, Mercury	Manometer Readings, Inches of Water			Ratio	√ Ratio	Error Per Cent
		Diam., Ins.	Volume, Cu. Ft.		At Orifice Head	At Orifice Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
8-19-21	1	....	.....	puls'less	0.900	4.33	4.33	....	......	...
	2	24	1.53	3.32	.883	5.09	5.18	1.200	1.094	9.4
	3	24	1.53	5.46	.892	4.61	4.66	1.077	1.038	3.8
	4	24	1.53	7.64	.911	4.78	4.73	1.093	1.046	4.6
	5	....	.....	puls'less	.900	4.25	4.25	....	......	...
	6	24	3.14	3.64	.900	4.40	4.40	1.036	1.018	1.8
	7	24	3.14	5.65	.901	4.28	4.28	1.010	1.005	0.5
	8	24	3.14	7.78	.906	4.27	4.24	1.000	1.000	0.0
	9	....	.....	puls'less	.900	4.31	4.31	....	......	...
	10*	24	6.28	2.78	.886	4.56	4.63	1.076	1.038	3.8
	11	24	6.28	3.82	.892	4.47	4.51	1.047	1.023	2.3
	12	24	6.28	5.82	.885	4.41	4.48	1.041	1.020	2.0
	13	....	.....	puls'less	.900	4.25	4.25	....	......	...
	14	24	7.86	1.97	.876	4.23	4.35	1.023	1.011	1.1
	15	24	7.86	2.61	.910	4.32	4.27	1.004	1.002	0.2
	16	....	.....	puls'less	.900	4.30	4.30	....	......	...
	17	24	15.73	2.07	.906	4.22	4.20	0.977	0.988	−1.2
	18	....	.....	puls'less	.900	4.33	4.33	....	......	...
	19	36	3.54	2.55	.898	4.93	4.94	1.140	1.068	6.8
	20	36	3.54	2.95	.909	4.88	4.83	1.115	1.057	5.7
	21	36	3.54	3.74	.885	4.44	4.51	1.040	1.020	2.0
	22	36	3.54	5.38	.888	4.42	4.48	1.033	1.016	1.6
	23	36	3.54	5.79	.904	4.50	4.48	1.033	1.016	1.6
	24	....	.....	puls'less	.900	4.34	4.34	....	......	...
	25	36	10.62	2.60	.892	5.10	5.14	1.183	1.088	8.8
	26	36	10.62	3.01	.880	5.05	5.16	1.190	1.090	9.0
	27	36	10.62	3.60	.896	5.04	5.06	1.166	1.080	8.0
	28	36	10.62	5.17	.916	5.22	5.14	1.183	1.088	8.8
	29	36	10.62	5.64	.886	5.06	5.13	1.181	1.087	8.7
	30	....	.....	puls'less	.900	4.33	4.33	....	......	...
	31	36	14.14	2.50	.915	4.52	4.45	1.028	1.014	1.4
	32	36	14.14	3.71	.905	4.43	4.41	1.017	1.008	0.8
	33	36	24.75	1.83	.902	4.46	4.45	1.028	1.014	1.4
	34	36	24.75	2.53	.905	4.55	4.53	1.043	1.022	2.2
	35	36	24.75	3.64	.913	4.58	4.52	1.041	1.020	2.0

* Throttled by one orifice at entrance to volume.

TABLE 26 EFFECT OF MUFFLERS FOR QUIETING MAXIMUM PULSATIONS WITH VENTURI METER (SEE P. 44, PAR. 80)

Date	Run No.	Kind of Muffler and Location	Drop Through Muffler, Inches, Mercury	Kind of Flow	Manometer Readings, Inches of Water			Ratio	$\sqrt{\text{Ratio}}$	Error Per Cent
					At Orifice Head	At Venturi Meter	Corrected to 0.9			
		1	2	3	4	5	6	7	8	9
7-21-21	1	57 ft. from comp., 15 ft. from meter..	....	puls'less	0.900	9.35	9.35	.....	.....	....
	2	Powell auto muffler..	0.10	pulsating	.908	21.70	21.50	2.300	1.516	51.6
	3	Judd's funnel muffler	0.25	pulsating	.899	8.92	8.94	0.957	0.979	−2.1
	4	....	....	puls'less	.900	9.68	9.68	.....	.....	....
	5	New muffler (direct)	6.40	pulsating	.892	10.38	10.47	1.081	1.039	3.9
	6	New muffler (reversed)	9.70	pulsating	.890	9.93	10.18	1.051	1.024	2.4
	7	New muffler (direct)	9.90	pulsating	.920	10.63	10.41	1.078	1.038	3.8
	8	43 ft. from comp., 27 ft. from meter..	....	puls'less	.900	9.10	9.10	.....	.....	....
	9	8 sect. muffler only..	0.25	pulsating	.906	24.20	24.05	2.645	1.623	62.3
	10	8 sect. muffler with special orifices....	2.00	pulsating	.905	12.20	12.12	1.350	1.160	16.0
	11	8 sect. muffler with special orifices....	3.70	pulsating	.900	10.60	10.60	1.165	1.080	8.0
	12	8 sect. muffler with special orifices....	4.90	pulsating	.901	9.73	9.72	1.070	1.034	3.4
	13	8 sect. muffler with special orifices....	4.70	pulsating	.911	9.80	9.70	1.067	1.032	3.2
	14	8 sect. muffler with special orifices....	4.70	pulsating	.910	9.50	9.40	1.030	1.015	1.5
	15	8 sect. muffler with special orifices....	3.20	pulsating	.920	10.20	10.00	1.100	1.050	5.0
	16	8 sect. muffler with special orifices....	2.10	pulsating	.914	11.40	11.24	1.236	1.110	11.0
	17	8 sect. muffler with special orifices....	2.30	pulsating	.905	11.50	11.45	1.260	1.123	12.3
	18	8 sect. muffler with special orifices....	1.20	pulsating	.908	13.00	12.90	1.420	1.193	19.3
	19	Muffler out.........	0.00	pulsating	.900	31.47	31.47	3.490	1.870	87.0

TABLE 27 EFFECT OF PULSATIONS ON FLOW-METER IN "DEAD-END" LINE
(SEE P. 46, PAR. 85)

Date	Run No.	Line Closed at Point:	Kind of Flow	Static Pressure in Line, Inches, Water	Manometer Readings, Inches of Water		Ratio	$\sqrt{\text{Ratio}}$	Error Per Cent	Equivalent Flow Based on Puls'less Flow, Per Cent
					At Orifice Head	At Meter				
		1	2	3	4	5	6	7	8	9
				VENTURI METER						
6–29–22	1		puls'less	9.96	0.900	9.29	....	......	...	.....
	2		pulsating	13.40	.900	30.15	3.250	1.850	85.0	.....
	3	8 ft. below meter	"dead-end"	3.50	....	1.91	0.206	0.454	...	45.4
	4		"dead-end"	7.90	....	2.29	0.248	0.498	...	49.8
	5		"dead-end"	13.40	....	2.81	0.303	0.552	...	55.2
	6		"dead-end"	20.70	....	3.25	0.350	0.592	...	59.2
	7		"dead-end"	40.80	....	4.06	0.437	0.661	...	66.1
6–29–22	8	18 ft. below meter	"dead-end"	3.4	....	3.85	0.414	0.644	...	64.4
	9		"dead-end"	5.1	....	7.87	0.847	0.920	...	92.0
	10		"dead-end"	8.0	....	12.60	1.358	1.162	...	116.2
	11		"dead-end"	10.9	....	14.83	1.596	1.262	...	126.2
	12		"dead-end"	13.4	....	16.10	1.732	1.315	...	131.5
	13		"dead-end"	17.4	....	17.63	1.900	1.378	...	137.8
	14		"dead-end"	20.5	....	18.26	1.960	1.402	...	140.2
	15		"dead-end"	40.8	....	20.43	2.205	1.484	...	148.4
	16	Without vol.	puls'less	9.96	.900	9.29	.....	......	...	....
	17	8 ft. below	"dead-end"	13.50	....	2.47	0.267	0.516	...	51.6
	18	18 ft. below	"dead-end"	13.50	....	8.52	0.916	0.957	...	95.7
	19	40 ft. below	"dead-end"	13.40	....	15.33	1.650	1.283	...	128.3
	20	45 ft. below	"dead-end"	13.40	....	10.00	1.077	1.038	...	103.8
				33% ORIFICE METER						
6–27–22	21	....	puls'less	27.7	.900	22.00	....	......	...	.....
	22	....	pulsating	32.0	.900	26.35	1.196	1.092	9.2	.....
	23	8 ft. below	"dead-end"	.....	....	0.86	0.039	0.198	...	19.8
	24	18 ft. below	"dead-end"	.....	....	6.03	0.274	0.522	...	52.2
	25	45 ft. below	"dead-end"	.....	....	6.20	0.281	0.530	...	53.0
	26	8 ft. below	"dead-end"	.....	....	−0.10	0.0045	0.021	...	−2.1
	27	18 ft. below	"dead-end"	.....	....	0.69	0.031	0.176	...	17.6
	28	45 ft. below	"dead-end"	.....	....	1.18	0.054	0.233	...	23.3

TABLE 27 — *Continued*

Date	Run No.	Line Closed at Point	Kind of Flow	Static Pressure in Line, Inches, Water	Manometer Readings, Inches of Water		Ratio	√Ratio	Error Per Cent	Equivalent Flow Based on Puls'less Flow, Per Cent
					At Orifice Head	At Meter				
		1	2	3	4	5	6	7	8	9
					70% ORIFICE METER					
6–28–22	29		puls'less	9.8	0.900	4.95	.....	.....	...	.....
	30	8 ft. below meter	pulsating	11.0	.900	25.75	5.200	2.280	128.0	.....
	31		"dead-end"	7.9	....	3.30	0.667	0.804	...	80.4
	32		"dead-end"	11.3	....	3.19	0.645	0.816	...	81.6
6–28–22	33	18 ft. below meter	"dead-end"	3.63	....	1.13	0.230	0.480	...	48.0
	34		"dead-end"	5.80	....	3.11	0.628	0.783	...	78.3
	35		"dead-end"	9.15	....	5.98	1.210	1.100	...	110.0
	36		"dead end"	11.10	....	6.81	1.375	1.172	...	117.2
	37		"dead-end"	17.30	....	9.33	1.865	1.366	...	136.6
	38		"dead-end"	23.10	....	10.97	2.215	1.487	...	148.7
	39		"dead-end"	40.80	....	12.30	2.480	1.575	...	157.5
8–11–22	40	Without volume	puls'less	9.0	.900	4.91	....	.....	...	.....
	41		pulsating	11.7	.900	25.45	5.190	2.280	128.0	.....
	42	8 ft. below	"dead-end"	11.7	....	4.22	0.860	0.930	...	93.0
	43	18 ft. below	"dead-end"	11.7	....	−0.79	0.161	0.400	...	−40.0
	44	40 ft. below	"dead-end"	11.7	....	8.31	1.690	1.300	...	13.0
	45	45 ft. below	"dead-end"	11.7	....	8.18	1.670	1.290	...	129.0
	46	With volume 18 ft. below	"dead-end"	11.7	....	6.66	1.357	1.160	...	116.0
	47	45 ft. below	"dead-end"	11.7	....	6.57	1.339	1.156	...	115.6
					80% ORIFICE METER					
6–28–22	48	....	puls'less	9.4	0.900	2.19	....	.....	...	.....
	49	....	pulsating	10.8	.900	15.54	7.010	2.655	165.5	.....
	50	8 ft. below	"dead-end"	10.8	....	3.17	1.440	1.200	...	120.0
	51	18 ft. below	"dead-end"	10.8	....	4.62	2.100	1.448	...	144.8
					90% ORIFICE METER					
6–29–22	52		puls'less	8.9	0.900	0.458	....	.....	...	.....
	53		pulsating	9.6	.900	5.740	12.53	3.543	254.3	.....
	54	8 ft. below meter	"dead-end"	3.8	....	0.990	2.16	1.470	...	147.0
	55		"dead-end"	6.6	....	2.550	5.56	2.310	...	231.0
	56		"dead-end"	9.6	....	2.550	5.56	2.310	...	231.0
	57		"dead-end"	12.3	....	2.860	6.24	2.500	...	250.0
	58		"dead-end"	17.4	....	3.110	6.78	2.600	...	260.0
	59		"dead-end"	23.1	....	3.210	7.00	2.640	...	264.0
	60		"dead-end"	40.8	....	4.510	9.82	3.123	...	312.3
	61	18 ft. below meter	"dead-end"	3.8	....	0.54	1.18	1.086	...	108.6
	62		"dead-end"	6.6	....	2.62	4.72	2.172	...	217.2
	63		"dead-end"	9.5	....	3.03	6.61	2.570	...	257.0
	64		"dead-end"	12.4	....	3.11	6.78	2.600	...	260.0
	65		"dead-end"	17.2	....	3.33	7.26	2.690	...	269.0
	66		"dead-end"	23.1	....	3.26	7.11	2.660	...	266.0
	67		"dead-end"	40.8	....	4.28	9.26	3.100	...	310.0

NOTE: — For runs 29 to 67 the inclined manometer was used. To reduce Column 5 to inches of water, multiply by 0.17.

TABLE 28 EFFECT OF PULSATIONS ON FLOW-METERS IN "DEAD-END" LINE
(SEE P. 46, PAR. 85)

Date	Run No.	Line Closed at Point:	Kind of Flow	Static Pressure in Line, Inches, Water	Manometer Readings, Inches of Water		Ratio	√Ratio	Error Per Cent	Equivalent Flow Based on Puls'less Flow, Per Cent
					At Orifice Head	At Meter				
		1	2	3	4	5	6	7	8	9
				3-Inch by 1⅛-Inch Flange Nozzle-Meter						
6–27–22	1	....	puls'less	13.0	0.900	5.59	....	.....	...	.....
	2	....	pulsating	16.3	.900	11.73	2.100	1.450	45.0	.....
	3	8 ft. below meter...	"dead-end"	16.3	....	0.70	0.125	0.354	...	35.4
	4	18 ft. below	"dead-end"	16.3	....	2.33	0.416	0.645	...	64.5
	5	45 ft. below	"dead-end"	16.3	....	2.74	0.490	0.700	...	70.0
				3-Inch by 1½-Inch Flange Nozzle-Meter						
6–27–22	6	....	puls'less	10.0	0.900	1.636	....	.....	...	.....
	7	....	pulsating	11.4	.900	6.160	3.770	1.942	94.2	.....
	8	8 ft. below meter...	"dead-end"	11.5	....	0.410	0.250	0.500	...	50.0
	9	18 ft. below	"dead-end"	11.5	....	0.923	0.564	0.752	...	75.2
	10	45 ft. below	"dead-end"	11.5	....	1.191	0.728	0.853	...	85.3
8–10–22	11	....	puls'less	9.8	.900	1.690	....	.....	...	.....
	12	....	pulsating	11.5	.900	6.100	3.610	1.900	90.0	.....
	13	8 ft. below meter...	"dead-end"	11.8	....	0.386	0.228	0.478	...	47.8
	14	18 ft. below with vols.	"dead-end"	11.8	....	1.108	0.655	0.810	...	81.0
	15	45 ft. below with vols.	"dead-end"	4.2	....	0.366	0.216	0.465	...	46.5
	16	45 ft. below with vols.	"dead-end"	7.1	....	0.597	0.353	0.594	...	59.4
	17	45 ft. below with vols.	"dead-end"	9.0	....	0.790	0.467	0.683	...	68.3
	18	45 ft. below with vols.	"dead-end"	11.8	....	1.097	0.648	0.805	...	80.5
	19	45 ft. below with vols.	"dead-end"	15.0	....	1.321	0.775	0.880	...	88.0
	20	45 ft. below with vols.	"dead-end"	18.0	....	1.450	0.858	0.926	...	92.6
				3-Inch by 2-Inch Flange Nozzle-Meter						
6–26–22	21		puls'less	9.3	0.900	2.767	...	.....	...	.....
	22		pulsating	9.6	.900	16.100	5.920	2.432	143.2	.....
	23	8 ft. below meter	"dead-end"	0.5	....	3.510	1.265	1.125	...	112.5
	24		"dead-end"	0.8	...	6.867	2.470	1.570	...	157.0
	25		"dead-end"	9.6	....	3.640	1.310	1.175	...	117.5
	26		"dead-end"	1.5	...	6.910	2.490	1.580	...	158.0
6–26–22	27	18 ft. below meter	"dead-end"	0.8	...	5.083	1.830	1.353	...	135.3
	28		"dead-end"	9.6	...	3.747	1.368	1.170	...	117.0
6–26–22	29	45 ft. below meter	"dead-end"	6.8	....	5.030	1.820	1.350	...	135.0
	30		"dead-end"	9.6	....	3.717	1.340	1.160	...	116.0

Note: — For runs 21 to 39 inclined manometer was used. To reduce Column 5 to inches of water, multiply by 0.17.

TABLE 28 EFFECT OF PULSATIONS ON FLOW-METERS IN "DEAD-END" LINE (SEE P. 46, PAR. 85) — *Continued*

Date	Run No.	Line Closed at Point:	Kind of Flow	Static Pressure in Line, Inches, Water	Manometer Readings, Inches of Water		Ratio	√Ratio	Error Per Cent	Equivalent Flow Based on Puls'less Flow, Per Cent
					At Orifice Head	At Meter				
		1	2	3	4	5	6	7	8	9
3-Inch by 2½-Inch Flange Nozzle-Meter										
6-27-22	31	....	puls'less	8.7	0.900	0.714	....	.....	...	.....
	32	....	pulsating	9.4	.900	6.400	8.960	2.990	199.0	.....
	33	8 ft. below	"dead-end"	9.6	....	3.510	4.920	2.220	...	222.0
	34	18 ft. below	"dead-end"	9.6	....	2.920	4.080	2.020	...	202.0
	35	45 ft. below	"dead-end"	9.6	....	3.050	4.280	2.070	...	207.0
6-27-22	36	8 ft. below	puls'less	8.5	.900	0.740	....	.....	...	.....
	37	with vols.	"dead-end"	10.0	....	2.740	3.700	1.923	...	192.3
	38	18 ft. below with vols.	"dead-end"	10.0	....	2.99	4.050	2.070	...	207.0
	39	45 ft. below with vols.	"dead-end"	10.0	....	1.30	1.760	1.327	...	132.7

TABLE 29 EFFECT OF PULSATIONS ON FLOW–METERS IN "DEAD–END" LINE (SEE P. 46, PAR. 85)

Date	Run No.	Line Closed at Point:	Kind of Flow	Static Pressure in Line, Inches, Water	Manometer Readings, Inches		Ratio	√Ratio	Error Per Cent	Equivalent Flow Based on Puls'less Flow, Per Cent
					At Orifice Head	At Meter				
		1	2	3	4	5	6	7	8	9
				Pitot Tube No. 1						
6–30–22	1	....	puls'less	8.8	0.900	0.982	....	.....	...	.....
	2	....	pulsating	9.2	.900	6.450	6.570	2.560	156.0	.....
6–30–22	3	8 ft. below meter	"dead-end"	3.2	....	0.660	0.672	0.820	...	82.0
	4			6.1	....	2.040	2.080	1.445	...	144.5
	5			9.0	....	1.910	1.945	1.393	...	139.3
	6			11.8	....	2.400	2.445	1.560	...	156.0
	7			14.3	....	2.720	2.770	1.663	...	166.3
	8			20.1	....	3.140	3.200	1.782	...	178.2
	9			40.8	....	4.680	4.760	2.180	...	218.0
6–30–22	10	18 ft. below meter	"dead-end"	3.7	....	0.45	0.458	0.677	...	67.7
	11			6.1	....	2.37	2.415	1.551	...	155.1
	12			9.1	....	3.30	3.360	1.830	...	183.0
	13			11.8	....	3.61	3.680	1.917	...	191.7
	14			14.5	....	3.88	3.950	1.987	...	198.7
	15			20.5	....	3.99	4.065	2.015	...	201.5
	16			40.8	....	5.26	5.350	2.315	...	231.5
8–9–22	17	....	puls'less	8.6	.900	0.935	....	.....	...	.....
	18	....	pulsating	9.2	.900	6.630	7.100	2.660	166.0	.....
8–9–22	19	8 ft. below meter with volumes	"dead-end"	9.3	....	0.62	0.663	0.815	...	81.5
	20			15.9	....	0.78	0.834	0.914	...	91.4
	21			18.6	....	1.18	1.262	1.123	...	112.3
	22			21.0	....	1.36	1.458	1.208	...	120.8
8–9–22	23	18 ft. below meter with volumes	"dead-end"	4.8	....	2.55	2.725	1.650	...	165.0
	24			7.0	....	2.71	2.900	1.702	...	170.2
	25			9.5	....	3.05	3.260	1.805	...	180.5
	26			12.3	....	3.56	3.810	1.950	...	195.0
	27			15.4	....	4.02	4.300	2.075	...	207.5
	28			20.7	....	4.19	4.480	2.220	...	222.0
8–9–22	29	45 ft. below meter with volumes	"dead-end"	3.1	....	2.36	2.525	1.590	...	159.0
	30			5.0	....	2.40	2.570	1.602	...	160.2
	31			9.3	....	3.28	3.510	1.873	...	187.3
	32			10.0	....	3.47	3.715	1.926	...	192.6
	33			12.7	....	3.98	4.260	2.035	...	203.5
				Pitot Tube No. 2						
8–9–22	34	....	puls'less	8.8	0.900	1.315	....	.....	...	.....
	35	....	pulsating	9.6	.900	5.870	4.460	2.115	111.5	.....
8–9–22	36	8 ft. below meter, with volumes	"dead-end"	6.7	....	−0.56	0.425	0.652	...	−65.2
	37			10.2	....	3.36	2.551	1.600	...	160.0
	38			12.9	....	5.20	3.950	1.990	...	199.0
	39			15.4	....	6.23	4.740	2.178	...	217.8
	40			20.8	....	6.66	5.060	2.250	...	225.0
8–9–22	41	18 ft. below meter, with volumes	"dead-end"	4.9	....	1.99	1.512	1.230	...	123.0
	42			7.0	....	2.13	1.620	1.272	...	127.2
	43			9.8	....	2.30	1.749	1.323	...	132.3
	44			12.0	....	2.47	1.878	1.370	...	137.0
	45			15.1	....	2.66	2.020	1.422	...	142.2
	46			18.0	....	2.71	2.060	1.464	...	146.4
8–9–22	47	45 ft. below meter	"dead-end"	4.8	....	2.03	1.543	1.242	...	124.2
	48			7.0	....	2.15	1.633	1.278	...	127.8
	49			9.8	....	2.41	1.832	1.353	...	135.3
	50			12.0	....	2.54	1.930	1.390	...	139.0

Note: — To reduce Column 5 to inches of water, multiply by 0.17.

TABLE 30 COMPARISON OF MANOMETERS UNDER SAME PULSATING FLOW CONDITIONS (SEE P. 38, PAR. 64)

Date	Run No.	Kind of Mano-meter	Static Pressure in Line, Inches, Water	Manometer Readings, Inches of Water							
				Kind of Flow		Line Closed at Points:					
						8 Ft. Below Meter		18 Ft. Below Meter		45 Ft. Below Meter	
				Manometer Connections							
				Direct	Reversed	Direct	Reversed	Direct	Reversed	Direct	Reversed
		1	2	3	4	5	6	7	8	9	10
1½-Inch Flange Nozzle Meter											
8–10–22	..	.....	....	puls'less	.....	.....	......	....	.....	.....	......
	1	Vertical	9.8	1.527	.....	.....	......	....	.....	.....	......
	2	U-tube	9.8	1.720	.....	.....	......	....	.....	.....	......
	3	Water	9.8	1.600	.....	.....	......	....	.....	.....	......
	4	Mano-	11.8	.....	.....	−0.250	−0.250	....	.....	.....	......
	5	meter	11.8	.....	.....	.....	......	....	.....	.....	0.875
				puls'less							
	6	Inclined	9.8	1.665	.....	.....	......	....	.....	.....	......
	7	U-tube	9.8	1.655	.....	.....	......	....	.....	.....	......
	8	Oil	9.8	1.680	.....	.....	......	....	.....	.....	......
	9	Mano-	11.8	.....	.....	0.468	0.525	....	.....	.....	......
	10	meter	11.8	.....	.....	0.138	0.251	....	.....	.....	......
	11	.....	11.8	.....	.....	.....	......	0.546	negative	.....	......
	12	.....	4.2	.....	.....	.....	......	....	.....	−0.077	−0.120
	13	.....	11.8	.....	.....	.....	......	....	.....	0.544	negative
				puls'less							
	14	.....	9.8	1.665	.....	.....	......	....	.....	.....	......
	15	Inclined	9.8	1.650	.....	.....	......	....	.....	.....	......
	16	One-leg	11.8	.....	.....	0.386	0.327	....	.....	.....	......
	17	Reser-	11.8	.....	.....	0.622	2.665	....	.....	.....	......
	18	voir	11.8	.....	.....	.....	......	1.108	0.114	.....	......
	19	Oil	4.2	.....	.....	.....	......	....	.....	0.366	0.109
	20	Mano-	11.8	.....	.....	.....	......	....	.....	1.117	0.097
	21	meter	11.8	.....	.....	−1.610	1.720	....	.....	.....	......
70% Orifice Meter											
8–11–22	..	.....	....	puls'less	.....	.....	......	....	.....	.....	......
	22	Vertical	9.0	0.713	.....	.....	......	....	.....	.....	......
		U-tube	....	pulsating	pulsating	.....	......	....	.....	.....	......
	23	Water	11.7	3.980	4.160	.....	......	....	.....	.....	......
	24	Mano-	....	.....	.....	0.070	0.320	....	.....	.....	......
	25	meter	....	.....	.....	.....	......	0.460	0.710	.....	......
	26	.....	....	.....	.....	.....	......	....	.....	±0.500	±0.500
				puls'less							
	27	Inclined	9.0	0.848	.....	.....	......	....	.....	.....	......
		U-tube									
		Oil									
		Mano-									
	28	meter	11.7	.....	.....	0.127	−0.006	....	.....	.....	......
				puls'less							
	29	Inclined	9.0	0.835	pulsating	.....	......	....	.....	....	......
	30	One-leg	11.7	.....	4.330	.....	......	....	....	.....	......
	31	Reser-	11.7	.....	.....	0.718	2.495	....	.....	.....	......
	32	voir Oil	....	.....	.....	.....	......	1.132	0.044	.....	......
	33	Mano-									
		meter	....	.....	.....	.....	......	....	.....	1.390	0.266
				puls'less							
	34	Vertical	9.0	0.813	.....	.....	......	....	.....	.....	......
	35	U-tube	11.7	.....	.....	−1.432	1.665	....	.....	.....	......
	36	2-liquid	....	.....	.....	.....	......	....	.....	1.540	−1.147
		Mano-									
		meter									

TABLE 31 SHOWING RATIO OF P_2 TO $(\sqrt{P_1}+\sqrt{D})^2$ FOR FLOW-METERS (SEE P. 47, PAR. 88)

Date	Run, No.	Static Pressure in Line, Inches, Water	Line Closed at Point:	Manometer Readings, Inches of Water			Ratio = $\frac{P_2}{(\sqrt{P_1}+\sqrt{D})^2}$			
				Puls'less Flow P_1	Pulsating Flow P_2	"Dead-end" Flow D	For 8 Ft.	For 18 Ft.	For 40 Ft.	For 45 Ft.
		1	2	3	4	5	6	7	8	9
VENTURI METER										
6-29-22	1	9.96	....	9.29	......	......	....	....	.....	....
	2	13.40	....	......	30.15	......	....	....	.....	....
	3	13.40	8 ft. below meter...	......	......	2.64	1.386	....	.....	....
	4	13.40	18 ft. below	......	......	16.10	....	0.606	.....	....
8-11-22	5*	13.40	....	......	......	8.52	....	0.847	.....	....
	6*	13.40	40 ft. below	......	......	15.33	....	....	0.624	....
	7*	13.40	45 ft. below	......	......	10.00	....	....	.....	0.784
33% ORIFICE METER										
6-27-22	8	11.70	....	22.00	......	......	....	....	.....	....
	9	13.70	....	......	26.35	......	....	....	.....	....
	10	13.70	18 ft. below	......	......	6.03	....	0.508	.....	....
	11	13.70	45 ft. below	......	......	6.20	....	....	.....	0.514
8-11-22	12*	13.70	18 ft. below	......	......	0.69	....	0.864	.....	....
	13*	13.70	45 ft. below	......	......	1.18	....	....	.....	0.790
70% ORIFICE METER										
6-28-22	14	9.80	....	4.93	......	......	....	....	.....	....
	15	11.00	....	......	25.60	......	....	....	.....	....
	16	11.00	8 ft. below	......	......	3.19	1.592	....	.....	....
	17	11.00	18 ft. below	......	......	6.68	....	....	.....	1.100
	18	11.00	45 ft. below	......	......	6.81	.	1.095	.....	....
8-11-22	19	11.00	18 ft. below	......	......	4.22	....	1.400	....	....
	20	11.00	40 ft. below	......	......	8.25	....	....	0.865	....
	21	11.00	45 ft. below	......	......	8.25	...	..	.....	0.865
80% ORIFICE METER										
6-28-22	22	9.40	....	2.19	.	......	..	...	....	....
	23	10.80	....	.	15.54	..	.	....	..	....
	24	10.80	8 ft. below	....	......	3.16	1.664	....	....	....
	25	10.80	18 ft. below		....	4.62	.	1.180	.....	....
90% ORIFICE METER										
6-29-22	26	9.00	...	0.458	...	...	..	...	....	....
	27	9.50	..	...	5.74		..	...	...	....
	28	9.50	8 ft. below	....	...	2.55	1.110		....	....
	29	9.50	18 ft. below	...	....	3.03	...	0.984	...	.

* Volume replaced by 3-inch line.

TABLE 31 SHOWING RATIO OF P_2 TO $(\sqrt{P_1}+\sqrt{D})^2$ FOR FLOW-METERS (SEE P. 47, PAR. 88) — *Continued*

Date	Run, No.	Static Pressure in Line, Inches, Water	Line Closed at Point:	Manometer Readings, Inches of Water			Ratio = $\frac{P_2}{(\sqrt{P_1}+\sqrt{D})^2}$			
				Puls'less Flow P_1	Pulsating Flow P_2	"Dead-end" Flow D	For 8 Ft.	For 18 Ft.	For 40 Ft.	For 45 Ft.
		1	2	3	4	5	6	7	8	9
			1⅛-INCH FLANGE NOZZLE METER							
6–27–22	30	13.00	....	5.59	......	......	....	....	.....	....
	31	16.30	....	......	11.73	......	....	....	.....	....
	32	16.30	8 ft. below	......	......	0.70	1.214	....	.....	....
	33	16.30	18 ft. below	......	......	2.33	....	0.813	.....	....
	34	16.30	45 ft. below	......	......	2.74	....	....	.....	0.760
			1½-INCH FLANGE NOZZLE METER							
6–27–22	35	10.0	....	1.663	......	......	....	....	.....	....
	36	11.4	....	......	6.130	......	....	....	.....	....
	37	11.4	8 ft. below	......	......	0.386	1.680	....	.....	....
8–10–22	38	11.4	8 ft. below	......	......	0.410	1.645	....	.....	....
	39	11.4	18 ft. below	......	......	0.923	....	1.210	.....	....
	40	11.4	18 ft. below	......	......	0.108	....	1.117	....	....
	41	11.4	45 ft. below	......	......	1.191	....	....	.....	1.090
	42	11.4	45 ft. below	......	......	1.097	....	....	.....	1.120
			2-INCH FLANGE NOZZLE METER							
6–26–22	43	9.3	....	2.767	......	......	....	....	.....	....
	44	9.6	....	......	16.10	......	....	....	.....	....
	45	9.6	8 ft. below	......	......	3.640	1.260	....	.....	....
	46	9.6	18 ft. below	......	......	3.750	....	1.240	.....	....
	47	9.6	45 ft. below	......	......	3.717	....	....	.....	1.245
			2½-INCH FLANGE NOZZLE METER							
6–27–22	48	8.4	....	0.727	......	......	....	....	.....	....
	49	9.6	....	......	6.40	......	....	....	.....	....
	50	9.6	8 ft. below	......	......	3.51	0.862	....	.....	....
	51	9.6	18 ft. below	......	......	2.92	....	0.975	.....	....
	52	9.6	45 ft. below	......	......	3.05	....	....	.....	0.947
8–11–22	53	9.6	18 ft. below	......	......	2.74	1.010	....	.....	....
	54	9.6	45 ft. below	......	......	2.99	....	0.960	....	....
			PITOT TUBE NO. 1							
6–30–22	55	8.9	....	0.982	......	......	....	....	.....	....
	56	9.2	....	......	6.45	......	....	....	.....	....
	57	9.2	8 ft. below	......	......	1.91	1.141	....	.....	....
	58	9.2	18 ft. below	......	......	3.30	....	0.820	.....	....
8–9–22	59	8.9	....	0.935	......	......	....	....	.....	....
	60	9.2	....	......	6.30	......	....	....	.....	....
	61	9.2	8 ft. below	......	......	0.62	2.040	....	.....	....
	62	9.2	18 ft. below	......	......	3.05	....	0.863	.....	....
	63	9.2	45 ft. below	......	......	3.28	....	....	.....	0.852
			PITOT TUBE NO. 2							
8–9–22	64	8.7	....	1.315	......	......	....	....	.....	....
	65	9.8	....	......	5.87	......	....	....	.....	....
	66	9.8	8 ft. below	......	......	3.36	0.661	....	.....	....
	67	9.8	18 ft. below	......	......	2.30	....	0.827	.....	....
	68	9.8	45 ft. below	......	......	2.41	....	....	.....	0.808

TABLE 32 (SEE P. 43, PAR. 75). QUIETING EFFECT OF VOLUMES FOR VENTURI AND ORIFICE METERS

Capacity of Volumes, Cu. Ft.					
0	5	10	15	20	25
Error, Per Cent (From Curve, Fig. 25)					
90	22	8	4	2	1

TABLE 33 (SEE P. 47, PAR. 86) APPARENT FLOW WHEN METERS ARE IN "DEAD-END" LINE UNDER STANDARD LINE PRESSURE CONDITIONS

Meter Used	Point of Line Closure Below Meter, Feet		
	8	18	45
	Apparent Flow Based on Pulsationless Flow, Per Cent		
Venturi	58	131	. . .
33% Orifice	19	. . .	. . .
70% Orifice	81	119	. . .
80% Orifice	120	145	. . .
90% Orifice	239	239	. . .
$1\frac{1}{2}''$ Flange Nozzle	49	81	. . .
Pitot No. 1	140	181	181
Pitot No. 2	50	136	136

APPENDIX D, CURVES AND ILLUSTRATIONS

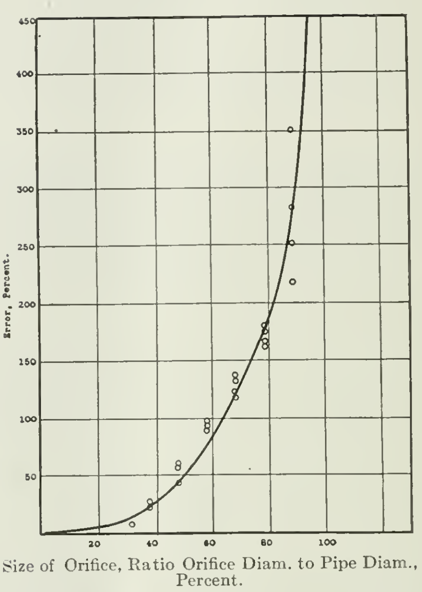


Fig. 33. Maximum Error for Orifice Meter. (See Table 16.)

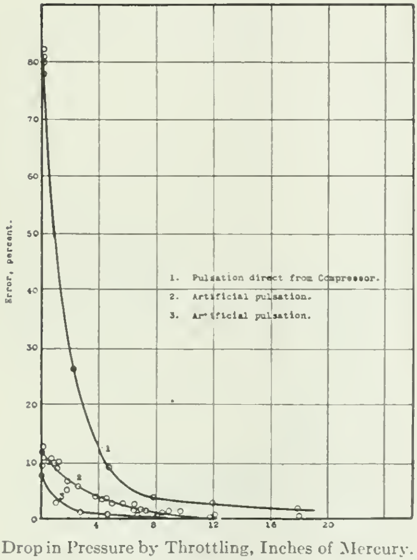


FIG. 34. PERCENT ERROR FOR VENTURI METER, PULSATIONS QUIETED BY THROTTLING. (SEE TABLE 17.)

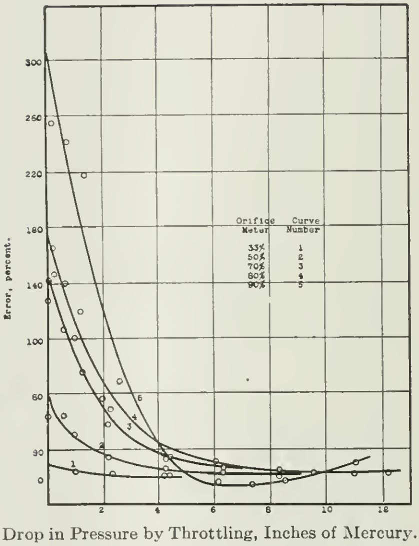
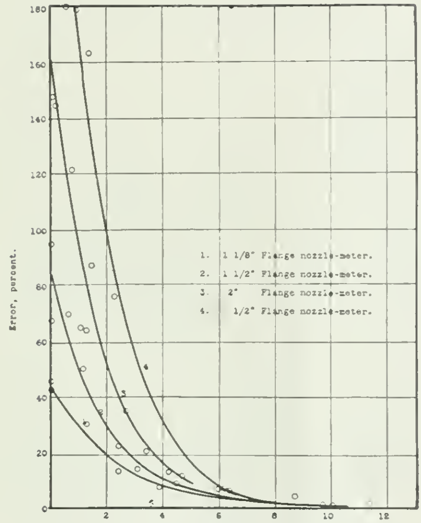
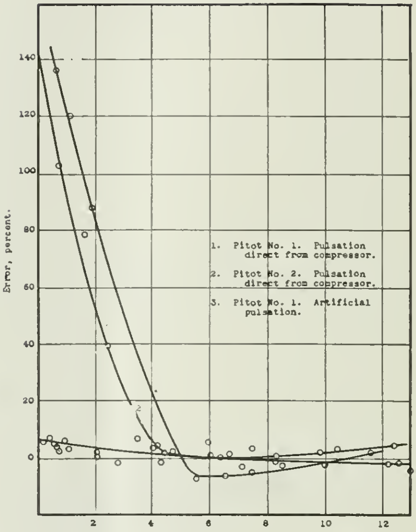


Fig. 35. Percent Error for Orifice Meters; Pulsation Quieted by Throttling. (See Table 18.)



Drop in Pressure by Throttling, Inches of Mercury.

Fig. 36. Percent Error for Flange-nozzle Meter. Pulsations Quieted by Throttling. (See Table 19.)



Drop in Pressure by Throttling, Inches of Mercury.

FIG. 37. PERCENT ERROR FOR PITOT METER. PULSATIONS QUIETED BY THROTTLING. (SEE TABLE 20.)

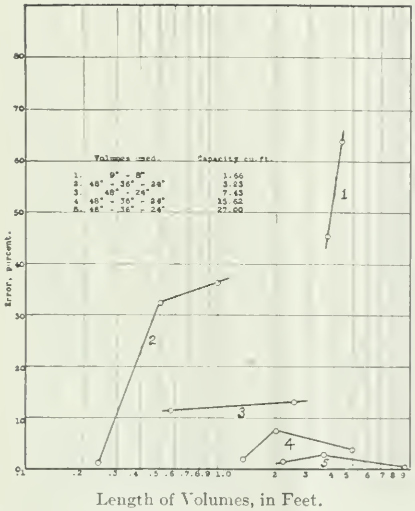


Fig. 38. Effect of Varying the Shape of the Quieting Volume. (See Page 43, Paragraph 77.)

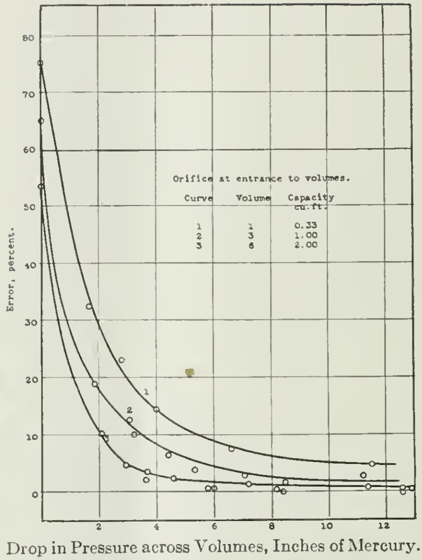
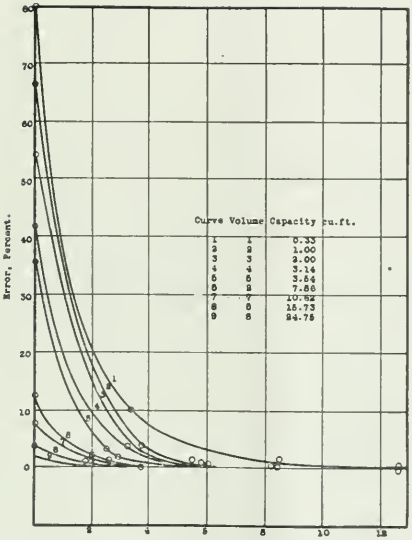
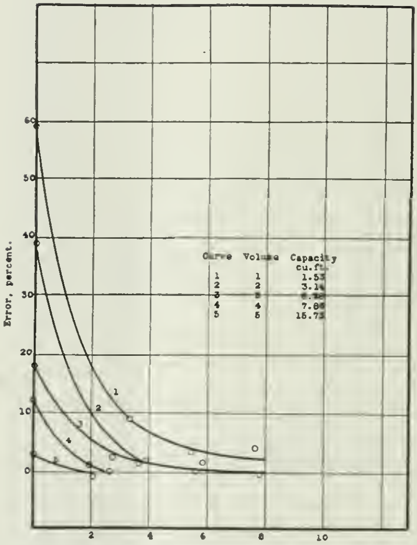


FIG. 39. EFFECT OF THROTTLING BY ORIFICES COMBINED WITH VOLUMES, FOR VENTURI METER. (SEE TABLE 24.)



Drop in Pressure across Volumes, Inches of Mercury.

FIG. 40. EFFECT OF THROTTLING BY ORIFICES AT ENTRANCE AND EXIT TO VOLUMES, FOR VENTURI METER. (SEE TABLE 24).



Drop in Pressure across Volumes, Inches of Mercury.

FIG. 41. EFFECT OF THROTTLING BY ORIFICES AT ENTRANCE AND EXIT TO 24-INCH VOLUMES FOR 70% ORIFICE METER. (SEE PAGE 43, PARAGRAPH 78.)

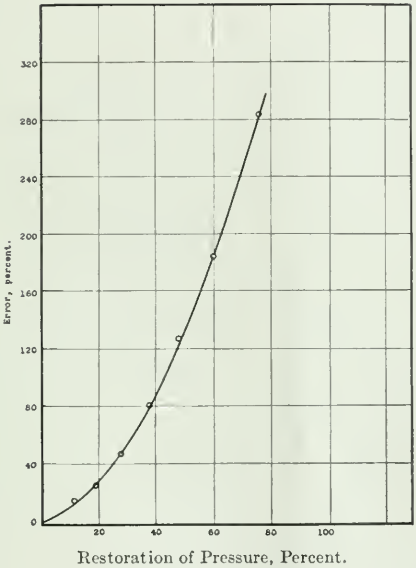


FIG. 42. RELATION OF ERROR DUE TO PULSATING FLOW TO RESTORATION OF PRESSURE BEYOND THE ORIFICE METER. (SEE TABLE 9, PAGE 39.)

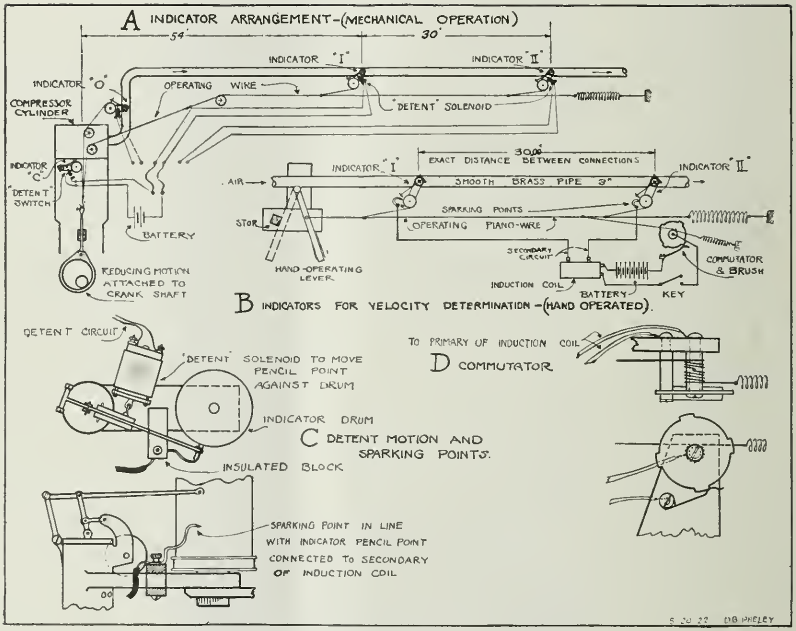

A INDICATOR ARRANGEMENT-(MECHANICAL OPERATION)
54'
30'
INDICATOR "I"
INDICATOR "II"
INDICATOR "O"
OPERATING WIRE
COMPRESSOR CYLINDER
"DETENT" SOLENOID
INDICATOR "C"
"DETENT" SWITCH
BATTERY
REDUCING MOTION ATTACHED TO CRANK SHAFT
30.00'
EXACT DISTANCE BETWEEN CONNECTIONS
INDICATOR "I"
INDICATOR "II"
AIR
SMOOTH BRASS PIPE 3"
SPARKING POINTS
STOP
"OPERATING PIANO-WIRE"
HAND-OPERATING LEVER
SECONDARY CIRCUIT
INDUCTION COIL
BATTERY
KEY
COMMUTATOR & BRUSH
B INDICATORS FOR VELOCITY DETERMINATION-(HAND OPERATED).
DETENT CIRCUIT
"DETENT" SOLENOID TO MOVE PENCIL POINT AGAINST DRUM
INDICATOR DRUM
C DETENT MOTION AND SPARKING POINTS.
INSULATED BLOCK
SPARKING POINT IN LINE WITH INDICATOR PENCIL POINT CONNECTED TO SECONDARY OF INDUCTION COIL
TO PRIMARY OF INDUCTION COIL
D COMMUTATOR
5 20 22 DB PHELEY

FIG. 43. DETAILS OF ARRANGEMENT OF INDICATORS

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TAPERED STRUTS
A THEORETICAL AND EXPERIMENTAL INVESTIGATION

BY
JAMES E. BOYD

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JAMES E. BOYD

PROFESSOR OF MECHANICS

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TAPERED STRUTS—A THEORETICAL AND EXPERIMENTAL INVESTIGATION

1. INTRODUCTION

Tapered struts of various forms have considerable application in engineering work. Solid tapered struts are employed in airplane construction. Connecting rods and other machine parts are frequently tapered. The masts and booms of large derricks are often built of latticed sections which converge from the middle toward each end. The compression members of dirigibles are made in a similar way. In general, whenever it is necessary to lower the weight of a structure or machine without diminishing its strength, it is advisable to consider the use of tapered compression members. It is desirable, therefore, to develop exact formulas for the computation of the strength and deflection of such tapered compression members, and to test the accuracy of these formulas by experimental investigations.

2. ACKNOWLEDGEMENTS

The experimental portion of these investigations was undertaken as a project of the Engineering Experiment Station of The Ohio State University, Dean E. A. Hitchcock, Director. The tests were made on machines of the Department of Mechanical Engineering, which were placed at the disposal of the author through the courtesy of Professor W. T. Magruder. The compression heads and some of the test pieces were made in the University Shops under the supervision of Professor W. A. Knight. Professor E. F. Coddington suggested methods of interpolation and did much of the laborious computation required in the preparation of the tables of Appendix D. Professor O. E. Williams prepared the drawings for the engraver. Professor R. D. Bohannon proposed a simplified method for solving one of the differential equations. To these and to all others who have contributed to the success of these investigations, the author acknowledges his indebtedness and expresses his gratitude.

3. TYPES OF TAPERED STRUTS INVESTIGATED

The types of tapered struts investigated in this paper may be designated in terms of the expression for the moment of inertia of the cross-sections. Figure 1 shows longitudinal sections. For each of these types the origin of co-ordinates is taken at a point O, which is at a distance kd from one end of the strut, and at a distance d from the middle. Any section in the lower half of the strut is at a distance x from the origin. The upper and lower halves are symmetrical with respect to the plane of the cross-section at the middle. The moment of inertia of any cross-section in the lower half may be expressed in terms of x and a constant.

The types investigated to which Fig. 1 applies are.

TYPE 1. Moment of Inertia Varies as the Fourth Power of x; $I=Cx^4$. If the transverse sections are circles and the longitudinal sections, through the axis, of each half of the strut are trapezoids, as is shown in Fig. 1_a, the moments of inertia of all sections of the lower half are proportional to the fourth power of x. The strut is made of two equal frustums of a cone with their common base at the middle

TYPE 2. Moment of Inertia Varies as the Third Power of x; $I=Cx^3$. If the transverse sections are rectangles of constant width perpendicular to the plane of the paper, and the thickness is proportional to x, as shown in Fig. 1_b, the moments of inertia of the cross-sections with respect to axes *perpendicular* to the plane of the paper are proportional to the third power of x. The strut is made of two equal truncated wedges with their common base at the middle.

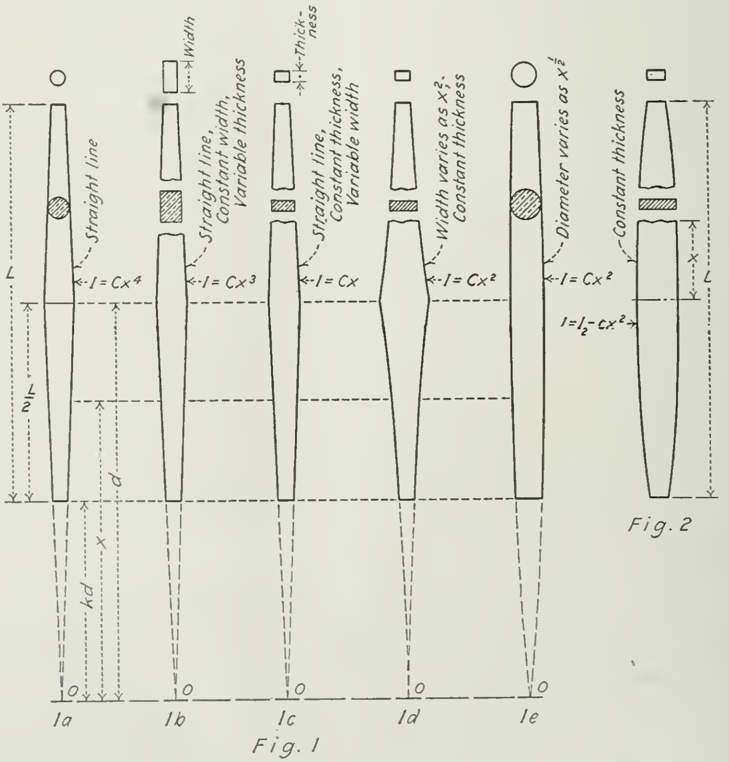
Width
Thick-ness
Straight line
Straight line, Constant width, Variable thickness
Straight line, Constant thickness, Variable width
Width varies as x^2, Constant thickness
Diameter varies as $x^{\frac{1}{2}}$
Constant thickness
$I = Cx^4$
$I = Cx^3$
$I = Cx$
$I = Cx^2$
$I = Cx^2$
$I = I_2 - cx^2$
L
$\frac{L}{2}$
d
x
kd
0
x
L
1a
1b
1c
1d
1e
Fig. 1
Fig. 2

FIG. 1
Four types of tapered struts.

FIG. 2
Tapered strut with halves tangent.

TYPE 3. Moment of Inertia Varies as the Second Power of x; $I=Cx^2$. If the transverse sections are rectangles of constant thickness perpendicular to the plane of the paper and the width is proportional to the square of x, as shown in Figure 1_d, the moments of inertia of the cross-sections with respect to axes *parallel* to the plane of the paper are proportional to the square of x. Or, if the sections are circular with diameters proportional to the square root of x, as shown in Fig. 1_e, the moments of inertia with respect to any diameters are proportional to the square of x. This last form may be made of similar parts of two equal parabaloids of revolution standing on a common base at the middle.

TYPE 4. Moment of Inertia Varies as the First Power of x; $I=Cx$ If the transverse sections are rectangles of constant thickness perpendicular to the plane of the paper, and the width is proportional to x, as shown in Fig. 1_c, the moments of inertia of the cross-sections with respect to axes *parallel* to the plane of the paper are proportional to the first power of x. The strut is made of two equal truncated wedges with their common base at the middle, and differs from Type 2 only in the direction of the bending.

In Fig. 2 the origin of co-ordinates is taken at the middle of the strut. The moment of inertia of any cross-section may be expressed in terms of the distance x from the middle.

TYPE 5. Moment of Inertia Equals I_2-cx^2, in which I_2 is the Moment of Inertia at the Middle and x is the Distance from the Middle. If the transverse sections are rectangles of constant thickness perpendicular to the plane of paper, and the width is reduced from the middle toward the ends proportionally to the square of the distance from the middle, as in Fig. 2, the moments of inertia of the cross-sections with respect to axes parallel to the plane of the paper meet the conditions of this type.

4. SCOPE OF THE INVESTIGATION

In this paper the differential equation is solved for each of of the five types. The constants are determined and the expression for the deflection at the middle is derived. *Methods are given for computing the ultimate strengths of slender struts of each of these types. The ratio of the ultimate load of the tapered strut to the ultimate load of a uniform strut having a moment of inertia equal to that of the maximum section of the tapered strut is calculated for a series of tapers for each of the five types. These *strength ratios* are plotted for each type and then brought together in Fig. 22. The ratio of the strength of the tapered strut to the strength of a uniform strut of equal volume is also calculated for a series of tapers for each type. The values of this ratio (called the *relative efficiency*) are plotted for all types in Fig. 23.

---

*(The theory for Type 3 has previously been published in Technologic Paper No. 152 of the Bureau of Standards. For purposes of comparison, this theory is given again with the form slightly changed to make it consistent with the other types.)

To determine the validity of the theory, steel struts, 36 inches in length, were made of Types 1, 2 and 4. These were tested with cylindrical ends rolling on plane surfaces. The deflections of each strut with loads of known eccentricity were measured and the results compared with the computed values.

Each strut was then tested under loads of very small eccentricity and the ultimate strength thus obtained was compared with the results of theory. For every strut there was close agreement between the experimental and the theoretical figures.

Types 3 and 5 are more difficult to construct accurately than Types 1, 2 and 4. On the other hand, the theoretical determinations for these two types are relatively simple. For these reasons no experiments were made with these two types. The theoretical values, however, were so consistent with each other and with those of the types which were experimentally checked as to leave no doubt as to their accuracy.

The solutions of the differential equations for Types 2 and 4 involve two functions in the form of infinite series. For the calculation of the deflection and the ultimate strength, these two series and two related series obtained from their first derivatives are required. These series may be put into such form that the same four functions may be made to apply to both Types 2 and 4. Since the differential equations for these types are relatively simple and probably occur in the investigation of physical problems other than those of tapered struts, a table of values of these four functions for arguments from 0.1 to 20.0 is included in Appendix D.

5. EXPERIMENTAL APPARATUS

To secure the condition of free rotation of the ends in a single plane, the struts were tested with hemi-cylindrical heads which rolled on plane surfaces. Figure 3 shows one of these heads. A steel rod, 1⅞ inches in diameter and 2¾ inches long, was planed down on the sides and top, to form two vertical planes each ¾ inch from the axis and a horizontal plane $\frac{9}{16}$ inch from the axis. The remainder of the upper half, for a length of 1¾ inches, was then milled down to the plane $D\ D'$, through the axis of the cylinder. The rest of the upper part, one-half inch long at each end, was left to preserve the centers and to afford attachment for four threaded rods which were required to support the clamp bars B. These clamp bars were held by nuts C. A little space was left between the lower surface of each clamp bar and the plane $D\ D'$ of the cylinder to insert a thin steel scale when necessary to determine the position of the strut. Rectangular brass bars, not shown in the drawing, were placed between the clamp bars and the strut. For the circular struts, one brass bar of each set was provided with a V notch at the middle of its length in order to hold the piece in a definite position. The cylindrical rod A was finally case hardened and the curved surface and the plane surface $D\ D'$, were finished by grinding.

To adjust the strut approximately, the distances from its surfaces to the vertical plane surfaces of the head were measured with a scale. The final adjustment was made with the deflection gages after the strut had been placed in position in the testing machine.

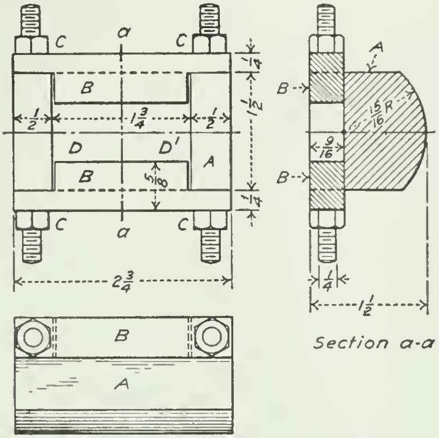

FIG. 3
Head for testing tapered struts.

Figure 4 shows two views of a strut in position for testing. For clearness, the clamp bars *B* and the nuts *C* have been left off the lower head. The cylindrical heads are compressed by the plates *E*, one of which rests on the weighing table and the other under the compression head of the testing machine. These plates are case-hardened and the surfaces in contact with the cylindrical heads are ground plane.

Figure 5 shows the apparatus for measuring the eccentricity and deflection of the strut. Screws *G* with hardened conical ends are threaded through the stirrups *F F'*. These conical ends are inserted in the centers of the compression heads and the screws are turned until the stirrups, while still rotating freely about the axes of the cylinders, can no longer be displaced laterally. The lock nuts *H* are then tightened to hold the screws firmly with reference to the stirrups.

The lower stirrup *F* is rigidly attached to a wooden bar *K*. The upper end of this bar has cone connection with the upper stirrup *F'*. This stirrup is placed at right angles with the length of the strut and bar, so that when it rotates slightly about the axis of the cylindrical head, as the load is applied, the horizontal distance between the end of the strut and the wooden bar remains practically unchanged.

Square brass bars *M*, fastened to the wooden bar, support three Ames dials, which are used to measure the deflection. With struts 36 inches long, the upper and lower dials were placed 6 inches from the upper and lower ends, respectively. The third gage was opposite the middle of the strut.

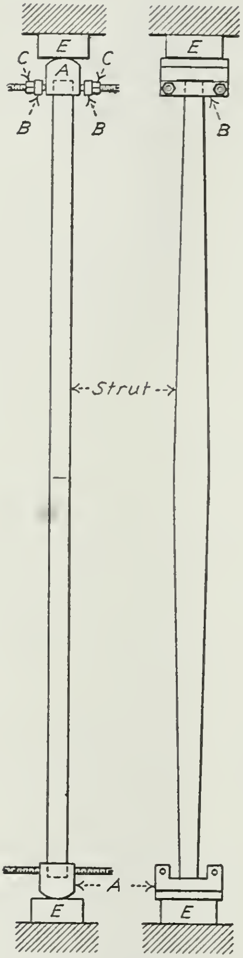

FIG. 4
Strut mounted for Compression with round ends.

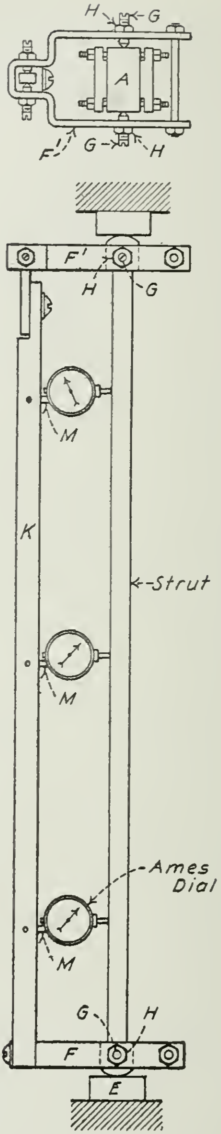

FIG. 5
Apparatus for measuring deflection.

METHOD OF TESTING

After the strut had been adjusted in the heads by measurement with a scale, it was placed in the 200,000 pound Olsen machine and subjected to a load. Usually it was found that the readings of the upper and lower dials differed considerably from each other. This showed that the eccentricity was not the same at both ends. The load was then released to about three hundred pounds. By turning the screw *C* of the compression heads (Fig. 3), the clamp bars were moved slightly and the strut was shifted with reference to the head by sliding on the plane surface *D*. This process was repeated until the upper and lower gages gave practically the same reading. With the eccentricity the same at both ends, the load was again released to about three hundred pounds and both ends of the strut shifted an equal amount as measured by the upper and lower gages. An interpolation between the deflection readings at the middle before and after the shift gave the eccentricity. With the eccentricity known, the strut was again shifted to make the eccentricity about 100 divisions or 0.0100 inch. After a run had been taken at this position, the strut was shifted about 200 divisions to a position with opposite eccentricity and another series of readings taken. If the shift was exactly 200 divisions, the sum of the readings at a given load was taken as twice the deflection for an eccentricity of 100 divisions.

For most of the struts, a series of tests was also made with an eccentricity of 20 divisions. For the final test, the strut was shifted to a position of as small eccentricity as possible, and the ultimate load was determined.

When the ultimate load was reached with little eccentricity, the deflection increased with no increase of load. With a small deflection of one-sixth inch or so in one direction, a slight horizontal push would throw the strut over and give an equal deflection in the opposite direction. With one strut the horizontal force required to reverse the deflection was two pounds. With every strut except two this result could be secured by a push with the little finger. The two struts which failed to meet this condition were found to have suffered a permanent set when the load was removed.

7. TEST OF A UNIFORM CIRCULAR ROD

A preliminary test was made on a uniform rod of cold-rolled steel 36 inches in length and 0.998 inch in diameter. As the rod was found to be not perfectly straight, it was placed in the heads and clamped lightly and the readings of the gages taken as it was rotated on its axis. By this means, a position was found at which the middle was out of line with the ends not over one-thousandth of an inch. (The same method was applied to all the circular struts.) With this rod the deflections with an eccentricity of about 20 divisions (0.0020 inch) were first taken and those with an eccentricity of 100 divisions (0.0100 inch) followed. Table I gives the readings.

TABLE I—Compression Test of Strut of Uniform Circular Section, Tested Sept. 13, 1921.

(Length, 36 inches; mean diameter, 0.998 inch. Tested on 200,000 pound Olsen machine. Deflections measured with Ames dials. Upper and lower dials each 6 inches from ends.)

Total load in pounds	Deflection in ten-thousandths of an inch						
	Upper		Middle		Lower		Average upper and lower
	Reading	Def. right	Reading	Def. right	Reading	Def. right	
400	2951	.... .	2240	....	2499	....	....
2000	2951	0	2236	4	2498	1	....
3000	2949	2	2231	9	2494	5	3.5
4000	2948	3	2227	13	2491	8	5.5
5000	2944	7	2220	20	2488	11	9.
6000	2940	11	2210	30	2482	17	14.
8000	2925	26	2173	67	2465	34	30
9000	2902	49	2135	105	2445	54	51.5
10000	2852	99	2030	210	2395	104	101.5
300	2951	0	2239	1	2500	—1	....

After the readings of Table I were taken, the strut was moved toward the left on the compression heads about 0.0040 inch, while the load on the machine was held at 300 pounds. The readings of Table II were then taken. The eccentricity, defined as the displacement of the axis of the strut from the line of the applied load, was now toward the left.

TABLE II—Compression Test of Strut of Uniform Circular Section, with Eccentricity toward the Left.

Total load in pounds	Deflection in ten-thousandths of an inch						
	Upper		Middle		Lower		Average upper and lower
	Reading	Def. left	Reading	Def. left	Reading	Def. left	
300	2991	....	2280	. ...	2539	....	....
1000	2995	4	2280	....	2541	2	3.
2000	2996	5	2285	5	2542	3	4.
3000	2996	5	2287	7	2543	4	4.5
4000	2999	8	2291	11	2545	6	7.
5000	3000	9	2297	17	2548	9	9.
6000	3006	15	2303	23	2551	12	13.5
7000	3012	21	2316	36	2559	20	20.5
8000	3025	34	2341	61	2570	31	32.5
8600	3031	40	2358	78	2577	38	39.
9000	3041	50	2379	99	2586	47	48.5
10000	3092	101	2419	199	2639	100	100.5
300	2991	0	2280	0	2540	1	....

The change of eccentricity from the experiment of Table I to that of Table II was calculated from the readings at the initial loads.

	Upper	Middle	Lower
Table II	2991	2280	2539
Table I	2951	2240	2499
	40	40	40

From these differences it is evident that the axis of the strut was shifted 0.0040 inch between the readings of Table I and those of Table II.

The deflections at the middle for an eccentricity of 0.0020 inch and the probable eccentricity at each reading are given in Table III.

TABLE III—Deflection at the Middle for an Eccentricity of 0.0020 Inch and the Eccentricity at each Load, Calculated from Tables I and II.

(Difference of Eccentricity, 0.0040 inch.)

Total load in pounds	Deflection in 0.0001 inch			Deflection for an eccentricity of 0 0020 inch	Eccentricity	
	Left	Right	Sum		Left	Right
2000	5	4	9	4.5	22	18
3000	7	9	16	8.	18	22
4000	11	13	24	12.	18	22
5000	17	20	37	18.5	18	22
6000	23	30	53	26.5	17	23
8000	61	67	128	64.	19	21
9000	99	105	204	102	19	21
10000	199	210	409	204.5	20	20

Since the total difference of eccentricity is 40 ten-thousandths, the deflection for an eccentricity of 20 divisions is one half of the sum of the deflections right and left. The eccentricity at a given load is found by dividing 40 into parts proportional to the deflections at that load. At a load of 6,000 pounds, for instance, the deflection left is 23, the deflection right is 30, and the sum of the two is 53. The eccentricity left is $\frac{23}{53} \times 40$, which is 17.4 ten-thousandths of an inch. The apparent change of the line of application of the load during these experiments is 5 divisions. After the first reading, for which the deflections are too small to give reliable results, the apparent change is only 3 divisions.

After the readings of Table II were taken, the strut was moved about 0.0080 inch toward the left to make the eccentricity about 0.0100 inch. Table IV gives the results of the next run.

TABLE IV—Compression Test of Strut of Uniform Circular Section, Continued, Eccentricity about 0.0100 inch toward the left.

Total load in pounds	Deflection in ten-thousandths of an inch						
	Upper		Middle		Lower		Average upper and lower
	Reading	Def. left	Reading	Def. left	Reading	Def. left	
400	3072	....	2362	....	2620	....	....
1000	3079	7	2371	9	2625	5	....
1500	3082	10	2378	16	2630	10	....
400	3072	0	2362	0	2620	0	....
0	3068	....	2356	....	2617	....	....
400	3072	4	2362	6	2620	3	3.5
1000	3079	11	2371	15	2625	8	9.5
1500	3082	14	2378	22	2630	13	13.5
2000	3086	18	2386	30	2634	17	17.5
2500	3089	21	2394	38	2636	19	20.0
3000	3094	26	2404	48	2642	25	25.5
3500	3100	32	2416	60	2649	32	32.0
4000	3104	36	2428	72	2654	37	36.5
4500	3112	44	2440	84	2660	43	43.5
5000	3122	54	2456	100	2668	51	52.5
5500	3132	64	2474	118	2676	59	61.5
6000	3149	81	2499	143	2690	73	77.0
6500	3160	92	2527	171	2704	87	89.5
7050	3180	112	2566	210	2725	108	110.0
7500	3203	135	2609	253	2746	129	132.0
8000	3235	167	2669	313	2780	163	165.0
8500	3279	211	2756	400	2821	204	207.5
9000	3345	277	2880	524	2884	267	272.0

Table IV gives a preliminary run up to a load of 1500 pounds. When the machine was run back to the initial load of 400 pounds, the readings were found to be the same as at the beginning. A similar preliminary run was made at the beginning of each experiment, though in the later tests the preliminary load was much larger than in Table IV. When the strut is shifted on the heads by moving the clamp bars, there may be considerable excess pressure on one side. It is possible that the vibration of the machine combined with this difference of pressure may cause a slight movement on the heads after the initial reading has been taken. In order to make this horizontal force as small as possible, the screws on one side were turned until the strut had moved one or two divisions too far. Then the screws on the other side were tightened until it was brought to the desired position. The preliminary run was then taken as a check. If there was a slight change of zero at the end of the preliminary run, this new zero was used in calculating the deflections. If there was a considerable change of zero, a second-preliminary run was made.

In Table IV the initial load is 400 pounds. The gage readings at zero load were calculated from the rate of change during the first intervals measured. With an eccentricity of 0.0020 inch, as in Tables I and II, it was not necessary to make this correction. In these tables, the zero is taken as the reading at the initial load.

In Table II and in Table IV, the deflections are toward the left. The eccentricity may be calculated from these tables by extrapolation. The displacement between the test of Table II and the test of Table IV is computed from the initial readings.

	Upper	Middle	Lower
Table IV	3068	2356	2617
Table II	2991	2280	2539
Difference	77	76	78

When the displacement readings were not alike, the average was used with double weight given to the reading at the middle. Fractions of a division were dropped. If the fraction was exactly one-half, the integer was used which was nearest the displacement at the middle. This method of calculation gives 76.75 as the displacement. This was taken to be 77 divisions in the calculations of Table V.

TABLE V—Calculation of Eccentricity of Uniform Strut by Extrapolation from Tables II and IV.

Total load in pounds	Deflection in 0.0001 inch			Eccentricity	
	Left from Table II	Left from Table IV	Difference	Left, Table II	Left, Table IV
2000	5	30	23	15	92
3000	7	48	41	13	90
4000	11	72	61	14	91
5000	17	100	83	16	93
6000	23	143	120	15	92
7000	36	206	170	16	93
8000	61	313	252	19	96
9000	99	524	425	18	95

A comparison with Table III shows that, for loads of 5,000 pounds and upwards, the eccentricity computed by extrapolation from deflections in the same direction does not differ from that computed by interpolation from deflections in opposite directions by more than three ten-thousandths of an inch. It is evident from these calculations, therefore, that a cylinder on a plane is equivalent to a knife edge of thickness not greater than three ten-thousandths of an inch. Errors in measurement may account for this difference, and the resultant pressure between cylindrical and plane surface may be regarded as applied on a line of negligible width provided the elastic limit of materials is not exceeded.

After the readings of Table IV were taken, the strut was shifted toward the right a distance of 202 divisions. Table VI gives the deflection for an eccentricity of 100 divisions and the eccentricity toward the left and toward the right computed from these readings and those of Table IV. Since the shift was 202 divisions, the deflection for 100 divisions was computed by dividing the sum of the deflections right and left by 2.02.

TABLE VI—Deflection at the Middle of a Uniform Strut for an Eccentricity of 0.0100 inch and Eccentricity at each Load.

—(Difference of Eccentricity, 0.0202 inch)

Total load in pounds	Deflection in 0.0001 inch			Deflection for an eccentricity of 0.0100 inch	Eccentricity	
	Left	Right	Sum		Left	Right
1000	15	14	29	14	105	97
1500	22	20	42	21	106	96
2000	30	27	57	28	106	96
2500	38	35	73	36	105	97
3000	48	46	94	47	103	99
3500	60	55	115	56	105	97
4000	72	68	140	69	104	98
4500	84	81	165	82	103	99
5000	100	102	202	100	100	102
5500	118	124	242	120	99	103
6000	143	149	292	145	99	103
6500	171	179	350	173	99	103
7050	210	215	425	220	100	102
7500	253	261	514	254	99	103
8000	313	328	641	318	99	103
8500	400	414	814	403	99	103
9000	524	538	1062	526	100	102

After the deflections toward the right which are given in the third column of Table VI were taken, the strut was moved about 100 divisions toward the left to the position of very small eccentricity. The results are given in Table VII.

TABLE VII—Compression Test of Strut of Uniform Section, Tested with Small Eccentricity, Sept. 13, 1921.

Total load in pounds	Deflection in ten-thousandths of an inch					
	Upper		Middle		Lower	
	Reading	Def. left	Reading	Def. left	Reading	Def. left
300	2971		2257		2519	
2000	2973	2	2258	1	2518	-1
4000	2973	2	2255	-2	2517	-2
6000	2973	2	2256	-1	2517	-2
7000	2973	2	2256	-1	2517	-2
8000	2973	2	2257	0	2517	-2
8500	2974	3	2258	1	2517	-2
9500	2976	5	2259	2	2515	-4
10500	2980	9	2268	11	2519	0
10800	2988	17	2278	19	2522	3
10900	2995	24	2290	33	2530	11
11000	3010	39	2312	55	2542	23
11200	3061	90	2425	168	2596	77
11250	3852	881	3996	1739	3372	853
Strut was pushed to the right with a light horizontal pressure applied with one finger at the middle. Load was not changed.						
11250	2176	−895	627	−1630	1704	-815
Machine was run with no increase of load.						
11250	1682	−1289	Off scale		1201	-1288
Released load.						
2000	2973	2	2260	3	2518	-1

8. THEORETICAL DEFLECTION OF A UNIFORM STRUT

The deflection of a round-end strut of uniform cross-section is computed by the well-known formula:

$$y = e \sec \sqrt{\frac{P}{EI}} \quad \frac{L}{2} \cos \sqrt{\frac{P}{EI}} \quad \left(\frac{L}{2} - x \right), \qquad (1)$$

in which y is the distance of the center of any section from the line of the load; e is the eccentricity or distance of the centers of all sections from the line of the load before the load is applied; and x is the distance of any section from one end of the strut. At the middle, where $x = \frac{L}{2}$, the deflection is

$$y_{\max} = e \sec \sqrt{\frac{P}{EI}} \quad \frac{L}{2} = e \sec \sqrt{\frac{PL^2}{4EI}} \quad . \qquad (2)$$

Since the secant of 90 degrees $\left(\text{or } \frac{\pi}{2} \text{ radians} \right)$ is infinite, the ultimate or critical load is that load for which

$$\sqrt{\frac{PL^2}{4EI}} = \frac{\pi}{2}, \tag{3}$$

$$P = \frac{\pi^2 EI}{L^2}, \tag{4}$$

which is Euler's formula.

The deflection at any point is $y - e$, and the deflection at the middle is $y_{\max} - e$.

If the angle $\sqrt{\frac{PL^2}{4EI}}$ in radians is equals to θ degrees,

$$\theta = 57.296 \sqrt{\frac{PL^2}{4EI}}, \tag{5}$$

$$P = \frac{4\theta^2 EI}{57.296^2 L^2} = R\theta^2 . \tag{6}$$

in which R is a convenient constant.

The modulus of elasticity of the uniform circular strut was determined by loading as a beam and measuring the deflection with apparatus which was used to measure the deflection as a column. The most careful measurements with two different spans each gave almost exactly 30,000,000. Other results were from one to two per cent higher. Measurements were also taken in direct compression with a gage length of 33 inches. The results of these tests were somewhat above 30,000,000.

Before these compression tests were made, the rod had been permanently distorted by an overload. For this reason, the results are not regarded as entirely reliable.

The modulus may be obtained from the ultimate load by Euler's formula. Since $L = 36$ inches, and $I = \frac{0.998^4 \times \pi^2}{64}$,

$$E = \frac{11250 \times 36^2 \times 64}{\pi^2 \times 0.998^4} = 30{,}340{,}000.$$

This modulus of elasticity has been used in the computation of all the struts of the type $I = Cx^4$, which were made from similar round rods of cold-rolled steel.

If the modulus of elasticity is taken from the ultimate load, then

$$R = \frac{\text{Ultimate Load}}{90^2} . \tag{7}$$

Appendix A gives the relation of load to deflection for a strut or column of uniform section. Column II gives the secants. The deflection at the middle for any experiment is obtained by subtracting unity from the secant and then multiplying the remainder by the eccentricity. The deflection at one-sixth the length from the end is calculated in a similar way from the data of Column V. Column VI gives the loads for the strut of the preceding experiments.

The middle and lower curves of Fig. 6 are drawn from the table of Appendix A. The ordinates are the loads from Column VI. The abscissas of the lower curve are obtained by subtracting unity from the secants of Column II and multiplying the remainder by 0.0100. The abscissas of the middle curve are computed in a similar way by multiplying by 0.0020. For example, for a load of 5000 pounds the secant is 2.0 and the deflection is (2—1)e. For this load the lower curve passes through the abscissa 100 and the middle curve passes through the abscissa 20.

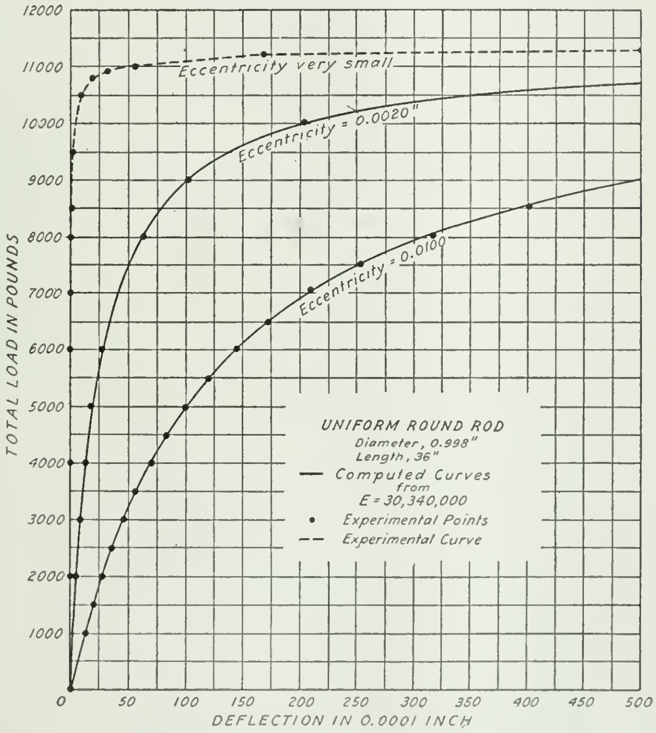

FIG. 6

Load-deflection curves for strut of uniform circular section.

The points on Fig. 6 are plotted from the experimental results. Those of the lower curve are from the fifth column and the first column of Table VI. For a load of 5000 pounds the deflection is 100; the experimental point falls exactly on the theoretical curve. For some of the loads, there is a slight variation from the theoretical curve. The difference, however, is within the limits of accuracy of the experiments.

The points on the middle curve are the experimental results from Table III. The upper curve of Fig. 6 is entirely experimental.

The upper curve of Fig. 7 gives the deflections of this same uniform strut at one-sixth the length from the ends. It is drawn from Columns V and VI of Appendix A. The points on the figure are taken from the last column of Table IV combined with a similar set of deflections toward the right. The close agreement of the experimental with the theoretical results shows that the strut must turn freely on the cylinders as knife edges. If there were any resistance to turning, which would partially fix the ends of the strut, the deflections near the ends would be smaller than the theoretical values. In the experiments it was found that the deflections at the sixth points were relatively smaller than the theoretical values when the deflection was in one direction, and relatively larger when the deflection was in the opposite direction. This variation was caused by slight crookedness of the strut. The ratio of the deflection at the sixth-points to the deflection at the middle is on one side of the theoretical ratio when the deflection is in one direction and on the other side of the theoretical ratio when the deflection is in the opposite direction. The difference between the theoretical and experimental ratios decreases as the load increases. For instance when the load in Table IV is 2000 pounds, the ratio is $\frac{17.5}{30} = 0.583$. The theoretical ratio from Appendix A for a load of 2006 pounds is

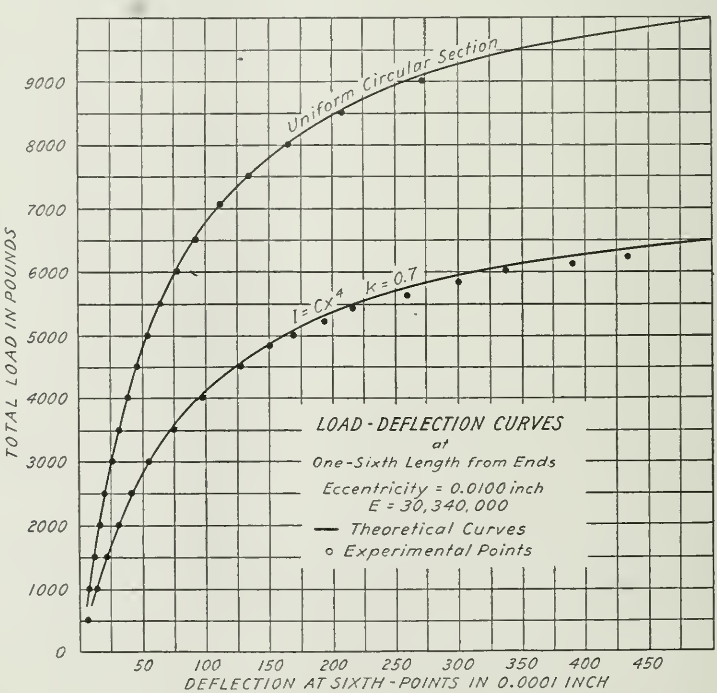

FIG. 7

Deflection at sixth-points of strut of uniform circular section.

$\frac{147}{269} = 0.546$. For a load of 8000 pounds the experimental ratio is $\frac{165}{313} = 0.527$, while the theoretical ratio is $\frac{1620}{3134} = 0.517$.

9. TAPERED STRUT FOR WHICH $I=Cx^4$

The tapered strut which is easiest to construct in metal is one with a conical taper. The diameter at a distance x from the vertex of the cone is a constant times x. The moment of inertia is another constant times the fourth power of the diameter. The moment of inertia for any section between the vertex and the middle of the strut may be expressed by the equation

$$I = Cx^4. \tag{1}$$

If d is the distance of the middle of the strut from the vertex, the maximum moment of inertia is

$$I_2 = Cd^4. \tag{2}$$

10. THEORY OF TAPERED STRUT FOR WHICH $I=Cx^4$

For any strut

$$EI\frac{d^2y}{dx^2} + Py = 0. \tag{1}$$

in which y is measured from the line of the load to the center of gravity of the cross-section, and is equal to the sum of the deflection and the eccentricity. When $I = Cx^4$,

$$ECx^4\frac{d^2y}{dx^2} + Py = 0. \tag{2}$$

$$x^4\frac{d^2y}{dx^2} + \frac{Py}{EC} = 0. \tag{3}$$

For convenience, $\frac{P}{EC}$ may be replaced by q^2, and Equation (3) is then

$$x^4\frac{d^2y}{dx^2} + q^2y = 0. \tag{4}$$

Equation (4) is a typical differential equation of the second order. The solution for the special form, $q=1$, is given in Johnson's Differential Equations in the form of infinite series which, by a little manipulation, may be transformed into the sum of two trigonometric functions. The equation may be solved more easily by means of some substitutions.*

---

*These substitutions, by which Equation (4) is transformed into the simple form of Equation (7) were suggested by Professor R. D. Bohannon.

If $z = \frac{1}{x}$, Equation (4) becomes

$$z \frac{d^2 y}{dz^2} + 2 \frac{dy}{dz} + q^2 (zy) = 0 \qquad (5)$$

If $zy = v$

$$z \frac{d^2 y}{dz^2} + 2 \frac{dy}{dz} = \frac{d^2 v}{dz^2}, \qquad (6)$$

and Equation (5) is

$$\frac{d^2 v}{dz^2} + q^2 v = 0, \qquad (7)$$

The solution of Equation (7) is

$$v = M \sin qz + B \cos qz,$$

$$\frac{y}{x} = M \sin \frac{q}{x} + B \cos \frac{q}{x},$$

$$y = Mx \sin \frac{q}{x} + Bx \cos \frac{q}{x}, \qquad (8)$$

in which M and B are arbitrary constants.

To put Equation (8) into a more convenient form, the second member may be multiplied and divided by $\sqrt{M^2 + B^2}$;

$$y = x \sqrt{M^2 + B^2} \left(\frac{M}{\sqrt{M^2 + B^2}} \sin \frac{q}{x} + \frac{B}{\sqrt{M^2 + B^2}} \cos \frac{q}{x} \right). \qquad (9)$$

Let $\frac{M}{\sqrt{M^2 + B^2}} = \cos \alpha$; then $\frac{B}{\sqrt{M^2 + B^2}} = \sin \alpha$, and $\frac{B}{M} = \tan \alpha$

The term in the parenthesis in Equation (9) is $\sin\left(\frac{q}{x} + \alpha\right)$, and

$$y = x \sqrt{M^2 + B^2} \sin\left(\frac{q}{x} + \alpha\right) = Ax \sin\left(\frac{q}{x} + \alpha\right), \qquad (10)$$

in which A $\left(\text{which equals} \sqrt{M^2 + B^3},\right)$ and α are the two arbitrary constants.

If the value of y from Equation (10) and its second derivative are substituted in Equation (4), the result is zero. Equation (10) is, therefore, a solution of Equation (4).

Figure 8 shows the strut. The curve at the right of Fig. 8 represents the axis of the strut with the eccentricity and deflection exaggerated. The ratio of the diameter at the ends to the diameter at the middle is k. The distance from the vertex to the middle is d, and, since the diameter is proportional to the distance from the vertex, the value of x at the lower end of the strut is kd. The strut is symmetrical with respect to the section at the middle.

The conditions for the elimination of the constants A and α are:

1. At the middle, $x=d$ and $\frac{dy}{dx} = 0$

2. At the end, $x=kd$ and $y=e$.

Differentiating Equation (10),

$$\frac{dy}{dx} = A \sin\left(\frac{q}{x} + \alpha\right) - \frac{Aq}{x}\cos\left(\frac{q}{x} + \alpha\right). \tag{11}$$

At the middle, where $x=d$,

$$0 = A \sin\left(\frac{q}{d} + \alpha\right) - \frac{Aq}{d}\cos\left(\frac{q}{d} + \alpha\right), \tag{12}$$

from which

$$\tan\left(\frac{q}{d} + \alpha\right) = \frac{q}{d}. \tag{13}$$

At the end, where $x=kd$, Equation (10) gives

$$e = A\ kd\ \sin\left(\frac{q}{kd} + \alpha\right). \tag{14}$$

$$A = \frac{e}{kd \sin\left(\frac{q}{kd} + \alpha\right)}. \tag{15}$$

At the middle, where $x=d$,

$$y_{\max} = \frac{ed \sin\left(\frac{q}{d} + \alpha\right)}{kd \sin\left(\frac{q}{kd} + \alpha\right)}. \tag{16}$$

At any point x,

$$y = \frac{ex \sin\left(\frac{q}{x} + \alpha\right)}{kd \sin\left(\frac{q}{kd} + \alpha\right)}. \tag{17}$$

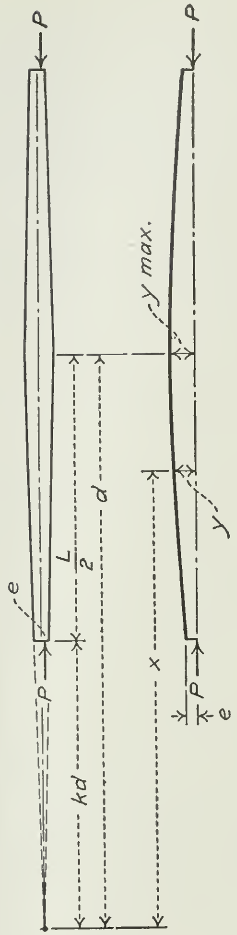

FIG. 8
Distances and dimensions for theoretical investigation.

Since $d - kd = \frac{L}{2}$,

$$d = \frac{L}{2(1-k)}. \tag{18}$$

If I_2 is the moment of inertia at the middle, where $x = d$, $I_2 = Cd^4$,

$$C = \frac{I_2}{d^4}. \tag{19}$$

Since $q^2 = \frac{P}{EC}$,

$$P=\left(\frac{q}{d}\right)^2\times\frac{EI_2}{d^2}=\left(\frac{q}{d}\right)^2\times\frac{4EI_2(1-k)^2}{L^2}. \qquad (20)$$

11. COMPUTATION OF DEFLECTION OF STRUT FOR WHICH $I=Cx^4$

Appendix B gives the computations for α and for $\sin\left(\frac{q}{d}+\alpha\right)$ for values of $\frac{q}{d}$ differing by 10 degrees. These constants are the same for all values of k.

EXAMPLE

Find the deflection at the middle of a strut 1 inch in diameter at the middle, 0.5 inch in diameter at the ends, and 36 inches long, if $E=30{,}340{,}000$ and the load is such that $\frac{q}{d}=100$ degrees.

$$\frac{q}{kd}=\frac{100^\circ}{0.5}=200^\circ.$$

From Appendix B, $\alpha=-39^\circ\ 48'.7$; hence $\frac{q}{kd}+\alpha=160^\circ\ 11'.3$.

From Equation (16) of Art. 10,

$$y_{\max}=\frac{e\sin\left(\frac{q}{d}+\alpha\right)}{0.5\sin 160^\circ\ 11'.3}=\frac{2\times 0.8677\,e}{0.3389}=5.1021\,e.$$

The deflection at the middle is then 4.1021 times the eccentricity.

To find the load P,

$$d=\frac{18''}{1-0.5}=36 \text{ inches.}$$

$$I_2=Cd^4=\frac{\pi}{64},$$

$$C=\frac{\pi}{64\times 36^4},$$

$$P=q^2EC=\frac{q^2}{36}\times\frac{30{,}340{,}000\,\pi}{64\times 36^2}=1149.2\left(\frac{q}{d}\right)^2$$

When $\frac{q}{d}=100$ degrees $=1.7453$ radians, $P=1149\times 1.7543^2=$ 3,500 pounds.

The critical load is that for which $\frac{q}{kd}+\alpha=180$ degrees, which makes the denominator of Equation (16), Art. 10, equal to zero and renders the deflection infinite.

For $\frac{q}{d}=110^\circ$, $\frac{q}{kd}+\alpha=220^\circ-47^\circ\ 36'.7=172^\circ\ 23'.3$;

For $\frac{q}{d}=120^\circ$, $\frac{q}{kd}+\alpha=240^\circ-55^\circ\ 31'.4=184^\circ\ 28'.6$.

An interpolation between these results gives $\frac{q}{d} = 116^\circ\ 296$ when $\frac{q}{kd} + \alpha = 180^\circ$. A second interpolation between $116^\circ.2$ and $116^\circ.3$ gives $116^\circ\ 243$ as the value of $\frac{q}{d}$ when $\frac{q}{kd} + \alpha = 180^\circ$. $116^\circ\ 243 = 2.02881$ radians. $2\ 02881^2 \times 1149\ 2 = 4730$ pounds, which is the ultimate load on this strut.

12. CORRECTION FOR VARIATION IN DIAMETER OF STRUT

On account of its flexibility, the rod as turned deviated somewhat from the true frustum of a cone. The correction might be made by finding the volume of the rod and then assuming that the material was so distributed as to form true frustums of a cone. But the moment of inertia at the middle has greater influence upon the deflection at the ends than does the moment of enertia near the ends. It is desirable, therefore, to use some method of correction which will take this effect in account. According to the "area moments" method for beams, the deflection at the end caused by the bending of any short portion is inversely proportional to the moment of inertia of the portion and directly proportional to the distance of the portion from the end. Table VIII shows the method of computing the correction which is based on this principle.

TABLE VIII—Size Correction for Round Strut for which $I = Cx^4$ and $k = 0.5$.

Distance from end	Diameter		Error		Relative error times distance
	Average	Nominal	Actual	Relative	
0	0.501	0.5000	0.0010	0.0020	0.0
2	0.548	0.5556	−0.0076	−0.0137	−0.0274
4	0.609	0.6111	−0.0021	−0.0034	−0.0136
6	0.6685	0.6667	0.0018	0.0027	0.0162
8	0.7265	0.7222	0.0043	0.0059	0.0472
10	0.785	0.7778	0.0072	0.0093	0.0930
12	0.841	0.8333	0.0077	0.0092	0.1104
14	0.901	0.8889	0.0121	0.0136	0.1904
16	0.960	0.9444	0.0156	0.0165	0.2640
18	0.997	1.0000	−0.0030	−0.0030	−0.0540
90	$0.6262 \div 90 = 0.00695$; $0.00695 \times 4 = 0.0278$ Size correction factor = 1.028.				0.6262

The rod of Table VIII may be regarded as equivalent to a rod for which $k = 0.5$ and the maximum diameter ~~is 1.00695~~ inch. Since the moment of inertia varies as the fourth power of the diameter, the correction for I is $0.00695 \times 4 = 0.0278$. The moment of inertia at the middle of the equivalent rod of uniform taper may be taken as 1.028 times the moment of inertia of a rod 1 inch in diameter. This coefficient may be called the *size correction factor.*

This method of correction for variation of diameter is not theoretically correct, but may be regarded as a fair engineering guess.

To get the total load for any given value of $\frac{q}{d}$ the coefficient 1149.2 is multiplied by the size correction factor. $1149.2 \times 1.028 = 1181.4$.

If $\frac{q}{d}$ is expressed in degrees, this constant must be divided by the square of 57.296,

$$1181.4 \div 57.296^2 = 0.35987.$$

When $\frac{q}{d}$ is 100 degrees, the theoretical load is 3599 pounds. The corrected ultimate load for this strut is $4730 \times 1.028 = 4862$ pounds.

The solid lines of Fig. 9 are drawn from the theoretical values corrected for the oversize of the strut.

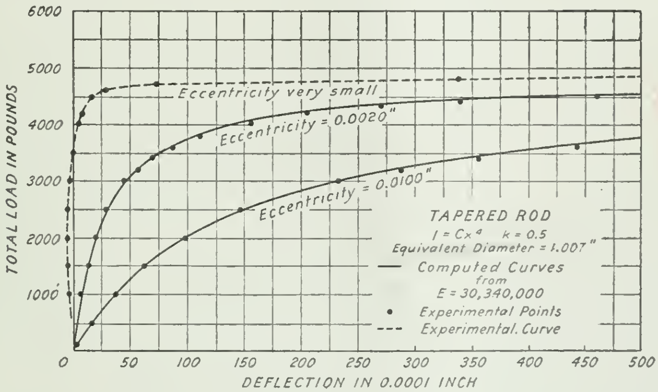

FIG. 9

Load-deflection curves for one tapered strut, $I = Cx^4$, $k = 0.5$.

13. EXPERIMENTAL RESULTS FOR STRUTS FOR WHICH $I = Cx^4$

Tables IX, X, and XI give the results of experiments on the strut for which $I = Cx^4$, $k = 0.5$, and size correction factor = 1.028. Table IX gives the loads and deflections at the middle for an eccentricity difference of 0.0199 inch; Table X gives the results for an eccentricity difference of 0.0038 inch; and Table XI gives the readings when the eccentricity was very small.

The points near the lower curve of Fig. 9 are plotted from Table IX, and those near the middle curve are plotted from Table X. The distances of the experimental points from the theoretical curves, while small, are a little greater than those of the uniform strut of Fig. 6. The modulus of elasticity was calculated from the ultimate load of the uniform strut, and may be slightly in error. The size correction factor is open to some question. It is evident, therefore, that the experiments agree with theory within the limits of accuracy of the measurements.

TABLE IX—Deflections at the Middle of a Tapered Strut for which $I=Cx^4$ and $k=0.5$. Tested Sept. 27, 1921.

(Difference of eccentricity, 0.0199 inch.)

Total load in pounds	Deflection in 0.0001 inch			Deflection for an eccentricity of 0.0100 inch	Eccentricity	
	Left	Right	Sum		Left	Right
500	21	12	33	17	125	74
1000	43	30	73	37	117	82
1500	74	52	126	63	117	82
2000	110	85	195	98	117	82
2500	157	133	290	146	108	91
3000	239	221	460	231	103	96
3200	294	282	576	289	102	97
3400	365	346	711	357	102	97
3600	463	424	887	446	102	97
3800	597	562	1159	582	102	97
4000	780	774	1554	781	100	99

TABLE X—Deflection at the Middle of Tapered Strut for which $I=Cx^4$ and $k=0.5$.

(Difference of Eccentricity, 0.0038 inch.)

Total load in pounds	Deflection in 0.0001 inch			Deflection for an eccentricity of 0.0020 inch	Eccentricity	
	Left	Right	Sum		Left	Right
500	5	0	5	3	... ...	.. ..
1000	12	0	12	6	....	....
1500	19	5	24	13	30	8
2000	28	10	38	20	28	10
2500	34	22	56	29	23	15
3000	44	41	85	45	20	18
3200	53	56	109	57	19	19
3400	63	70	133	70	18	20
3600	77	91	168	88	17	21
3800	101	114	215	113	18	20
4000	147	158	305	161	18	20
4200	179	226	405	213	17	21
4300	234	280	514	271	17	21
4400	299	348	647	341	18	20
4500	404	470	874	460	18	20
4600	678	800	1478	778	17	21

TABLE XI – Compression Test of Strut for which $I = Cx^4$ and $k = 0.5$, Tested with Small Eccentricity.

(Nominal diameter, 1 inch: size correction factor for I, 1.028.)

Total load in pounds	Deflection in ten-thousandths of an inch					
	Upper		Middle		Lower	
	Reading	Def. right	Reading	Def. right	Reading	Def. right
300	1399		2809		960	
1000	1401	−2	2811	−2	962	−2
1500	1402	−3	2813	−4	962	−2
2000	1402	−3	2814	−5	963	−3
2500	1402	−3	2814	−5	962	−2
3000	1400	−1	2811	−2	960	0
3500	1399	0	2808	1	957	3
4000	1394	5	2803	6	951	9
4200	1392	7	2800	9	950	10
4500	1384	15	2791	18	942	18
4600	1374	25	2780	29	937	23
4700	1340	59	2735	74	909	51
4800	1190	209	2470	339	670	290
	Changed	while	reading			
4800	991	408	2192	619	572	388
4325	11	1388	670	2139	Off scale	

The points of the upper curve of Figure 9 are plotted from the deflections at the middle of the strut, which are given in Table XI. The positive direction in this experiment is toward the right. It will be seen from the curve, as well as from the table, that the deflections were toward the left until the load reached 3000 pounds. The reason for this change in direction of deflection is evident from Tables IX and X. Each table shows that the eccentricity shifted from left toward right and became practically stable when the load reached 3000 pounds. With the small eccentricity, the line of load was to the right of the axis of the strut at the small loads and the deflection was toward the left. As the load increased, the line of the resultant shifted toward the left which caused the eccentricity to move toward the right and thus changed the direction of the deflection.

When it was desired to make the eccentricity a minimum to find the critical load, the amount of shift necessary was calculated from the eccentricity under relatively large loads It frequently happened that there was considerable deflection in one direction at first and afterward the deflection changed to the opposite direction under the larger loads.

The ultimate load as measured for this strut was 4825 pounds, as compared with a computed load of 4862 pounds.

14. DEFLECTION AT THE SIXTH-POINTS

Appendix C gives the calculations for the deflections at the middle and at the sixth-points for the strut for which $k = 0.7$. (For plotting the curves, twice as many points were calculated as are given in Appendix

C.) Tables XII and XIII give the experimental results for this strut. The last column of each of these tables gives the ratio of the average deflection at the sixth-points to the deflection at the middle. Column XI of Appendix C gives the theoretical ratio. In Table XII, with the deflection toward the left, the ratios for the small loads are too low; while in Table XIII, with the deflection toward the right, the ratios for the small loads are too high. For the larger loads, the ratios approach very closely to the theoretical values.

In Table XII the readings of the upper instrument are slightly larger than those of the lower instrument. In Table XIII, also, the readings of the upper instrument are the larger. Usually, the upper instrument gives the larger reading when the deflection is in one direction, and the lower instrument gives the larger reading when the deflection is in the opposite direction. This is caused by difference of eccentricity at the two ends. The exception to this rule in Tables XII and XIII was caused by a slight inequality in the dimensions of the upper and lower portions of the strut. The lower half of the length averaged a little larger than the upper half. The upper part was, therefore, more easily bent.

TABLE XII Compression Test of Tapered Strut for which $I = Cx^4$ and $k = 0.7$, Tested Sept. 20, 1921.

(Size correction factor, 1.0182)

Total load in pounds	Deflection in ten-thousandth of an inch							Ratio of average divided by middle
	Upper		Middle		Lower		Average of upper and lower	
	Read.	Def. left	Read.	Def. left	Read.	Def. left		
0	2181	....	2681	....	1738	....	....	....
100	2182	1	2683	2	1739	1	. . ..	....
500	2188	7	2694	13	1743	5	6	0.461
1000	2196	15	2707	26	1750	12	13.5	0.519
1500	2200	19	2716	35	1754	16	17.5	0.500
2000	2206	25	2728	47	1761	23	24.0	0 511
2500	2215	34	2742	61	1768	30	32.0	0.524
3000	2230	49	2761	80	1780	42	45.5	0.569
3500	2245	64	2788	107	1797	59	61.5	0.575
4000	2265	84	2819	138	1817	79	81.5	0.590
4500	2292	111	2861	180	1840	102	106.5	0.592
4800	2314	133	2896	215	1861	123	128.0	0.596
5000	2331	150	2928	247	1880	142	146.0	0.591
5200	2349	168	2958	277	1896	158	163.0	0.588
5400	2371	190	2993	312	1916	178	184.0	0 589
5600	2410	229	3050	369	1950	212	220.5	0.595
5700	2424	243	3077	396	1964	226	234.5	0.592
5800	2449	268	3115	434	1988	250	259.0	0.597
5900	2470	289	3159	478	2010	272	280.5	0.587
6000	2491	310	3203	522	2033	295	302.5	0.579
6100	2519	338	3257	576	2060	322	330.5	0.573
6200	555	374	3321	640	2095	357	365.5	0.571
2000	2209	28	2730	49	1763	25	26.5	....
100	2182	1	2683	2	1739	1	... .	......

TABLE XIII—Compression Test of Tapered Strut for which $I = Cx^4$ and $k = 0.7$ with Deflection toward the Right.

Total load in pounds	Deflection in ten-thousandths of an inch							Ratio of average divided by middle
	Upper		Middle		Lower		Average of upper and lower	
	Read.	Def. right	Read	Def. right	Read	Def. right		
0	1981	....	2480	......	1538		...	..
500	1975	6	2471	9	1531	7	6.5	....
1000	1968	13	2459	21	1522	16	14.5	0.690
1500	1953	28	2442	38	1512	26	27.0	0.710
2000	1942	39	2423	57	1500	38	38.5	0.675
2500	1930	51	2404	76	1488	50	50.5	0.664
3000	1915	66	2376	104	1472	66	66.0	0.634
3500	1890	91	2337	143	1449	89	90.0	0.629
4000	1862	119	2292	188	1427	111	115.0	0.612
4500	1827	154	2229	251	1392	146	150.0	0.593
4800	1802	179	2180	300	1367	171	175.0	0.583
5000	1780	201	2139	341	1346	192	196.5	0.576
5200	1749	232	2093	387	1314	224	228.0	0.589
5400	1723	258	2050	430	1288	250	254.0	0.591
5600	1672	309	1967	513	1240	298	203.5	0.592
5700	1649	332	1922	558	1215	323	327.5	0.587
5800	1630	351	1887	593	1195	342	347.0	0.585
5900	1593	383	1832	648	1163	375	379.0	0.585
6000	1552	429	1759	721	1118	420	424.5	0.589
6100	1520	461	1695	785	1086	452	456.5	0.582
6200	1474	507	1614	866	1040	498	502.0	0.580
3000	1902	79	2361	119	1461	74	76.5	0.643
2000	1936	45	2417	63	1497	41	43.0	0.683
1000	1968	13	2452	28	1520	18	15.5	. ...
100	1979	2	2478	2	1537	1	....	....

Table XIV gives the average deflections at the sixth-points for the strut for which $k=0.7$ from the readings of Tables XII and XIII. The lower curve of Fig. 7 is drawn from the theoretical deflections as given in Column X of Appendix C. The curve represents the theoretical deflections of a strut 36 inches long, for which $E=30{,}340{,}000$, the nominal diameter $=1$ inch, and the size correction factor $=1.0182$. The points near this curve are drawn from the fifth column of Table XIV. For most of the distance, the experimental points fall a little to the right of the theoretical curve. A slightly smaller modulus of elasticity would account for the difference.

15. DEFLECTION AT THE MIDDLE OF STRUT FOR WHICH $I=Cx^4$

Table XV gives the deflections at the middle of strut for which $k=0.7$, taken from the readings of Tables XII and XIII. The third curve from the bottom of Fig. 10 gives the theoretical deflections. Part of these deflections may be calculated from Column VI of Appendix C. For instance, when the angle $\frac{q}{d}$ is 180 degrees, the reading of Column VI is 2.6813. The deflection is 1.6813 multiplied by 0.0100. The corresponding load from Column XII is 4158 pounds.

TABLE XIV—Deflection at Sixth-points of Tapered Strut for which $I = Cx^4$ and $k = 0.7$, from Data of Tables XII and XIII.

(Difference of eccentricity, 0.0201 inch)

Total load in pounds	Deflection in 0.0001 inch			Deflection for an eccentricity of 0.0100 inch	Eccentricity	
	Left	Right	Sum		Left	Right
500	6.0	6.5	12.5	6	97	104
1000	13.5	14.5	28.0	14	97	104
1500	17.5	27.0	44.5	22	79	122
2000	24.0	38.5	62.5	31	77	124
2500	32.0	50.5	82.5	41	78	123
3000	45.5	66.0	111.5	55	82	119
3500	61.5	90.0	151.5	75	82	119
4000	81.5	115.0	196.5	98	83	118
4500	106.5	150.0	256.5	128	83	118
4800	128.0	175.0	303.0	151	85	116
5000	146.0	196.5	342.5	170	86	115
5200	163.0	228.0	391.0	195	86	115
5400	184.0	254.0	438.0	218	84	117
5600	220.5	303.5	524.0	261	85	116
5700	234.5	327.5	562.0	280	84	117
5800	259.0	347.0	606.0	301	84	117
5900	280.5	379.0	659.5	328	85	116
6000	302.5	424.5	727.0	362	84	117
6100	330.0	456.5	786.5	391	84	117
6200	365.5	502.5	868.0	432	85	116

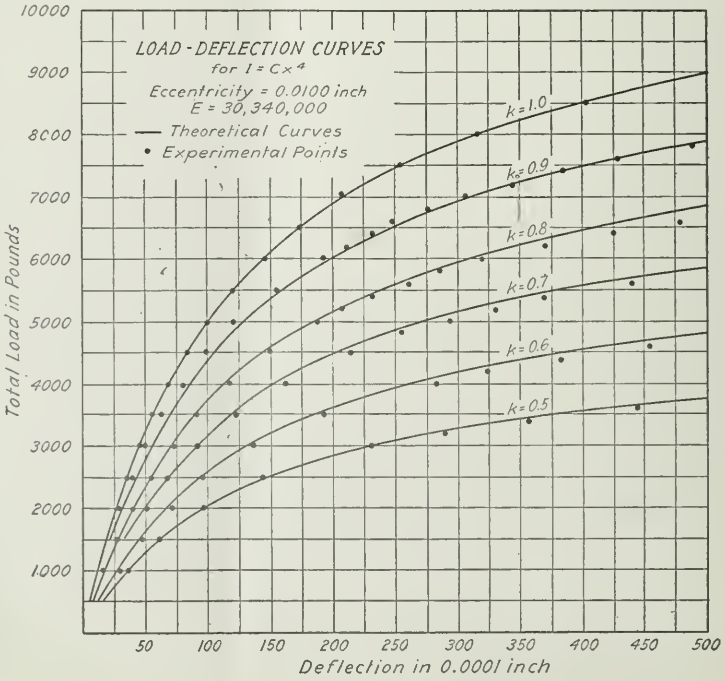

FIG. 10

Load-deflection curves with eccentricity=0.0100 inch for struts, $I=Cx^4$

Appendix C gives only a portion of the figures which were used in plotting the curve of Fig. 10. The points near this curve, k = 0.7, of Fig. 10 represents the experimental values of Table XV. Like the deflections at the sixth-points, the deflections at the middle are slightly larger than the computed values.

The modulus of elasticity was computed from the cold-rolled rod of uniform section, which included all the denser material of the the outer portions. In turning the tapered struts, most of this denser material was removed. It is possible that the modulus of elasticity of the remainder is smaller than that of the uniform rod.

TABLE XV—Deflections at the Middle of the Tapered Strut for which $I = Cx^4$ and $K = 0.7$, from Data of Tables XII and XIII.

(Difference of eccentricity, 0.0201 inch.)

Total load in pounds	Deflection in 0.0001 inch			Deflection for an eccentricity of 0.0100 inch	Eccentricity	
	Left	Right	Sum		Left	Right
500	13	9	22	11	119	82
1000	26	21	47	23	111	90
1500	35	38	73	36	96	105
2000	47	57	104	52	91	110
2500	61	76	137	68	90	111
3000	80	102	184	91	87	114
3500	107	143	250	124	86	115
4000	138	188	326	162	85	116
4500	180	251	431	214	84	117
4800	215	300	515	256	84	117
5000	247	341	588	293	84	117
5200	277	387	664	330	84	117
5400	312	430	742	369	85	116
5600	369	513	882	439	84	117
5700	396	558	954	475	84	117
5800	434	593	1027	511	85	116
5900	478	648	1126	560	85	117
6000	522	721	1243	618	84	117
6100	576	785	1361	677	85	116
6200	640	866	1506	749	85	116

Figure 10 gives the theoretical curves and the experimental points for the series of struts for which $I = Cx^4$ from $k = 0.5$ to $k = 0.9$, inclusive. For comparison, the curve and experimental points are shown for the uniform rod also.

Figure 11 gives the deflections and ultimate loads with small eccentricities. To avoid confusion, the small deflections at the lower loads have been omitted. The curves of Figs. 10 and 11, or those of Figs. 6 and 9, give a vivid picture of the great loss of strength in a tapered strut as compared with one of uniform section.

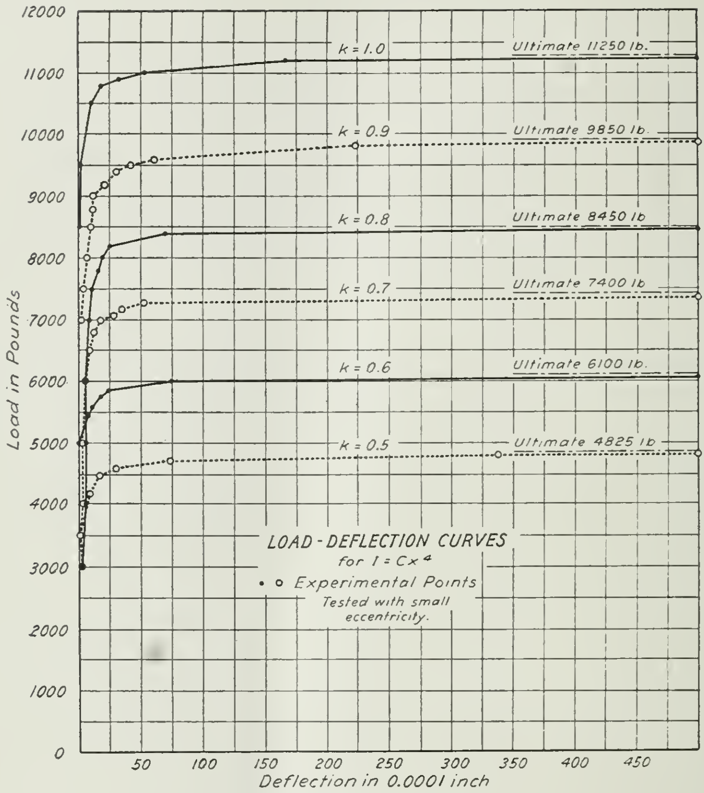

FIG. 11

Load-deflection curves for struts, $I = Cx^4$, Eccentricity very small.

16. STRENGTH RATIO OF STRUTS FOR WHICH $I=Cx^4$

The ratio of the strength of a tapered strut to the strength of a uniform strut the cross-section of which is equal to the maximum cross-section of the tapered strut may be called the *strength ratio*. From Equation (20) of Art. 10,

$$P_{max} = \left(\frac{q}{d}\right)^2 \times \frac{4EI_2(1-k)^2}{L^2}, \tag{1}$$

in which $\frac{q}{d}$ is the maximum value. The maximum load of a relatively long uniform strut is given by Euler's formula:

$$P = \frac{\pi^2 EI_2}{L^2} \tag{2}$$

Dividing the load of Equation (1) by that of Equation (2)

$$\text{strength ratio} = \left(\frac{q}{d}\right)^2 \times \frac{4(1-k)^2}{\pi^2}. \tag{3}$$

in which $\frac{q}{d}$ is the maximum angle in radians.

Since radians are reduced to degrees by multiplying by $\frac{\pi}{180}$

$$\text{strength ratio} = \left(\frac{q}{d} \times \frac{1-k}{90}\right)^3 \qquad (4)$$

in which $\frac{q}{d}$ is the maximum angle in degrees.

Table XVI gives the computations for a series of strength ratios.

TABLE XVI—Strength Ratio of Struts for which I = Cx^4, computed by Equation (4).

k	*1−k*	*max* $\frac{q}{d}$	$\frac{q}{d}(1-k)$	$\frac{q}{d} \times \frac{1-k}{90}$	*Strength ratio*
0.01	0.99	1°.800	1.7820	0.01980	0.0004
0.05	0.95	9°.004	8.5538	0.09504	0.0090
0.08	0.92	14°.423	13.2692	0.14743	0.0217
0.10	0.90	18°.055	16.2495	0.18055	0.0326
0.15	0.85	27°.273	23.1820	0.25758	0.0663
0.20	0.80	36°.820	27.4560	0.32739	0.1071
0.30	0.70	57°.757	40.4299	0.44922	0.2018
0.40	0.60	83°.064	49.8384	0.55376	0.3067
0.50	0.50	116°.243	58.1215	0.64579	0.4170
0.60	0.40	163°.90	65.560	0.72844	0.5306
0.70	0.30	241°.17	72.351	0.80390	0.6463
0.80	0.20	393°.14	78.628	0.87364	0.7632
0.90	0.10	844°.93	84.493	0.93881	0.8814
1.00	....	....	....	....	1.0000

Table XVII gives the experimental strength ratios. The numbers in the second column are the measured ultimate loads. In the fourth column these loads have been reduced for a diameter of one inch by division by the size correction factor. The fifth column gives the strength ratio obtained by dividing the reduced load for the tapered strut by the reduced load for the uniform strut. For purpose of comparison the theoretical strength ratios of Table XVI are given in the last column.

TABLE XVII—Experimental Strength Ratio of Struts for which I = Cx^4.

k	Ultimate measured load	Size correction factor	Load for 1-inch section	Strength ratio	
				Experimental	Theoretical
0.5	4825	1.028	4693	0.414	0.417
0.6	6100	1.021	5975	0.527	0.531
0.7	7100	1.018	7269	0.611	0.646
0.8	8950	0.988	8553	0.754	0.763
0.9	9850	0.989	9960	0.878	0.881
1.0	11250	0.992	11340	1.0	1.0

The solid line of Fig. 12 is drawn through the theoretical strength ratios of Table XVI. The points are the experimental ratios of Table XVII. The experimental points agree with the theoretical curve within the limits of accuracy of the measurements. It may, therefore, be inferred that the theoretical equations of Art. 9 are correct.

Above $k = 0.6$ the curve of Fig. 12 is practically straight. (The differences from Table XVI show a slight curvature upward, which is too small to be recognized in the drawing.) The broken line of Fig. 12 is a continuation of the straight portion of the theoretical curve. The equation of this broken line is strength ratio $= \frac{20}{17}(k-0.15)$.

This empirical formula may be used in the calculation of struts for which $I = Cx^4$, provided they are sufficiently long to fail by bending before the elastic limit is reached. Struts with small ends are liable to fail by crushing near the ends.

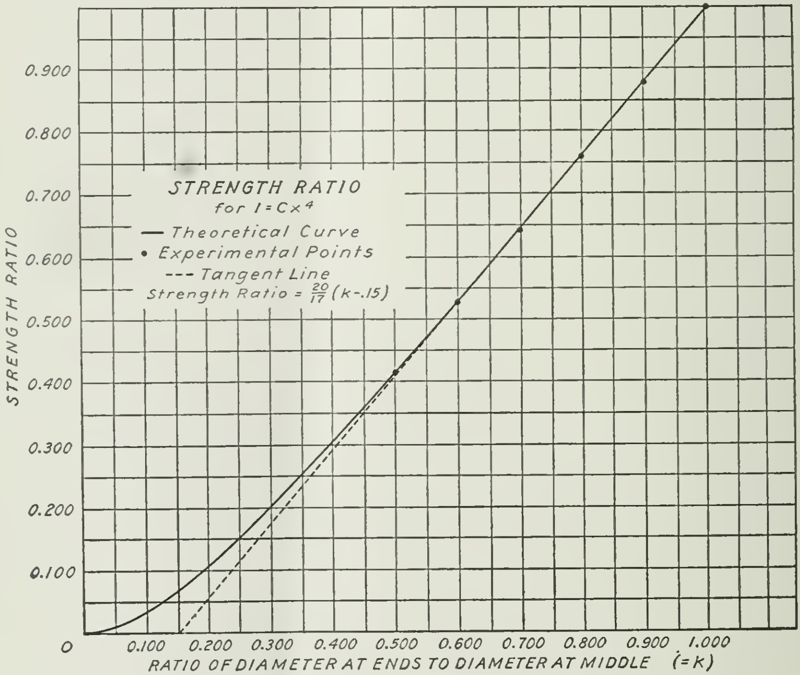

FIG. 12

Strength ratio for struts, $I=Cx^4$.

The theory indicates that a strut of this type with very small ends would deflect without limit under a very small load. If it were possible to make a very long strut of this type with extremely small ends, its strength might possibly be greater than the theory indicates. In the derivation of these formulas, as in the common derivation of all beam and column formulas, it has been assumed that the slope of the tangent is every where so small that $1+\left(\frac{dy}{dx}\right)^2$ is equal to unity. For a strut of this type which is very slender at the ends, the slope at the ends under a small load would be so great that this approximation would no longer be warranted.

17. RELATIVE EFFICIENCY OF STRUTS FOR WHICH $I=Cx^4$

The relative efficiency of a tapered strut may be defined as the ratio of the strength of the strut to the strength of a uniform strut of equal volume. Each half of a tapered strut for which $I=Cx^4$ and all sections are similar is frustum of a cone or pyramid. If unity is the area at the middle, the area at each end is k^2, and the average area is $\frac{k^2+k+1}{3}$. If $k=0.4$, for instance, the average area is $\frac{0.16+0.4+1}{3}=0.52$. Since the moment of inertia is proportional to the square of the area, the strength of a uniform strut of volume equal to the tapered strut $k=0.4$ is 0.52^2 times the strength of a strut of uniform section equal to the maximum section of the tapered strut. From Table XVI the strength ratio of the strut, $k=0.4$, is 0.3067. The relative efficiency is then

$$\frac{0.3067}{0.52^2}=\frac{0.3067}{0.2704}=1.134.$$

TABLE XVIII—Computation of Relative Efficiency of Struts for which $I=Cx^4$

k	$\frac{k^2+k+1}{3}$	Relative I for uniform strut	Strength ratio from Table XVI	Relative efficiency
0.1	0.3700	0.1369	0.0326	0.238
0.2	0.4133	0.1708	0.1071	0.621
0.3	0.4633	0.2147	0.2108	0.981
0.4	0.5200	0.2704	0.3067	1.134
0.5	0.5833	0.3403	0.4170	1.225
0.6	0.6533	0.4268	0.5306	1.243
0.7	0.7300	0.5309	0.6463	1.217
0.8	0.8133	0.6615	0.7632	1.153
0.9	0.9033	0.8160	0.8814	1.080

The table shows that when the diameter at the end is less than three-tenths of the diameter at the middle, the strength of the tapered strut is less than that of a uniform strut of equal volume. The maximum efficiency is secured when the diameter at the ends is about six-tenths of the diameter at the middle. The efficiency is only a little lower when $k=0.7$, and there is less danger of failure near the ends if the strut is relatively short.

18. TAPERED STRUTS FOR WHICH $I=Cx^3$

The equation of this strut is $I=Cx^3$, in which x is the distance from the origin of any section between one end and the middle, and the origin is beyond the end. The two halves of the strut are symmetrical with respect to the section at the middle. At the middle, $x=d$ and $I_2=Cd^3$. At the end which is between the origin of coordinates and the middle, $x=kd$ and $I_1=C(kd)^3$.

19. THEORY OF TAPERED STRUTS FOR WHICH $I=Cx^3$

$$EI\frac{d^2y}{dx^3}+Py=0. \tag{1}$$

Since $I=Cx^3$,

$$x^3\frac{d^2y}{dx^3}+\frac{Py}{EC}=0. \tag{2}$$

If $\frac{P}{EC}$ is represented by a single letter b, Equation (2) may be written

$$x^3\frac{d^2y}{dx^2}+by=0. \tag{3}$$

The solution to equation (3) is

$$y=(A+B\log x)\left[1-\frac{bx^{-1}}{\lfloor 2}+\frac{b^2x^{-2}}{\lfloor 2\,\lfloor 3}-\frac{b^3x^{-3}}{\lfloor 3\,\lfloor 4}+\frac{b^4x^{-4}}{\lfloor 4\,\lfloor 5}-\text{etc.}\right]$$
$$+\frac{Bx}{b}-B\left[\frac{bx^{-1}}{\lfloor 2}\left(\frac{1}{1}+\frac{1}{2}\right)-\frac{b^2x^{-2}}{\lfloor 2\,\lfloor 3}\left(\frac{1}{1}+\frac{2}{2}+\frac{1}{3}\right)+\frac{b^3x^{-3}}{\lfloor 3\,\lfloor 4}\right.$$
$$\left(\frac{1}{1}+\frac{2}{2}+\frac{2}{3}+\frac{1}{4}\right)-\frac{b^4x^{-4}}{\lfloor 4\,\lfloor 5}\left(\frac{1}{1}+\frac{2}{2}+\frac{2}{3}+\frac{2}{4}+\frac{1}{5}\right)+$$
$$\left.\text{etc.}\right], \tag{4}$$

in which $\lfloor 4\,\lfloor 5$ means the product of factorial 4 multiplied by factorial 5, etc.

The value of y and its second derivative from Equation (4) together satisfy Equation (3). Equation (4) is, therefore, a solution of Equation (3).

If each term in each bracket of Equation (4) is multiplied and divided by $\frac{x}{b}$, the Equation becomes

$$y=\frac{x}{b}(A+B\log x)\left[bx^{-1}-\frac{b^2x^{-2}}{\lfloor 2}+\frac{b^3x^{-3}}{\lfloor 2\,\lfloor 3}-\frac{b^4x^{-4}}{\lfloor 3\,\lfloor 4}+\text{etc.}\right]$$
$$+\frac{Bx}{b}-\frac{Bx}{b}\left[\frac{b^2x^{-2}}{\lfloor 2}\left(\frac{1}{1}+\frac{1}{2}\right)-\frac{b^3x^{-3}}{\lfloor 2\,\lfloor 3}\left(\frac{1}{1}+\frac{2}{2}+\frac{1}{3}\right)+\text{etc.}\right]. \tag{5}$$

If the infinite series in the first bracket of Equation (5) is represented by $f_1\ (bx^{-1})$, and the series in the second bracket is represented by $f_2(bx^{-1})$, Equation (5) may then be written

$$y = \frac{x}{b}\,(A+B\log x)\,f_1(bx^{-1}) + \frac{Bx}{b} - \frac{Bx}{b}f_2(bx^{-1}). \tag{6}$$

When Equation (6) is differentiated with respect to x

$$\frac{dy}{dx} = \frac{A+B+B\log x}{b}\,f_1\,(bx^{-1}) + \frac{B}{b} - \frac{B}{b}f_2\,(bx^{-1}) +$$

$$\frac{x}{b}(A+B\log x)\frac{d}{dx}\,f_1\,(bx^{-1}) - \frac{Bx}{b}\frac{d}{dx}\,f_2\,(bx^{-1}). \tag{7}$$

$$\frac{d}{dx}\,f_1\,(bx^{-1}) = -bx^{-2} + b^2\,x^{-3} - \frac{b^3x^{-4}}{\lfloor 2\,\lfloor 2} + \frac{b^4x^{-5}}{\lfloor 3\,\lfloor 3} - \text{etc.} \tag{8}$$

$$\frac{x}{b}\,(A+B\log x)\frac{d}{dx}\,f_1\,(bx^{-1}) = \frac{A+B\log x}{b}\Bigg[bx^{-1} - b^2x^{-2} +$$

$$\frac{b^3x^{-3}}{\lfloor 2\,\lfloor 2} - \frac{b^4x^{-4}}{\lfloor 3\,\lfloor 3} + \frac{b^5x^{-5}}{\lfloor 4\,\lfloor 4} - \text{etc.}\Bigg]. \tag{9}$$

$$\frac{d}{dx}f_2\,(bx^{-1}) = -\,b^2x^{-3}\left(\frac{1}{1} + \frac{1}{2}\right) + \frac{b^3x^{-4}}{\lfloor 2\,\lfloor 2}\left(\frac{1}{1} + \frac{2}{2} + \frac{1}{3}\right)$$

$$-\text{ etc.} \tag{10}$$

$$-\frac{Bx}{b}\frac{d}{dx}\,f_2\,(bx^{-1}) = \frac{B}{b}\Bigg[\;b^2x^{-2}\left(\frac{1}{1} + \frac{1}{2}\right) - \frac{b^3x^{-3}}{\lfloor 2\,\lfloor 2}\Bigg(\frac{1}{1} +$$

$$\frac{2}{2} + \frac{1}{3}\Bigg) + \frac{b^4x^{-4}}{\lfloor 3\,\lfloor 3}\left(\frac{1}{1} + \frac{2}{2} + \frac{2}{3} + \frac{1}{4}\right) - \frac{b^5x^{-5}}{\lfloor 4\,\lfloor 4}\Bigg(\frac{1}{1} + \frac{2}{2}$$

$$+\frac{2}{3} + \frac{2}{4} + \frac{1}{5}\Bigg)\Bigg] \tag{11}$$

If the series in the bracket of Equation (9) is represented by $f_3\ (bx^{-1})$, and the series in the bracket of Equation (11) is represented by $f_4\ (bx^{-1})$, Equation (7) may then be written

$$\frac{dy}{dx} = \frac{A+B+B\log x}{b}\ f_1\ (bx^{-1}) + \frac{B}{b} - \frac{B}{b}\ f_2\ (bx^{-1}) -$$

$$\frac{A+B\log x}{b}\ f_3\ (bx^{-1}) + \frac{B}{b}\,f_4\ (bx^{-1}). \quad (12$$

Appendix D gives the values of $f_1(x)$, $f_2(x)$, $f_3(x)$, and $f_4(x)$ for values of x from 0 to 20. From 0 to 10, the functions were computed by means of the series. From 10 to 20, the functions for integral values of x were computed by the series, while the functions for the fractional values were calculated by Professor Coddington by Stirling's method of interpolation. All the computations were carried to seven decimal places and all the results were checked by differences. It is believed, therefore, that the tables as given to five places are correct.

For struts with small values of k, it was necessary to compute these functions for intervals of hundredths. This was done by Newton's method of interpolation from the values for tenths as given in the tables.

The known conditions for the elimination of the constants are:

1. At the middle, $x = d$ and $\frac{dy}{dx} = 0$.
2. At the end, $x = kd$ and $y = e$.

From the first condition,

$$(A+B+B\log d)\, f_1\left(\frac{b}{d}\right) + B - B f_2\left(\frac{b}{d}\right) = (A+B\log d)\, f_3\left(\frac{b}{d}\right) - B f_4\left(\frac{b}{d}\right). \qquad (13)$$

From the second condition,

$$e = \frac{kd}{b}\left[(A+B\log kd)\, f_1\left(\frac{b}{kd}\right) + B - B f_2\left(\frac{b}{kd}\right)\right] \qquad (14)$$

At the middle, where $x = d$, the deflection is

$$y_{\max} = \frac{d}{b}\left[(A+B\log d)\, f_1\left(\frac{b}{d}\right) + B - B f_2\left(\frac{b}{d}\right)\right]. \qquad (15)$$

At any distance x from the origin, provided x is not greater than d,

$$y = \frac{x}{b}\left[(A+B\log x)\, f_1\left(\frac{b}{x}\right) + B - B f_2\left(\frac{b}{x}\right)\right]. \qquad (16)$$

From Equation (13)

$$\frac{A}{B} + \log d = \frac{f_1\left(\frac{b}{d}\right) + f_4\left(\frac{b}{d}\right) - f_2\left(\frac{b}{d}\right) + 1}{f_3\left(\frac{b}{d}\right) - f_1\left(\frac{b}{d}\right)} = \frac{S_1}{S_2}, \qquad (17)$$

In which S_1 and S_2 are brief symbols for the numerator and denominator of the fraction.

From Equation (14)

$$B = \frac{\frac{eb}{kd}}{\left(\frac{A}{B} + \log kd\right) f_1\left(\frac{b}{kd}\right) - f_2\left(\frac{b}{kd}\right) + 1}; \qquad (18)$$

$$\text{in which } \frac{A}{B} + \log kd = \frac{A}{B} + \log d + \log k. \qquad (19)$$

From Equation (15)

$$y_{\max} = \frac{Bd}{b}\left(\frac{A}{B} + \log d\right) f_1\left(\frac{b}{d}\right) - f_2\left(\frac{b}{d}\right) + 1. \qquad (20)$$

Substituting the value of B from Equation (18)

$$y_{\max} = \frac{e}{k}\,\frac{\left(\frac{A}{B} + \log d\right) f_1\left(\frac{b}{d}\right) - f_2\left(\frac{b}{d}\right) + 1}{\left(\frac{A}{B} + \log kd\right) f_1\left(\frac{b}{kd}\right) - f_2\left(\frac{b}{kd}\right) + 1} = \frac{e}{k}\,\frac{S_4}{S_3}. \qquad (21)$$

A similar expression is easily found for the deflection at a distance x from the origin.

20. CALCULATION OF DEFLECTION AND LOAD

Appendix E illustrates the computation of deflection. The values S_1, S_2, and S_4, and of $\frac{A}{B} + \log d$ apply to all tapers. The lower part of the table, applies only to the taper $k = 0.6$. To save space in the table, $\frac{b}{d}$ and $\frac{b}{kd}$ are represented by u and v respectively.

It will be noticed that $\frac{A}{B} + \log d$ changes sign by passing through infinity near the point $\frac{b}{d} = 6.6$.

When S_3 becomes zero, $y_{\max}$ is infinite. To find the value of $\frac{b}{d}$ for this critical load, it is necessary to solve for $S_3 = 0$ by trial and error. It is better, however, to find the value of $\frac{b}{d}$ at which the ratio $\frac{S_3}{S_4}$ (which is proportional to the reciprocal of the deflection) passes through zero, since the values of this ratio as functions of $\frac{b}{d}$ vary more uniformly than the values of S_3.

For $k = 0.6$ some of the values of $\frac{S_3}{S_4}$ are

$\frac{b}{d}$	$\frac{S_3}{S_4}$	Differences	
7.2	3505	−857	28
7.8	2648	−829	32
8.4	1819	−797	33
9.0	1022	−766	31
9.6	256	−735	
10.2	−479		

The second differences are nearly constant. Interpolation between 9.6 and 10.2 on the assumption that the line is straight throughout that interval gives $\frac{b}{d} = 9.817$ when $\frac{S_3}{S_4} = 0$. Interpolation by Newton's method for intervals of 0.06 gives $\frac{S_3}{S_4} = 0.0038$ when $\frac{b}{d} = 9.78$ and $\frac{S_3}{S_4} = -0.0034$ when $\frac{b}{d} = 9.84$. A linear interpolation between these results gives 9.8117 as the critical value of $\frac{b}{d}$.

At the middle $I_2 = Cd^3$; hence $C = \frac{I_2}{d^3}$.

$$P = bEC = bE\frac{I_2}{d^3} = \frac{b}{d}E\frac{I_2}{d^2}. \qquad (1)$$

$$d = \frac{L}{2(1-k)}.$$

$$P = \frac{b}{d} \times \frac{4EI_2(1-k)^2}{L^2}. \qquad (2)$$

The tested struts of this type were 36 inches long, 1.25 inches wide, and 1 inch thick at the middle. The nominal $I_2 = \frac{5}{48}$.

The modulus of elasticity was taken to be 30,880,000. For $k = 0.6$ the size correction factor was found to be 0.9677. For this strut

$$P = \frac{b}{d} \frac{30\,380\,000 \times 5 \times 4 \times 0.16 \times 0.9677}{48 \times 36 \times 36} = 1537.2 \frac{b}{d}.$$

The loads given in Appendix E were calculated from this constant. The maximum load for this strut is 1537.2 × 9.8117 = 15,436 pounds.

21. DETERMINATION OF E.

The struts of the type $I = Cx^3$ were made from cold-rolled bars which were 1.25 inches wide and 1 inch thick. One of these bars was tested in compression for the modulus of elasticity. The bar was placed on the cylindrical heads and adjusted to the position of minimum eccentricity. The deformation was measured directly by two Ames dials at equal distances from the test piece in the plane of the axis. (Three gages at the vertices of an equilateral triangle with the axis of the bar at the center is the ideal arrangement, but with minimum eccentricity in one direction and very little eccentricity in the other, it was not thought necessary to make this adjustment.)

Table XIX gives the results of one run. Other runs were consistent. The compression in the gage length of 33 inches for a difference of load amounting to 12,058 pounds per square inch is given in the last column of the table. As the load increases, this difference in compression increases from 126 to 130.5 divisions. The average is 128.25. The average modulus from these readings is

$$E = \frac{12058 \times 33}{0.012825} = 31,030,000.$$

Figure 13 is the stress-strain diagram from the unit stresses and unit deformations of Table XIX. The line has a slight curvature which is scarcely noticeable.

Table XX gives the ultimate load for the uniform rectangular strut of Table XIX.

By Euler's formula, the modulus of elasticity is found to be

$$E = \frac{24300 \times 36^2 \times 12}{9.8696 \times 1.246 \times 0.9984^3} = 30,880,000,$$

which is slightly smaller than the result obtained from the average of the compression readings and slightly larger than the result from the readings at the larger loads. This value of 30,880,000 is probably as reliable as any and is used in the calculations of all struts of the type $I = Cx^3$, which were made from similar 1.25 by 1-inch cold-rolled bars.

TABLE XIX—Compression Test of Uniform Strut of Rectangular Section for the Determination of E. Tested July 10, 1922.

(Length 36 inches, breadth 1.246 inches; thickness 0.9984 inch; area 1.244 square inches. Compression measured with two Ames dials on a gage length of 33 inches.)

Load		Gages				Average comp.		Comp. for 12058 lb. per sq. in. 0.0001
		Left		Right				
Total	Unit	Read	Comp.	Read	Comp.	In gage length. 0.0001 inch	per inch 0.000001	
625	502	849		1625				
1250	1005	855	6	1631	6	6	18	....
1875	1507	860	11	1637	12	11.5	35	....
2500	2010	865	16	1641	16	16	48	....
3125	2512	870	21	1648	23	22	67	....
3750	3014	876	27	1652	27	27	82	....
4375	3517	881	32	1658	33	32.5	98	....
5000	4020	889	40	1662	37	38.5	117	....
5625	4522	892	43	1668	43	43	130	....
6250	5024	899	50	1672	47	48.5	147	....
6875	5526	903	54	1679	51	54	164	....
7500	6029	908	59	1682	57	58	176	....
8125	6531	912	63	1688	63	63	191	....
9375	7536	923	74	1700	75	74.5	226	....
10000	8039	930	81	1704	79	80	242	....
10625	8541	934	85	1710	85	85	258	....
11875	9546	943	94	1720	95	94.5	286	....
12500	10048	950	101	1726	101	101	306	....
13125	10551	953	104	1730	105	104.5	317	....
13750	11053	960	111	1736	111	111	336	....
14375	11555	965	116	1741	116	116	352	....
15000	12058	970	121	1746	121	121	367	....
15625	12560	976	127	1751	126	126.5	383	126.5
16250	13063	981	132	1757	132	132	400	126.0
16875	13565	988	139	1762	137	138	418	126.5
17500	14067	993	144	1769	144	144	436	128.0
18125	14570	1000	151	1775	150	150.5	456	128.5
18750	15072	1003	154	1781	156	155	470	128.0
19375	15575	1010	161	1788	163	162	491	129.5
20000	16077	1016	167	1793	168	167.5	508	129.0
20625	16580	1021	172	1799	174	173	524	130.0
21250	17082	1026	177	1806	181	179	542	130.5
10000	8039	931	82	1704	79	80.5	Average	128.25
5000	4020	890	41	1660	35	38	....	....
2500	2010	870	21	1640	15	18	....	....
625	502	851	2	1622	-3	-0.5	....	....

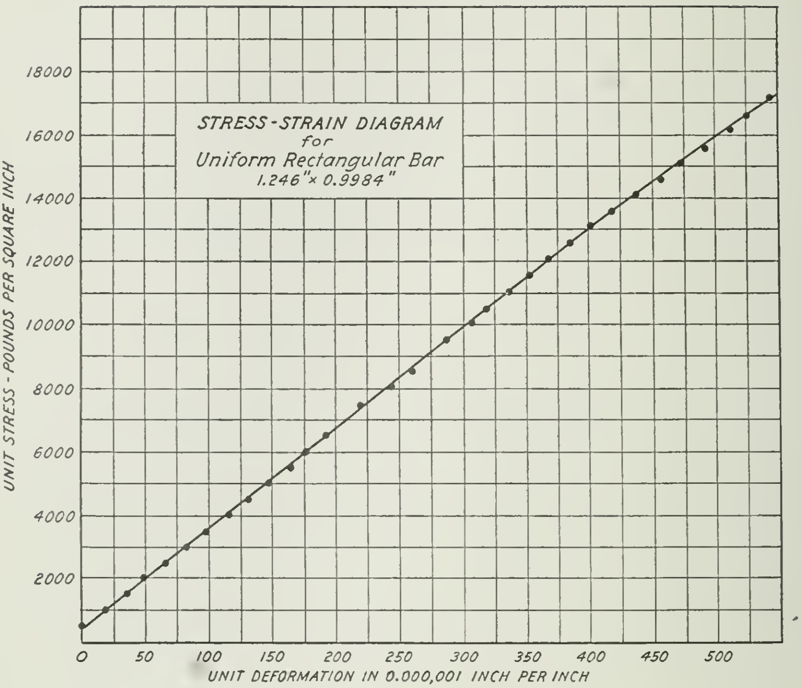

Fig. 13
Stress-strain diagram for uniform rectangular bar.

Table XXI gives the deflection of the uniform bar of Tables XIX and XX for an eccentricity of 0.0100 inch. The upper curve of Fig. 14 is drawn from the theoretical deflections computed from $E=30{,}880{,}000$. The points are the experimental results of Table XXI. Except near the beginning and end, these points lie above the curve. If the modulus of elasticity were taken a little higher, to correspond with the smaller compression readings of Table XIX, the agreement would be closer.

22. TEST OF UNIFORM RECTANGULAR STRUT

Table XX gives the ultimate load for the uniform rectangular bar of Table XIX.

TABLE XX—Compression Test of Strut of Uniform Rectangular Section, Tested with Small Eccentricity, March 11, 1922.

Total load in pounds	Deflection in ten-thousandths of an inch					
	Upper		Middle		Lower	
	Reading	Def.	Reading	Def.	Reading	Def.
400	3280		1722		2882	
2000	3280	0	1721	-1	2882	0
4000	3280	0	1721	-1	2882	0
6000	3280	0	1721	-1	2882	0
8000	3279	-1	1721	-1	2882	0
10000	3279	-1	1721	-1	2882	0
12000	3279	-1	1721	-1	2882	0
13000	3278	-2	1721	-1	2884	2
14000	3278	-2	1722	0	2884	2
15000	3277	-3	1721	-1	2885	3
16000	3276	-4	1721	-1	2885	3
17000	3276	4	1721	-1	2885	3
18000	3277	-3	1722	0	2886	4
19000	3277	-3	1723	1	2887	5
20000	3277	-3	1724	2	2888	6
21000	3279	-1	1727	5	2889	7
22000	3280	0	1730	8	2890	8
23000	3281	1	1734	12	2892	10
23200	3282	2	1738	16	2894	12
23400	3283	3	1740	18	2896	14
23600	3287	7	1747	25	2899	17
23800	3289	9	1747	25	2899	17
24000	3290	10	1753	31	2902	20
24200	3300	20	1779	57	2912	30
24300	4030	750	3252	1530	3639	757
			Shifted easily with one finger			
400	3283	3	1728	6	2884	2

The upper curve of Fig. 15 is plotted from Table XX with the small deflections for loads below 20,000 pounds omitted.

TABLE XXI—Deflection at the Middle of Strut of Uniform Rectangular Section. Tested March 11, 1922.

(Dimensions 1.246 in. by 0.9984 inch. Difference of Eccentricity 0.0200 inch.)

Total load in pounds	Deflection in 0.0001 inch			Def. for an eccentricity of 0.0100 inch	Eccentricity	
	Left	Right	Sum		Left	Right
1000	9	1	10	5		
2000	12	9	21	10	119	91
3000	16	18	34	17	94	106
4000	21	24	45	22	93	107
5000	29	30	59	29	98	102
6000	36	39	75	37	96	104
7000	45	50	95	47	95	105
8000	52	60	112	56	93	107
9000	62	70	132	66	94	106
10000	73	83	156	78	94	106
11000	86	100	186	93	93	107
12000	100	119	219	109	91	109
13000	121	140	261	130	93	107
14000	146	172	318	159	92	108
15000	176	206	382	191	92	108
15500	192	226	418	209	92	108
16000	210	246	456	228	92	108
16500	233	270	503	251	93	107
17000	253	305	558	279	91	109
17500	282	335	617	380	91	109
18000	320	383	703	351	91	109
18500	358	425	783	391	91	109
19000	402	482	884	442	91	109
19200	420	508	928	464	91	109
19400	440	533	973	486	90	110

23. TEST PIECES OF THE TYPE I = Cx^3

The form of these test pieces is shown in Fig. 1_b. All the struts were made from 1.25-inch by 1-inch cold-rolled bars. The width was left as rolled and the thickness was varied by planing. On account of the flexibility of the bars, there were some variations from the nominal dimensions. Table XXII shows the magnitude of these variations and the method of computing the size correction factor.

TABLE XXII—Size Correction for Strut for which $I = Cx^3$ and $k = 0.6$.

Dist. from end	Thickness				Error		Relative times distance
	Upper	Lower	Average	Nominal	Actual	Relative	
2	0.637	0.635	0.6360	0.6444	— 84	130	— 260
4	0.677	0.674	0.6755	0.6889	—134	—194	— 776
6	0.718	0.714	0.7160	0.7333	—173	—235	—1410
8	0.763	0.758	0.7605	0.7778	—173	—222	—1776
10	0.809	0.806	0.8075	0.8222	—147	—179	—1790
12	0.857	0.855	0.8560	0.8667	—107	—123	—1476
14	0.904	0.902	0.9030	0.9111	— 81	— 89	—1246
16	0.953	0.953	0.9530	0.9556	— 26	— 27	— 432
18	0.997	0.997	0.9970	1.0000	— 30	— 30	— 540
							—9702

—0.9702 ÷ 90 = —0.01078; —0.01078 × 3 = —0.03234

Size correction factor = 1—0.03234 = 0.9677.

24. TEST OF STRUTS FOR WHICH $I = Cx^3$

Table XXIII gives the results of the test of the strut for which $k = 0.6$. The theoretical curve for this strut, part of the data for which is given in Appendix E, is plotted in Fig. 14. The points, most of which are below the curve, are plotted from the data of the first and fifth columns of Table XXIII.

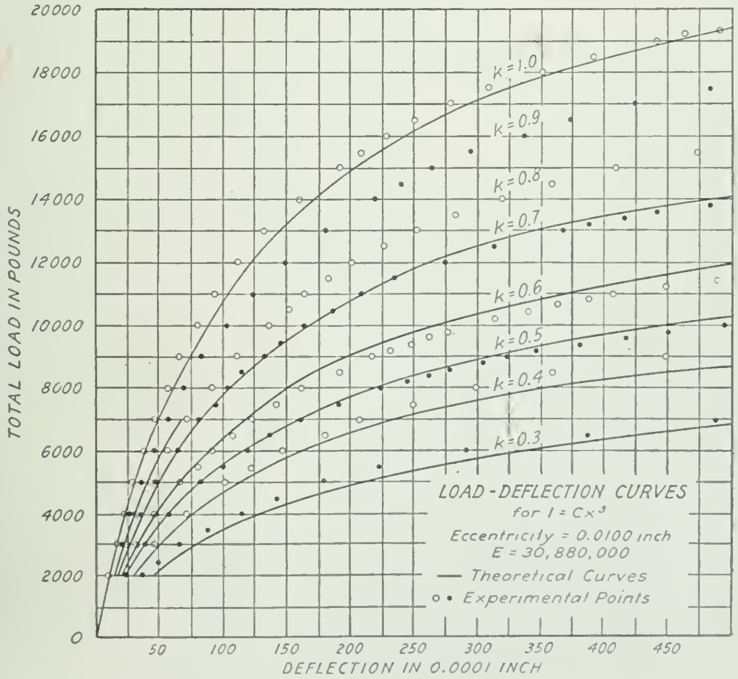

FIG 14

Load-deflection curves with eccentricity=0.0100 inch for struts, $I = Cx^3$.

Figure 14 also gives the curves for the other struts of this type. No theoretical curve is drawn for $k = 0.9$, and only a short portion for $k = 0.8$. These could not be computed completely on account of the limitation of the values of $f_1(x), f_2(x)$, etc, which are given in Appendix D. To calculate the deflections for $k = 0.8$ for all loads, the tables must be carried to values of x above 60, and to calculate the deflections for $k = 0.9$, the tables must be carried to functions of 250. The tables are not sufficient to calculate the ultimate load for $k = 0.7$, but go far enough to carry the deflections beyond the limits required by the figure.

TABLE XXIII—Deflection at the Middle of Strut of the Type $I=Cx^3$ and $k=0.6$. Tested Jan. 14, 1922.

(Nominal dimensions, width 1.25 inches; thickness at the middle, 1 inch; thickness at the ends, 0.600 inch. Size correction factor for I, 0.9677. Difference of eccentricity, 0.0194 inch.)

Total load in pounds	Deflection in 0.0001 inch			Deflection for eccen. of 0.0100 in.	Eccentricity	
	Left	Right	Sum		Left	Right
1000	9	8	17	9	103	91
2000	16	19	35	18	89	105
3000	30	34	64	33	91	103
4000	46	48	94	48	95	99
5000	67	66	133	68	98	96
5500	82	77	159	82	100	94
6000	93	89	182	94	99	95
6500	104	105	209	108	97	97
7000	120	120	240	124	97	97
7500	139	139	278	143	97	97
8000	157	161	318	164	96	98
8500	184	187	371	191	96	98
9000	214	209	423	218	98	96
9200	227	226	453	233	97	97
9400	241	240	481	248	97	97
9600	257	252	509	262	98	96
9800	267	268	535	276	97	97
10000	288	284	572	295	98	96
10200	302	310	612	315	96	98
10400	325	337	662	341	95	99
10600	349	356	705	363	96	98
10800	375	378	753	388	97	97
11000	402	388	790	407	99	95
11200	430	441	871	449	96	98
11400	469	478	945	487	96	98

The experimental and theoretical results agree well for $k = 0.7$. For $k=0.6$ and $k=0.5$, the deflections are larger than the theory indicates. A slightly smaller modulus of elasticity would account for the difference. For $k=0.4$ and $k=0.3$, the deflections are smaller than required by theory. These struts were made by cutting down the struts $k=0.7$ and $k=0.6$. They were tested last, after the cylindrical heads had been subjected to loads of nearly 30,000 pounds. It is possible that these large loads slightly flattened the heads, so that they no longer rolled as freely as at first.

Table XXIV gives the test of the strut $k=0.6$ with small eccentricity

TABLE XXIV—Compression Test of Strut for which $I=Cx^3$ and $k=0.6$. Tested with Small Eccentricity, January 14, 1922.

Total load in pounds	Deflection in ten-thousandths of an inch					
	Upper		Middle		Lower	
	Reading	Def. right	Reading	Def. right	Reading	Def. right
400	1425		1833		1011	
1000	1423	2	1831	2	1011	0
2000	1421	4	1831	2	1010	1
3000	1420	5	1831	2	1010	1
4000	1419	6	1831	2	1004	7
5000	1418	7	1831	2	1002	9
6000	1414	11	1832	1	1000	11
7000	1412	13	1833	0	998	13
8000	1410	15	1834	−1	996	15
9000	1408	17	1831	2	990	21
10000	1402	23	1829	4	986	25
11000	1398	27	1821	12	979	32
12000	1388	37	1791	42	958	53
13000	1369	62	1769	64	942	69
13500	1338	87	1725	108	918	93
14000	1281	144	1640	193	865	146
14500	922	503	1035	798	522	489
14700	0	1425	Off		Off	

A second run gave an ultimate load of 14800 pounds.

Table XXIV shows greater deflections at the sixth-points than at the middle until the load passed 12,000. This was caused by lack of straightness of the strut. A measurement from a plane surface showed that more material had been cut from one side than from the other. A similar effect was shown with the eccentricities of 0.0100 inch and 0.0020 inch. With both eccentricities, when the deflection was toward the right, as in Table XXIV, the deflections at the sixth-points were relatively large. When deflections were toward the left, on the other hand, the readings of the upper and lower gages were relatively small compared with the deflections at the middle.

Figure 15 shows the curves for the tests of the struts $I=Cx^3$ with small eccentricities. The entire curve is given for the strut $k=0.4$, for which the negative deflection was relatively large. To avoid confusion, the readings at the lower loads are omitted for the heavier struts. For the struts $k=0.4$, when tested with an eccentricity of about 0.0100 inch, the shift of eccentricity was from the left toward the right. When this strut was tested with small eccentricity, the deflection was first toward the left and changed to the right at a load of 7,000 pounds.

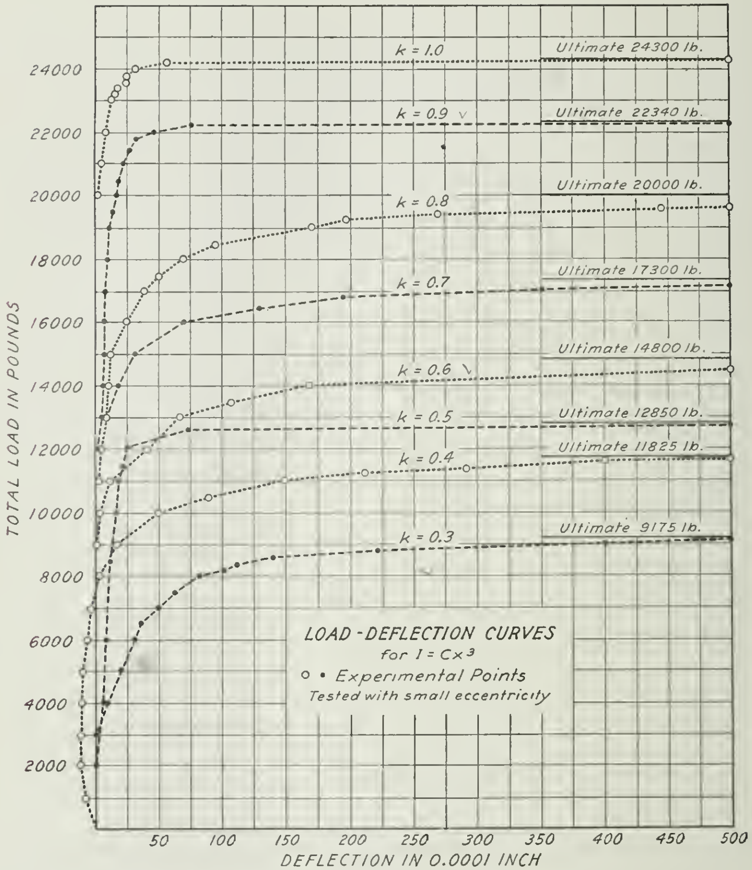

FIG. 15

Load-deflection curves for struts, $I = Cx^3$. Eccentricity very small.

When the struts of the type $I = Cx^4$ were tested, the cylindrical heads were used in the same position for all experiments, and the shift of eccentricity was always in the same direction. For the later experiments, the heads were frequently removed from the stirrups and were replaced at random. As a result, the amount and direction of the shift of eccentricity varied. For the strut $I = Cx^3$, $k = 0.7$, the eccentricity toward the left was 78 when the load was 5,000 pounds, 96 when the load was 9,000 pounds, and 102 when the load was 13,000 pounds. When this strut was tested with small eccentricity, the deflection was 8 units towards the right for a load of 6,000 pounds and for a load of 8,000, and was toward the left for a load of 12,000. It is evident that the line of the resultant load shifted from left toward right, so that the eccentricity (which is here defined as the displacement of the axis of the strut from the line of the load) shifted from right to left. With the small eccentricity the line of the load was at first to the left of the axis, the eccentricity was toward the right, and the deflection was also toward the right. As the line of the load shifted toward the right, the eccentricity decreased and finally changed sign. For the strut $k = 0.5$ there was little change in eccentricity. When this strut was tested with little eccentricity, the deflection at the middle for all loads was in one direction. This strut, like the one for $k = 0.7$, was lacking in symmetry, which made the deflections at the sixth-points negative until the load reached 12,000 pounds

25. STRENGTH RATIO OF STRUTS FOR WHICH $I = Cx^3$

From Equation (2) of Art. 20

$$P = \frac{b}{d} \times \frac{4EI_2(1-k)^2}{L^2}$$

Dividing by the ultimate strength of a uniform strut of moment of inertia I_2 as given by Euler's formula,

$$\text{strength ratio} = \frac{b}{d}\ \frac{4EI_2\ (1-k)^2}{L^2} \times \frac{L^2}{\pi^2\ EI_2} = \frac{b}{d} \times \frac{4\ (1-k)^2}{\pi^2}$$

The strength ratios are given in Table XXV,

TABLE XXV—Strength Ratio of Struts for which $I = Cx^3$

k	Maximum $\frac{b}{d}$	$1-k$	$4(1-k)^2$	$4\frac{b}{d}(1-k)^2$	Strength ratio	
					From formula	From eq'n of curve
0.01	0.0358	0.99	3.9204	0.14305	0.01422	....
0 05	0.1878	0.95	3.61	0.67796	0.06869	0.0708
0.10	0.3960	0.90	3.24	1.28304	0.13000	0.1306
0.15	0.6412	0.85	2.89	1.85306	0.18775	0.1874
0.20	0.9326	0.80	2.56	2.38746	0.24190	....
0.25	1.2921	0.75	2.25	2.90720	0.29456	0.2946
0.30	1.7414	0.70	1.96	3.41344	0.34585	....
0.35	2.3237	0.65	1.69	3.92708	0.39789	0.3960
0.40	3.0515	0.60	1.44	4.39416	0.44522	....
0.45	4.0125	0.55	1.21	4.85508	0 49192	0.4937
0.50	5.3457	0.50	1.00	5.34570	0 54163	....
0.60	9.8062	0.40	0.64	6.27596	0 63589	....
0.65	13.7540	0.35	0.49	6.73949	0.68285	0.6823
0.70	....	....	....	....	....	0 7284
0.80	....	....	....	....	....	0.8194
0.90	....	....	....	....	....	0 9085

The maximum values of $\frac{b}{d}$ were not calculated for values of k greater than 0.65, on account of the labor required to extend the tables of Appendix D. The strength ratios for larger values of k were computed by means of the equation of the curve of k and strength ratio through the points $k = 0.2, 0.3, 0.4, 0.5, 0.6$, and 1.0. To avoid decimals, the values of k were taken as integers instead of tenths, and the values of the strength ratio were regarded as whole numbers to five significant figures. With the origin at 4 for symmetry, the equation of the fifth degree is

$$\text{strength ratio} = 44522 + bx + cx^2 + dx^3 + ex^4 + fx^5 .$$

For $k = 0.2, x = -2$ and the equation becomes

$$-2b + 4c - 8d + 16e - 32f = 24190 - 44522 = -20332 \quad (1)$$

For $k = 0.3$, $x = -1$; for $k = 0.5$, $x = 1$; for $k = 0.6$, $x = 2$; and for $k = 1.0, x = 6$. These values give four additional equations similar to Equation (1). From these five equations, the constants are found to be; $b = 9769.98$, $c = -144.625$, $d = 18.7084$, $e = -3.3750$, and $f = 0.30833$. The values of the strength ratio in the last column of Table XXV were computed from these constants. Except for $k = 0.05$, which lies at a point where there is considerable curvature, the error is less than two in the third place. For the portion above $k = 0.6$, the line is nearly straight. It is believed, therefore, that the values of the strength ratio for $k = 0.7$, 0.8, and 0.9 are correct to within a small fraction of one per cent.

Table XXVI gives the experimental strength ratios and a comparison with the theoretical values. The measured ultimate loads are given in the third column. In the fourth column, the loads are reduced for the nominal dimensions by dividing the ultimate measured loads by the size correction factor. The ratio of the reduced load to the reduced load for the uniform strut is the experimental strength ratio.

The solid line of Fig. 16 gives the theoretical strength ratio. The line is concave downward with the greatest curvature at the lower end. At the upper end it is nearly straight. The points represent the experimental results. The broken line of Fig. 16, which is tangent to the upper part of the curve, gives a close approximation to the correct strength ratio for all values of k above $k = 0.3$. The equation of this straight line is

$$\text{strength ratio} = 0.090 + 0.91\,k.$$

TABLE XXVI—Experimental Strength Ratio for Struts for which $I = Cx^3$.

(Nominal dimensions of struts: length 36 inches; width, 1.25 inches; thickness at middle, 1 inch.)

k	Size correction factor	Ultimate load		Strength ratio	
		Actual	Reduced for nominal size	Experimental	Theoretical
0.3	1.0210	9175	8990	0.367	0.346
0.4	1.0326	11825	11450	0.467	0.445
0.5	0.9891	12850	12990	0.530	0.542
0.6	0.9677	14800	15290	0.624	0.636
0.7	1.0076	17300	17160	0.700	0.728
0.8	0.9897	20000	20210	0.825	0.819
0.9	1.0000	22340	22340	0.912	0.910
1.0	0.9920	24300	24500		1.000

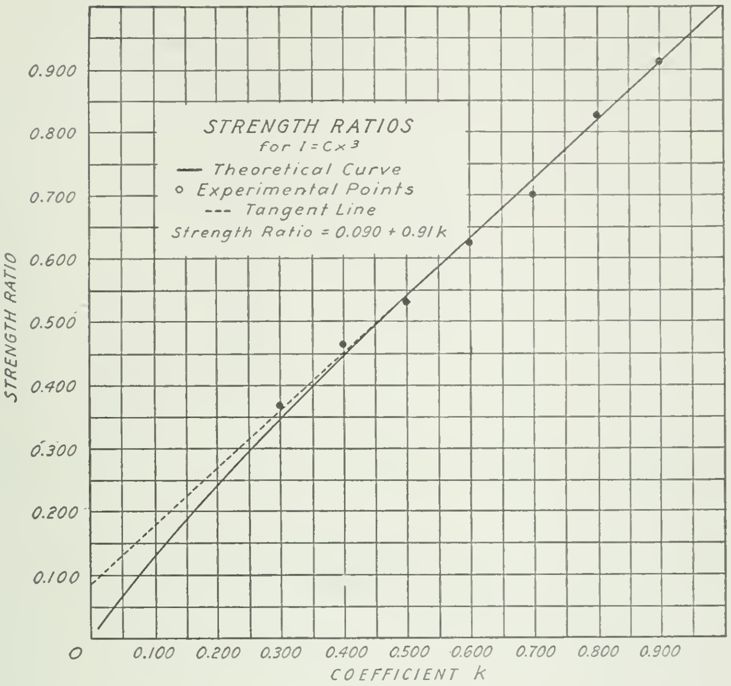

FIG. 16
Strength ratio for struts, $I=Cx^3$.

26. RELATIVE EFFICIENCY OF STRUTS FOR WHICH $I=Cx^3$

Table XXVII gives the relative efficiency of struts of constant width and variable thickness for which $I=Cx^3$. When $k=0.4$, for instance, the average thickness is seven-tenths of the thickness at the middle. The moment of inertia of a uniform strut containing the same amount of material is 0.343 as great as the moment of inertia of a strut of uniform section equal to that of the tapered strut at the middle. The theoretical strength ratio of the tapered strut $k=0.4$ is 0.44522. The relative efficiency, which is the ratio of the strength of the tapered strut to the strength of a uniform strut of equal volume is $0.44522 : 0.343 = 1.2980$.

The last column of Table XXVII shows that the greatest efficiency is obtained when k is about 0.4, that is when the ratio of the moment of inertia at the end to the moment of inertia at the middle is 0.064. Long struts of this type, for which the ratio of the radius of gyration at the middle to the length is large (150 or more), may have a value of k as low as 0.4. Struts which are relatively shorter, and, therefore, likely to fail by crushing before the critical load is reached, should be larger at the ends.

TABLE XXVII—Relative Efficiency of Struts for which I=Cx³ and Breadth is Constant.

k	Average thickness	Relative I for uniform strut=t³	Strength ratio from Table XXV	Relative efficiency
0.1	0.55	.166375	0.13000	0.7814
0.2	0.60	0.216	0.24190	1.1199
0.3	0.65	0.274625	0.34585	1.2593
0.4	0.70	0.343	0.44522	1.2980
0.5	0.75	0.421875	0.54163	1.2838
0.6	0.80	0.512	0 63589	1.2419
0.7	0.85	0.614125	0.7284	1.1861
0.8	0.90	0.729	0.8194	1.1240
0.9	0.95	0.857375	0.9085	1.0596

A strut may be made with all sections similar and with moment of inertia proportional to the cube of x.

$$I=Cx^3=C_1 r^4, \qquad (1)$$

in which C_1 is a constant which depends upon the form of the sections, and r is any transverse dimension.

$$r=C_2 x^{\frac{3}{4}}, \qquad (2)$$

$$\text{area}=C_3 x^{\frac{3}{2}}, \qquad (3)$$

in which C_2 and C_3 are constants which depend upon the form of the sections and upon the dimension which is taken as r.

$$\text{volume} =C_3 \int x^{\frac{3}{2}} dx = \frac{2}{5} C_3 \Big[x^{\frac{5}{2}} \Big]_{kd}^{d} = C_3 \frac{2}{5} d^{\frac{5}{2}} (1-k^{\frac{5}{2}}). \quad (4)$$

The average area is obtained by dividing the volume by one-half the length.

$$\frac{L}{2} =d(1-k),$$

$$\text{average area} =\frac{2}{5} \frac{C_3 d^{\frac{3}{2}} (1-k^{\frac{5}{2}})}{1-k}. \qquad (5)$$

The area at the middle is $Cd^{\frac{3}{2}}$. The ratio of the average area to the area at the middle is $\frac{2}{5} \frac{(1-k^{\frac{5}{2}})}{1-k^2}$ · Since the moments of inertia of similar figures are proportional to the squares of the areas, the ratio of the strength of a uniform strut of volume equal to that of a given tapered strut to the strength of a uniform strut of section equal to the maximum section of the tapered strut is $\frac{4(1-k^{\frac{5}{2}})^2}{25(1-k)^2}$.

Table XXVIII gives the calculations for this form of strut. The relative efficiencies are obtained by dividing the strength ratios of the sixth column of Table XXV (or the fourth column of Table XXVII) by $\frac{4(1-k^{\frac{5}{2}})^2}{25(1-k)^2}$.

TABLE XXVIII—**Relative Efficiency of Struts for which $I=Cx^3$ and all Sections are Similar.**

k	k^5	$k^{\frac{5}{2}}$	$1-k^{\frac{5}{2}}$	$\frac{1-k^{\frac{5}{2}}}{1-k}$	$\frac{4(1-k^{\frac{5}{2}})^2}{25(1-k)^2}$	Relative efficiency
0.1	0.00001	0.00316	0.99684	1.1076	0.19629	0.6623
0.2	0.00032	0.01789	0.98211	1.2276	0.24112	1.0029
0.3	0.00243	0.04929	0.95071	1.3582	0.29515	1.1718
0.4	0.01024	0.10195	0.89805	1.4967	0.35842	1.2422
0.5	0.03125	0.17678	0.82322	1.6464	0.43370	1.2489
0.6	0.07776	0.27886	0.72114	1.8028	0.52001	1.2229
0.7	0.16807	0.40996	0.59004	1.9668	0.61893	1.1769
0.8	0.32768	0.57243	0.42757	2.1378	0.73123	1.1206
0.9	0.59049	0.76843	0.23157	2.2316	0.85800	1.0585

With this form, the maximum efficiency is obtained when k is between 0.4 and 0.5. Since the ratio of the moment of inertia at the end to the moment of inertia at the middle is proportional to the cube of x, and, therefore, is proportional to the cube of k, the maximum efficiency is secured when the ratio of the moment of inertia at the end to the moment of inertia at the middle lies between 0.064 and 0.125.

27. TAPERED STRUTS FOR WHICH $I=Cx^2$

The theory of this form of tapered strut has been published in Technologic Paper No. 152 of the Bureau of Standards. For purpose of comparison, however, this theory is here given again in somewhat abridged form. In order to be consistent with the other types of this reference bulletin, the method has been slightly changed. In Technologic Paper No. 152, the ratio represented by k is the reciprocal of the k which is used here.

On occount of the difficulty of accurate construction, no tests were made on struts of this type.

28. THEORY OF TAPERED STRUTS FOR WHICH $I=Cx^2$

(1)

The differential equation for the strut $I=Cx^2$ is $x^2\frac{d^2y}{dx^2}+by=0$,

in which $b=\frac{P}{EC}$.

One solution of this equation is

$$y=A\left(\frac{x}{d}\right)^{\frac{1}{2}}\sin\left(B+\sqrt{4b-1}\ \log\left(\frac{x}{d}\right)^{\frac{1}{2}}\right), \qquad (2)$$

in which x and d have the same meanings as in the preceding types, and refer to distances as shown in Fig. 1 and in Fig. 8.

If the values of y and its second derivative are substituted in Equation (1), the equation is satisfied. Equation (2) is, therefore, a solution of Equation (1).

In order that Equation (2) may be a real solution of Equation (1), the term under the radical must be positive; that is, $4b$ must be greater than unity. If $4b$ is less than unity, a solution of the exponential form is used. However, for the ultimate loads, the values of P as compared with the product EC are sufficiently large so that their quotients are always greater than one-fourth. If it is desired to find the deflections at all loads, the solution for $4b$ less than unity, and the solution for $4b$ equal to unity must also be used.

To eliminate the constants A and B the conditions are:

1. At the middle, $x=d$ and $\dfrac{dy}{dx} = 0$.

2. At the end, $x=kd$ and $y=e$.

Differentiating Equation (2),

$$\frac{dy}{dx} = \frac{A}{2\sqrt{xd}} \sin\left(B + \sqrt{4b-1}\ \log\left(\frac{x}{d}\right)^{\frac{1}{2}}\right) + \frac{A}{2\sqrt{xd}} \sqrt{4b-1} \cos\left(B + \sqrt{4b-1}\ \log\left(\frac{x}{d}\right)^{\frac{1}{2}}\right). \tag{3}$$

Substituting $x=d$ in (3) and equating to zero,

$$\tan B = -\sqrt{4b-1}\ . \tag{4}$$

From the second condition,

$$e = Ak \sin\left(B + \sqrt{4b-1}\ \log k^{\frac{1}{2}}\right), \tag{5}$$

$$A = \frac{e}{k \sin\left(B + \sqrt{4b-1}\ \log k^{\frac{1}{2}}\right)}, \tag{6}$$

in which B is found from Equation (4).

At the middle,

$$y_{\max} = \frac{e \sin B}{k \sin\left(B + \sqrt{4b-1}\ \log k^{\frac{1}{2}}\right)}\ . \tag{7}$$

The critical load is reached when the denominator of Equation (6) or Equation (7) becomes zero. This condition is met when

$$B = -\sqrt{4b-1}\ \log k^{\frac{1}{2}}\ . \tag{8}$$

To obtain the relations between b and k, arbitrary values may be given to b and the values of k found which satisfy Equation (8). For instance, let $\sqrt{4b-1} = 6$, then

$$\tan B = -6;\ B = 99^\circ.462 = 1.7359 \text{ radians.}$$

$$\frac{\log_e k}{2} = \frac{1.7359}{-6},\ \log_e k = -0.5786,$$

$$\log_{10} k = -0.25128 = \bar{1}.75872,$$

$$k = 0.5607.$$

In this way, values of k are found for a series of values of $\sqrt{4b-1}$. The values of $\sqrt{4b-1}$ which correspond to a particular value of k are then calculated by interpolation.

Appendix F gives the last two steps of the calculation of $\sqrt{4b-1}$ in terms of k. A table was first made, which gave k in the terms of $\sqrt{4b-1}$, as shown above. From this table, the value of $\sqrt{4b-1}$ corresponding to a given value of k was determined by interpolation. Usually three or four calculations, such as are shown in Appendix F, were required. To save space, only the last two of these calculations are here given. Instead of dividing the angle B, in degrees, by 57.296 to reduce to radians, the value of log k was multiplied by one-half of 57.296, the product thus obtained was multiplied by $\sqrt{4b-1}$, and the result was then added to the angle in degrees. Column VII of Appendix F gives the value of $\sqrt{4b-1}$ from the final interpolation.

29. STRENGTH RATIO OF STRUTS FOR WHICH $I=Cx^2$.

$$\text{Since } b = \frac{P}{EC}, \text{ then } \frac{4P}{EC} = 4b = \left(\sqrt{4b-1}\right)^2 + 1\ . \qquad (1)$$

At the middle $I_2 = Cd^2$,

$$d = \frac{L}{2(1-k)}, \qquad (2)$$

$$C = \frac{I_2}{d^2} = \frac{4I_2(1-k)^2}{L^2}. \qquad (3)$$

$$P = \frac{EC}{4}\left(\frac{4P}{EC}\right) = \frac{EI_2(1-k)^2}{L^2} \times \frac{4P}{EC}, \qquad (4)$$

Dividing this value of P by the ultimate strength of a uniform strut of moment of inertia I_2 ,

$$\text{strength ratio} = \frac{EI_2(1-k)^2}{L^2} \times \frac{4P}{CE} \times \frac{L^2}{\pi^2 EI_2} =$$

$$\frac{4P}{EC}\,\frac{(1-k)^2}{\pi^2}, \qquad (5)$$

in which $\frac{4P}{EC}$ is the value of $4b$ when Equation (8) of Art. 28 is satisfied.

TABLE XXIX—Strength Ratio of Struts for which $I = Cx^2$.

k	Ultimate $\sqrt{4b-1}$	$\frac{4P}{EC} = 4b$	$(1-k)^2$	$\frac{4P}{EC}\ (1-k)^2$	Strength ratio
0.01	1.09160	2.19159	0.9801	2.14798	0.21764
0.05	1.45145	3.10672	0.9025	2.80382	0.28409
0.1	1.80401	4.25445	0.81	3.44611	0.34916
0.2	2.43602	6.93422	0.64	4.43759	0.44965
0.3	3.12397	10.75875	0.49	5.27179	0.53414
0.4	3.96752	16.74121	0.36	6.02685	0.61065
0.5	5.09189	26.92704	0.25	6.73176	0.68207
0.6	6.72773	46.26235	0.16	7.40192	0.74997
0.7	9.40214	89.40024	0.09	8.04602	0.81523
0.8	14.68802	216.73734	0.04	8.66950	0.87840
0.9	30.44097	927.65266	0.01	9.27653	0.93991

Figure 17 shows the strength ratio for struts of this type.

30. RELATIVE EFFICIENCY OF STRUTS FOR WHICH $I = Cx^2$ AND ALL SECTIONS ARE SIMILAR.

For similar sections, the moment of inertia varies as the fourth power of homologous dimensions and the area varies as the square of these dimensions. Since $I = Cx^2$,

$$A = Qx, \tag{1}$$

in which A is the area of cross-section and Q is constant which depends upon the form of the sections. For one half the strut, from $x = kd$ to $x = d$,

$$\text{volume} = \int Qxdx = \frac{Q}{2}\Big[x^2\Big]_{kd}^{d} = \frac{Qd^2}{2}(1-k^2). \tag{2}$$

Since $\frac{L}{2} = d(1-k)$,

$$\text{average area} = \frac{Qd^2}{2}(1-k^2) \div d(1-k) = \frac{Qd}{2}(1+k). \tag{3}$$

At the middle, where $x = d$, $A_2 = Qd$. The ratio of the average area to the area at the middle is $\frac{Qd}{2}(1+k) \div Qd = \frac{1+k}{2}$.

If I_3 is the moment of inertia of a section of a uniform strut of volume equal to that of a given tapered strut,

$$\frac{I_3}{I_2} = \frac{(1+k)^2}{4} \tag{4}$$

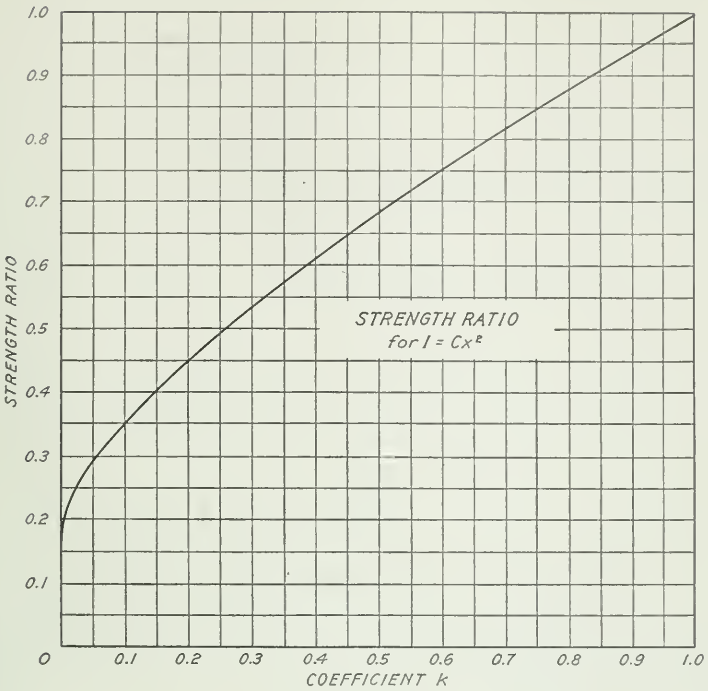

FIG. 17

Strength ratio for struts, $I=Cx^2$.

Table XXX gives the relative efficiency of struts of this type. The maximum efficiency is obtained when k is about 0.3.

TABLE XXX—Relative Efficiency of Tapered Struts for which $I=Cx^2$ and all Sections are Similar.

k	$\frac{(1+k)^2}{4}$	Strength ratio from Table XXIX	Relative Efficiency
0.01	0.255025	0.21764	0.8534
0.05	0.275625	0.28409	1.0307
0.1	0.3025	0.34916	1.1542
0.2	0.36	0.44965	1.2490
0.3	0.4225	0.53414	1.2642
0.4	0.49	0.61065	1.2462
0.5	0.5625	0.68207	1.2125
0.6	0.64	0.74997	1.1718
0.7	0.7225	0.81523	1.1283
0.8	0.81	0.87840	1.0844
0.9	0.9025	0.93991	1.0415

31. THEORY OF TAPERED STRUTS FOR WHICH $I=Cx$

The differential equation is

$$ECx\frac{d^2y}{dx^2}+Py=0, \qquad (1)$$

$$x \frac{d^2y}{dx^2} + by = 0\,, \tag{2}$$

in which $b = \dfrac{P}{EC}$.

The solution of Equation (2) is

$$y = (A + B \log x)\left[b\,x - \frac{b^2x^2}{\lfloor 2} + \frac{b^3x^3}{\lfloor 2 \lfloor 3} - \frac{b^4x^4}{\lfloor 3 \lfloor 4} + \right.$$

$$\left. \text{etc.} \right]$$

$$- B + B\left[\frac{b^2x^2}{\lfloor 2}\left(\frac{1}{1} + \frac{1}{2}\right) - \frac{b^3x^3}{\lfloor 2 \lfloor 3}\left(\frac{1}{1} + \frac{2}{2} + \frac{1}{3}\right) \right.$$

$$+ \frac{b^4x^4}{\lfloor 3 \lfloor 4}\left(\frac{1}{1} + \frac{2}{2} + \frac{2}{3} + \frac{1}{4}\right)$$

$$\left. - \frac{b^5x^5}{\lfloor 4 \lfloor 5}\left(\frac{1}{1} + \frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \frac{1}{5}\right) + \text{etc.} \right] \ . \tag{3}$$

The terms in brackets of Equation (3) are indentical with those of Equation (5) of Art. 18 when bx is subtituted for bx^{-1}.

If the infinite series in the first bracket of Equation (3) is represented by $f_1\ (bx)$ and that in the second series is represented by $f_2\ (bx)$, the equation is then written

$$y = (A + B \log x)\ f_1\ (bx) - B + f_2\ (bx). \tag{4}$$

The value of y and its second derivative satisfy Equation (2). Equation (3) is, therefore, a solution of Equation (2).

When Equation (4) is differentiated with respect to x,

$$\frac{dy}{dx} = \frac{B}{x}\ f_1(bx)\ + (A + B \log x)\left[b - b^2x + \frac{b^3x^2}{\lfloor 2 \lfloor 2} - \frac{b^4x^3}{\lfloor 3 \lfloor 3} \right.$$

$$\left. + \text{ etc.} \right]$$

$$+ \ B\left[b^2\ x\left(\frac{1}{1} + \frac{1}{2}\right) - \frac{b^3x^2}{\lfloor 2 \lfloor 2}\left(\frac{1}{1} + \frac{2}{2} + \frac{1}{3}\right) + \right.$$

$$\left. \frac{b^4x^3}{\lfloor 3 \lfloor 3}\left(\frac{1}{1} + \frac{2}{2} + \frac{2}{3} + \frac{1}{4}\right) - \text{ etc.} \right], \tag{5}$$

$$\frac{dy}{dx} = \frac{B}{x}\ f_1\ (bx)\ + \frac{A + B \log x}{x}\left[\ bx - b^2x^2 + \frac{b^3x^3}{\lfloor 2 \lfloor 2} - \text{ etc.} \right]$$

$$+\frac{B}{x}\left[\,b^2x^2\left(\frac{1}{1}+\frac{1}{2}\right)-\frac{b^3x^3}{\llcorner 2\,\llcorner 2}\left(\frac{1}{1}+\frac{2}{2}+\frac{1}{3}\right)+\right.$$

$$\left.\frac{b^4x^4}{\llcorner 3\,\llcorner 3}\left(\frac{1}{1}+\frac{2}{2}+\frac{2}{3}+\frac{1}{4}\right)-\text{etc.}\right] \qquad (6)$$

Comparison with Equations (9) and (11) of Art. 18 for struts for which $I = Cx^3$ shows that the first bracket of Equation (6) is $f_3\,(bx)$ and the second bracket is $f_4\,(bx)$. Equation (6) may then be written

$$\frac{dy}{dx}=\frac{B}{x}\,f_1(bx)+\frac{A+B\log x}{x}\,f_3(bx)+\frac{B}{x}\,f_4(bx). \qquad (7)$$

Since the functions are the same for $I = Cx$ as for $I = Cx^3$, provided bx is taken as the argument instead of bx^{-1}, the tables of Appendix D may be used for both types.

The conditions for the elimination of the constants are:

1. At the middle, $x = d$ and $\frac{dy}{dx} = 0$.
2. At the end, $x = kd$ and $y = e$.

From the first condition, Equation (7) gives

$$B\,f_1(bd)+(A+B\log d)\,f_3\,(bd)+Bf_4\,(bd)=0, \qquad (8)$$

$$\frac{A}{B}+\log d=-\frac{f_1\,(bd)+f_4\,(bd)}{f_3\,(bd)}=-\frac{S_1}{S_2} \qquad (9)$$

From the second condition, Equation (4) gives

$$(A+B\log kd)\,f_1\,(bkd)+B\,f_2\,(bkd)-B=e, \qquad (10)$$

$$B=\frac{e}{\left(\frac{A}{B}+\log kd\right)f_1\,(bkd)+f_2\,(bkd)-1}. \qquad (11)$$

From Equation (4),

$$y_{\max}=(A+B\log d)\,f_1\,(bd)+B\,f_2\,(bd)-B, \qquad (12)$$

$$y_{\max}=\frac{\left(\frac{A}{B}+\log d\right)f_1\,(bd)+f_2\,(bd)-1}{\left(\frac{A}{B}+\log kd\right)f_1\,(bkd)+f_2\,(bkd)-1}\,e=\frac{S_4e}{S_3}\cdot \qquad (13)$$

The upper half of Appendix G gives the computation for $\frac{A}{B}+\log d$ and for S_4 for a few values of bd. This portion of the computation is the same for all values of k. The lower half of this table completes the calculation for $k = 0.4$. The critical load is reached when the denominator S_3 becomes zero. From Appendix G, it is evident that this point is between

$bd = 5.0$ and 5.5. The true value may be found approximately by interpolation. Instead of using the values of S_3, the ratio $S_3 \div S_4$, which is proportional to the reciprocal of $y_{\max}$ may be taken. From Appendix G, this reciprocal is 6654 when $bd = 5.0$, is — 475 when $bd = 5.5$.

The linear interpolation gives $bd = 5 + \frac{6554}{7029} \times 0{\cdot}5 = 5.4662$.

On account of the large interval, the linear interpolation is questionable. With the interval 0.5 divided into ten parts, Newton's method of interpolation gives $\frac{S_3}{S_4} = 0.00210$ when $bd = 5.45$. A linear interpolation between this and $-\ 0.00475$ at $bd = 5.5$ gives $bd = 5.45 + \frac{210}{685} \times 0.05 = 5.4653$.

Since the value falls near the end of the interval from 5.0 to 5.5, the error of the first linear interpolation is small.

The lower line of Appendix G gives the loads for a strut 1 inch thick, 1.5 inches wide at the middle, 0.6 inch wide at the ends, and 36 inches long, computed for a modulus of elasticity of 31,090,000. $P = bEC$, $C = \frac{I_2}{d}$, hence $P = \frac{bdEI_2}{d^2}$.

When $k = 0.4$ and $L = 36$ inches, $d = 18 \div 0.6 = 30$ inches. With $E = 31{,}090{,}000$ and the maximum section 1.5 inches by 1 inch,

$$P = \frac{bd \times 31{,}090{,}000 \times 1.5}{30 \times 30 \times 12} = 4318\ bd.$$

32. TEST PIECES OF THE TYPE I=Cx

The test pieces of this type were made in the form of Fig. 1_c from rectangular cold-rolled bars, 1.5 inches by 1 inch. Since these bars were stiffer in the plane of the taper than those for which $I = Cx^3$, the error in construction was smaller. Moreover, since the deflections were perpendicular to the parallel surfaces, which were planed accurately, it was not thought necessary to make corrections for the slight variations in the dimensions.

33. TEST OF UNIFORM BAR FOR ULTIMATE STRENGTH AND MODULUS OF ELASTICITY

The maximum load for a uniform strut 1.5 inches wide, 1 inch thick, and 36 inches long, was found to be 29,800 pounds. After this load had been applied and the bar had been deflected about one-third of an inch, it was found impossible to go higher than 29,600 pounds. The load of 29,600 pounds was, therefore, taken as the ultimate strength. A calculation by Euler's formula gives 31,090,000 as the value E. This figure has been used in the computation of all the struts of the type $I = Cx$.

34. TESTS OF STRUTS FOR WHICH I=Cx

Table XXXI gives the results of the tests of the strut $k = 0.4$ for an eccentricity of 0.0100.

TABLE XXXI—Deflection at the Middle of Strut of Type I=Cx and k=0.4. Tested April 11, 1922.

(Thickness, 1 inch; width at the middle, 1.5 inches; width at the ends, 0.6 inch; length 36 inches. Difference of eccentricity 0.0200 inch.)

Total load in pounds	Deflection in 0.0001 inch.			Deflection for eccen. 0.01000	Eccentricity	
	Left	Right	Sum		Left	Right
2000	9	10	19	10	95	105
3000	14	19	33	17	85	115
4000	22	26	48	24	92	108
5000	29	34	63	32	92	108
6000	40	42	82	41	97	103
7000	54	51	105	53	103	97
8000	67	61	128	64	105	95
9000	80	75	155	78	103	96
10000	97	93	190	95	102	98
11000	116	111	227	114	102	98
12000	136	131	267	134	102	98
12500	147	141	288	144	102	98
13000	160	153	313	157	102	98
13500	177	166	343	172	103	97
14000	194	185	379	190	101	99
14500	210	201	411	206	102	98
15000	233	224	459	230	102	98
15500	258	246	504	252	102	98
16000	285	273	558	279	102	98
16500	316	302	618	309	102	98
17000	344	341	685	343	100	100
17500	390	372	762	381	102	98
18000	446	431	877	438	102	98
18500	500	478	978	489	102	98

The lower curve of Fig. 18 is drawn for $k = 0.4$. The solid line is from the theoretical values, part of which are computed in Appendix G. The points are from the experimental values of Table XXXI. The other curves of this figure are drawn in a similar way for $k = 0.6$, $k = 0.8$ and for the uniform strut. On account of the limits of the tables of Appendix D, it was possible to compute the deflections for $k = 0.8$ for only a small part of the loads. The curves of Fig. 19 are drawn for $k = 0.3, 0.5, 0.7, 0.9$, and for the uniform strut.

Table XXXII gives the results of the test of the strut $k = 0.4$ with small eccentricity. At the end of each run there was some set. The unit stress near the ends evidently exceeded the elastic limit. At the end of the second run, the ease with which the strut could be pushed to the left or right showed that the theoretical conditions were closely approximated.

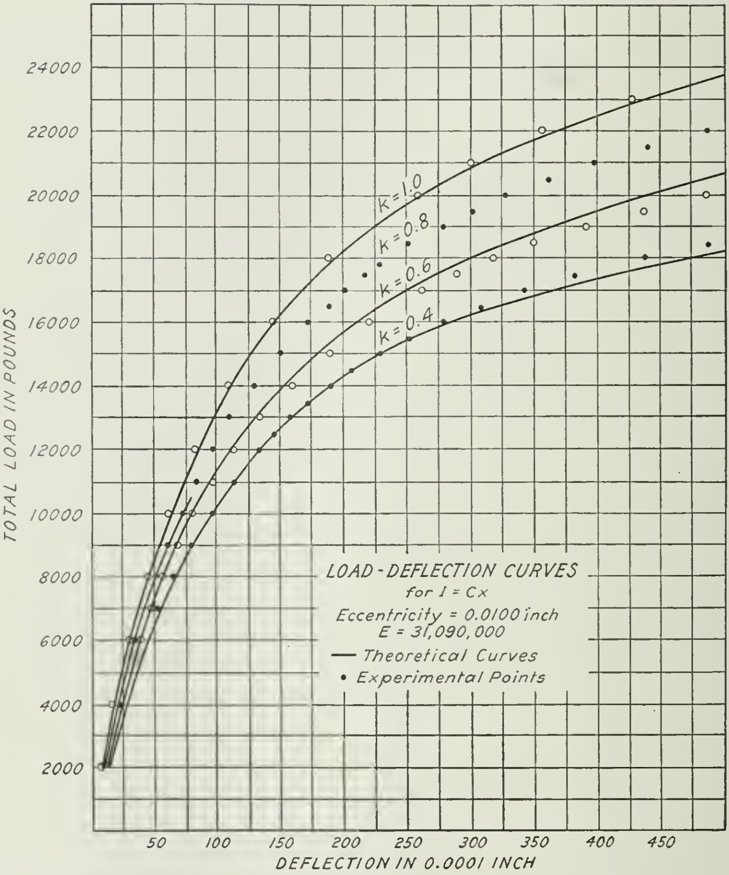

FIG. 18

Load-deflection curves for part of struts, I=Cx, Eccentricity=0.0100 inch.

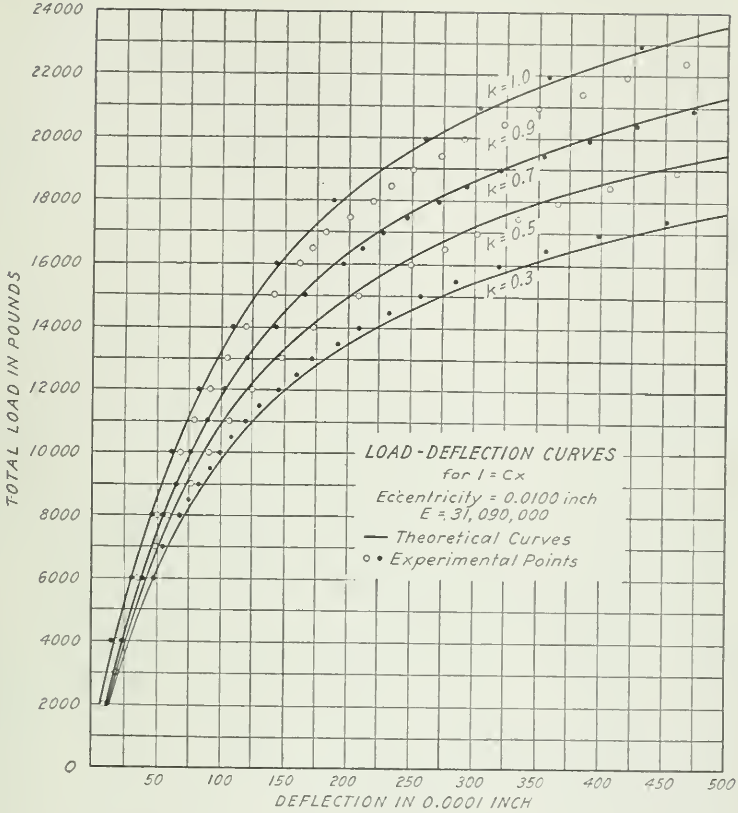

FIG. 19

Load-deflection curves for remainder of struts, I=Cx, Eccentricity=0.0100 inch.

TABLE XXXII—Compression Test of Strut for which I=Cx and k=0.4. Tested with Small Eccentricity, April 17, 1922.

Total load in pounds	Deflection in ten-thousandths of an inch.					
	Upper		Middle		Lower	
	Reading	Def. left	Reading	Def. left	Reading	Def. left
500	2806		1967		2830	
4000	2806	0	1963	−4	2830	0
6000	2806	0	1962	−5	2830	0
8000	2805	−1	1963	−4	2830	0
10000	2805	−1	1963	−4	2831	1
12000	2805	−1	1963	−4	2831	1
14000	2805	−1	1962	−5	2832	2
16000	2804	−2	1962	−5	2833	3
18000	2805	−1	1962	−5	2835	5
20000	2804	−2	1962	−5	2837	7
21000	2805	−1	1965	−2	2839	9
21500	2805	−1	1965	−2	2839	9
22000	2805	−1	1969	2	2840	10
22200	2805	−1	1969	2	2841	11
22600	2808	2	1973	6	2845	15
22800	2811	5	1982	15	2849	19
23000	2820	14	2000	37	2859	29
23200	3110	304	2560	593	3162	332
23400	3470	664	3230	1263	3522	692
	Pushed over with one finger, but would not stay,					
400	2811	6	1975	8	2836	6
	Shifted heads toward the right.					
400	2805		1963		2828	
12000	2799	−6	1957	−6	2828	0
16000	2795	−10	1951	−12	2828	0
20000	2788	−17	1939	−24	2822	−6
22000	2770	−35	1905	−58	2808	−20
23000	2670	−135	1740	−223	2722	−108
23400	1860	−945	200	−1763	1900	−928
	Pushed right and left easily with one finger,					
500	2788	−17	1950	−13	2817	−11

The difference between the eccentricity of the upper and lower ends was greater in the experiment of Table XXXII than in most of the tests.

Figure 20 shows the curves for small eccentricity for the struts for which $I = Cx$. Many of these deflected first in one direction and then in the other, as in the first run of Table XXXII. These deflections were small, and, to avoid confusion, have been omitted from the figure except those for the strut $k = 0.3$. The direction of deflection for this strut reversed twice. The first reversal, like those of other struts, was caused by shifting the line of the resultant force on the heads as they were flattened by the load. The second reversal was caused by elastic failure of the strut near the ends. When the load was removed from this strut, the upper gage showed a set of 0.0085 inch, the middle gage showed a set of 0.0099 inch, and the lower gage showed a set of 0.0101 inch.

35. STRENGTH RATIO OF STRUTS FOR WHICH I = Cx

The load is given by the equations

$$P = bEC = \frac{bdEI_2}{d^2} = \frac{4bdEI_2(1-k)^2}{L^2} \ . \qquad (1)$$

Dividing by the ultimate load for a uniform strut as given by Euler's formula,

$$\text{strength ratio} = \frac{4bd(1-k)^2}{9.8696}\ , \qquad (2)$$

in which bd is the maximum value.

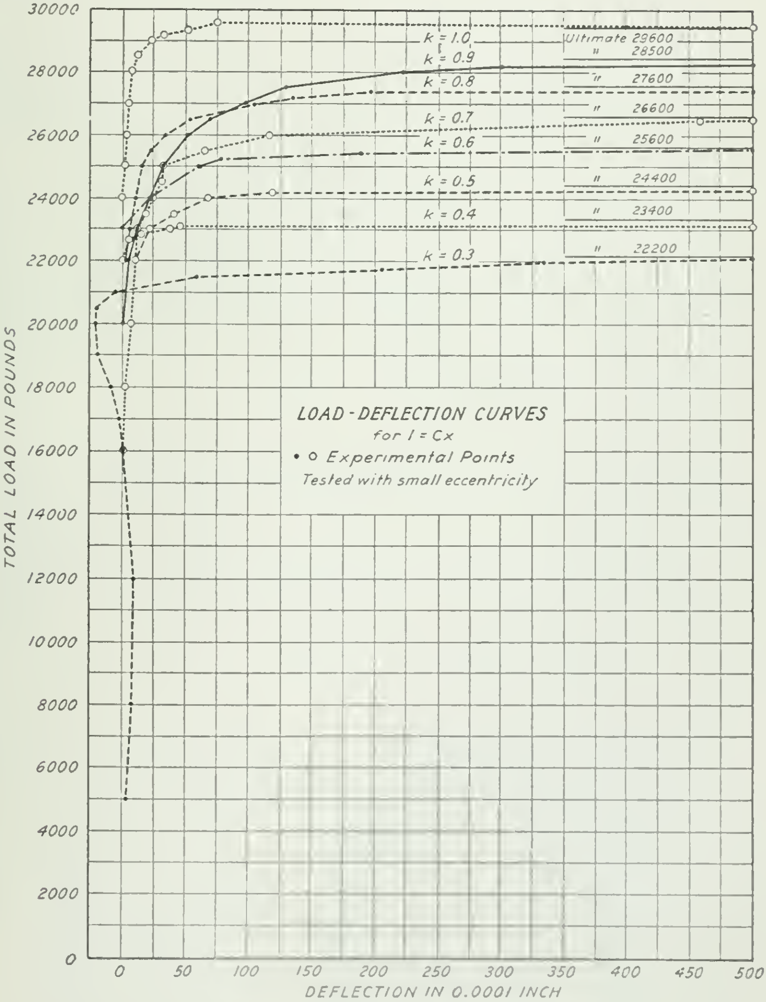

FIG. 20

Load-deflection curves for struts, I = Cx. Eccentricity very small.

TABLE XXXIII—Strength Ratio of Struts for which I=Cx.

k	Maximum bd	$4(1-k)^2$	$4bd(1-k)^2$	Theoretical str. ratio	Experimental	
					Ult. load	Str. ratio
0.	1.4458	4.0	5.7832	0.5860		
0.01	1.4983	3.9920	5.8739	0.5952		
0.05	1.7105	3.61	6.1749	0.6257		
0.1	2.0013	3.24	6.4842	0.6570		
0.2	2.7377	2.56	7.0085	0.7101		
0.3	3.8068	1.96	7.4613	0.7560	22200	0.750
0.4	5.4653	1.44	7.8700	0.7974	23400	0.790
0.5	8.2484	1.0	8.2484	0.8357	24400	0.824
0.6	13.4431	0.64	8.6036	0.8717	25600	0.865
0.7				0.9058	26600	0.897
0.8				0.9382	27600	0.932
0.9				0.9684	28500	0.963
1.0				1.0000	29600	1.000

The fifth column of Table XXXIII gives the theoretical strength ratio. On account of the limits of the tables of Appendix D, the values of *bd* could not be calculated for values of k above 0.65. The strength ratio for $k = 0.7$, 0.8, and 0.9 were computed by means of the equation of the curve passing through the values for $k = 0.2$, 0.3, 0.4, 0.5, 0.6, and 1.0.

The sixth column of Table XXXIII gives the ultimate loads obtained in the tests with small eccentricity. The last column gives the strength ratio obtained by dividing the ultimate load by 29,600 pounds, which was the ultimate load for the uniform strut. The theoretical and experimental ratios agree within reasonable limits. The curve of Fig. 21 is drawn from the theoretical ratios of Table XXXIII. The points are from the experimental values of the last column.

36 RELATIVE EFFICIENCY OF STRUTS FOR WHICH I=Cx

With constant thickness and variable width, as in Fig.1c, the average width is $\frac{1+k}{2}$, and the ratio of the moment of inertia of a uniform strut of equal volume to the moment of inertia at the middle of the tapered strut is also $\frac{1+k}{2}$. Table XXXIV gives the relative efficiency for struts of this form.

TABLE XXXIV—Relative Efficiency of Tapered Struts for which I=Cx and Thickness is Constant.

k	$\frac{1+k}{2}$	Strength ratio from Table XXXIII	Relative efficiency
0.	0.50	0.5860	1.172
0.1	0.55	0.6570	1.194
0.2	0.60	0.7101	1.183
0.3	0.65	0.7560	1.165
0.4	0.70	0.7974	1.138
0.5	0.75	0.8357	1.114
0.6	0.80	0.8717	1.090
0.7	0.85	0.9058	1.065
0.8	0.90	0.9382	1.042
0.9	0.95	0.9684	1.019

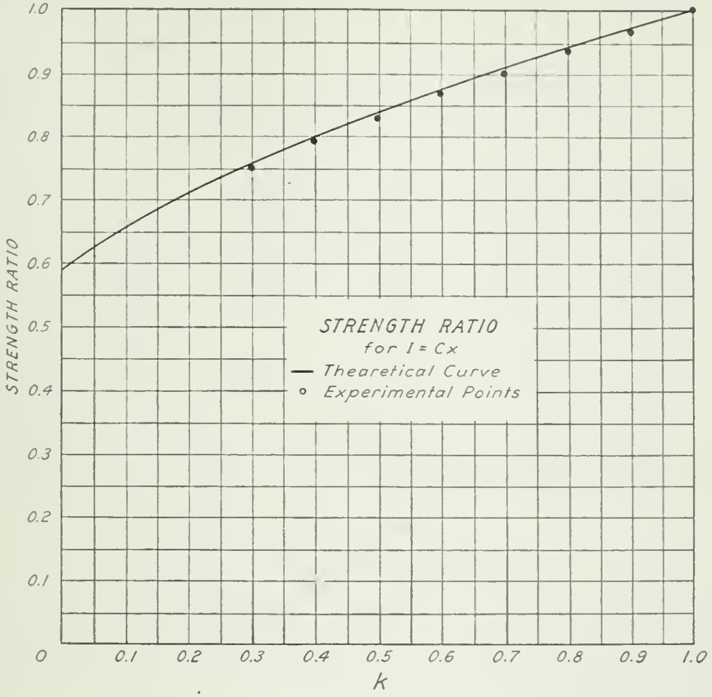

FIG. 21

Strength ratio for struts, $I = Cx$.

If $I = Cx$ and all sections are similar,

$$A = Qx^{\frac{1}{2}}, \tag{1}$$

in which A is the area of any section, and Q is a constant which depends upon the form of the sections. For one-half the length, from $x = kd$ to $x = d$,

$$\text{volume} = \int Qx^{\frac{1}{2}}dx = \frac{2Q}{3}\left[x^{\frac{3}{2}}\right]_{kd}^{k} = \frac{2Qd^{\frac{3}{2}}}{3}(1-k^{\frac{3}{2}}). \tag{2}$$

Since $\frac{L}{2} = d(1-k)$,

$$\text{average area} = A_3 = \frac{2Qd^{\frac{1}{2}}(1-k^{\frac{3}{2}})}{3(1-k)}. \tag{3}$$

At the middle, $A_2 = Qd^{\frac{1}{2}}$,

$$\frac{A_3}{A_2} = \frac{2(1-k^{\frac{3}{2}})}{3(1-k)}. \tag{4}$$

$$\frac{I_3}{I_2} = \frac{4(1-k^{\frac{3}{2}})^2}{9(1-k)^2}. \tag{5}$$

TABLE XXXV—Relative Efficiency of Tapered Struts for which I=Cx and all Sections are Similar.

k	$k^{\frac{3}{2}}$	$1-k^{\frac{3}{2}}$	$\frac{2(1-k^{\frac{3}{2}})}{3(1-k)}$	$\frac{4(1-k^{\frac{3}{2}})^2}{9(1-k)^2}$	Strength ratio	Relative Efficiency
0.	0.	1.0	0.6667	0 4444	0.5860	1.319
0.01	0.001	0.999	0.6727	0.4525	0.5952	1.315
0.05	0.01118	0.98882	0.6940	0.4816	0.6257	1.299
0.1	0.03162	0.96838	0.7175	0.5145	0.6570	1.277
0.2	0.08944	0 91056	0.7588	0.5758	0.7101	1.233
0.3	0.16432	0.83568	0 7959	0.6335	0.7560	1.193
0.4	0.25298	0.74702	0.8300	0.6889	0.7974	1.157
0.5	0.35355	0.64645	0.8619	0 7429	0.8357	1.125
0.6	0.46476	0.53524	0.8921	0.7958	0.8717	1.095
0.7	0.58566	0.41434	0.9207	0.8477	0.9058	1 068
0.8	0.71555	0.28445	0.9481	0.8989	0 9382	1.044
0.9	0.85381	0.14619	0.9476	0.9498	0.9684	1.020

Table XXXV gives the relative efficiency for struts for which all sections are similar. If the struts are sufficiently long to fail by bending, the maximum efficiency is obtained when the ends are very small.

37. STRUTS OF THE TYPE $I=I_2-Cx^2$

On struts of the first four types the surfaces of the two halves intersect at an angle at the middle. On many actual tapered struts, the surfaces of the two halves are tangent at the middle. Figure 2 shows a strut the thickness of which is constant while the width decreases as the square of the distance from the middle. The equation of the moment of inertia is

$$I = I_2 - Cx^2,$$

in which I_2 is the moment of inertia at the middle and x is the distance from the middle. Struts of this type are not usually made of constant thickness but have sections which are approximately similar.

38. THEORY OF TAPERED STRUTS FOR WHICH $I=I_2-Cx^2$

The differential equation is

$$E(I_2 - Cx^2)\frac{d^2y}{dx^2} + Py = 0. \tag{1}$$

If $\frac{I_2}{C} = a$, and $\frac{P}{EC} = b$,

$$(a-x^2)\frac{d^2y}{dx^2} + by = 0. \tag{2}$$

The solution of Equation (2) is

$$y = A\left(1 - \frac{bx^2}{2a} + \frac{b(b-2)x^4}{\lfloor 4a^2} - \frac{b(b-2)(b-3\times4)x^6}{\lfloor 6a^3} + \right.$$

$$\left.\frac{b(b-2)(b-12)(b-5\times6)x^8}{\lfloor 8a^4} - \frac{b(b-2)(b-12)(b-30)(b-7\times8)x^{10}}{\lfloor 10a^5} + \text{etc.}\right)$$

$$+ B\left(x - \frac{bx^3}{\lfloor 3a} + \frac{b(b-2\times 3)x^5}{\lfloor 5a^2} - \frac{b(b-6)(b-4\times 5)x^7}{\lfloor 7a^3} +\right.$$

$$\left.\frac{b(b-6)(b-20)(b-6\times 7)x^9}{\lfloor 9a^4} - \frac{b(b-6)(b-20)(b-42)(b-8\times 9)x^{11}}{\lfloor 11a^5} + \text{etc.}\right) \qquad (3)$$

This value of y and its second derivative together satisfy Equation (2). Equation (3) is, therefore, a solution of Equation (2).

The conditions for the elimination of the constants are:

1. At the middle, $x = 0$ and $\frac{dy}{dx} = 0$.

2. At one end, $x = \frac{L}{2}$ and $y = e$.

From the first condition, $B = 0$. From the second condition,

$$A = \frac{e}{1 - \frac{b}{\lfloor 2a}\left(\frac{L}{2}\right)^2 + \frac{b(b-2)}{\lfloor 4a^2}\left(\frac{L}{2}\right)^4 - \frac{b(b-2)(b-12)}{\lfloor 6a^3}\left(\frac{L}{2}\right)^6 + \text{etc.}} \qquad (4)$$

At the middle, where $x = 0$, $y = y_{\max}$; hence

$$y_{\max} = A. \qquad (5)$$

The critical load for the strut is reached when the denominator of Equation (4) becomes zero.

39. COMPUTATION OF CRITICAL LOAD FOR STRUT FOR WHICH $I=I_2-Cx^2$

If I_1 is the moment of inertia at either end,

$$I_1 = I_2 - C\left(\frac{L}{2}\right)^2, \text{ and } C = \frac{I_2 - I_1}{\left(\frac{L}{2}\right)^2}. \qquad (1)$$

$$a = \frac{I_2}{C} = \frac{I_2\left(\frac{L}{2}\right)^2}{I_2 - I_1}, \qquad (2)$$

$$\frac{\left(\frac{L}{2}\right)^2}{a} = \frac{I_2 - I_1}{I_2} = 1 - \frac{I_1}{I_2}. \qquad (3)$$

If z is substituted for $1 - \frac{I_1}{I_2}$, the denominator of Equation (4) of Art. 38 becomes

$$1-\frac{bz}{2}+\frac{bz}{2}\,\frac{(b-2)z}{3\times4}-\frac{bz}{2}\,\frac{(b-2)z}{12}\,\frac{(b-12)z}{5\times6}+\frac{bz}{2}\,\frac{(b-2)z}{12}$$

$$\frac{(b-12)z}{30}\,\frac{(b-30)z}{7\times8}-\frac{bz}{2}\,\frac{(b-2)z}{12}\,\frac{(b-12)z}{30}\,\frac{(b-30)z}{56}\,\frac{(b-56)z}{9\times10}+\text{etc.} \tag{4}$$

The sum of the series of expression (4) becomes zero at the critical load.

For the condition $\frac{I_1}{I_2}=0.9$, $z=0.1$. If $b=24.34$,

$\frac{bz}{2}=1.217$; $\frac{bz}{2}\,\frac{(b-2)z}{12}=1.217\times\frac{2.234}{12}=0.22656$:

$\frac{bz}{2}\,\frac{(b-2)z}{12}\,\frac{(b-12)z}{30}=0.22656\times\frac{1.234}{30}=0.00932.$

The series to five decimal places is
$1-1.217+0.22656-0.00932-0.00009=0.00015$
If $b=24.35$ when $z=0.1$, the series is
$1-1.2175+0.22676-0.00934-0.00009=-0.00017.$
An interpolation between these two results gives

$b=24.34+\frac{15}{32}\times0.005=24.345$ as the value at which the sum of the series is zero.

When $z=0.2$, if b is taken as 12, the series becomes $1-1.2+0.2=0$. In this case the series is finite and is equal to zero.

When $\frac{I_1}{I_2}=0$, $z=1$. If $b=2$ when $z=1$, the series is $1-1+0=0$.

As the value of z approaches unity, the series (when not finite) converges more and more slowly. For $z=0.9$, thirty terms are required to secure a sum which is correct to the fifth decimal place. To diminish the labor of the trial and error method, the reciprocals of b, which vary less rapidly than b, were calculated for the smaller values of z. The reciprocal of b for the next higher value of z was then computed by differences. The figure thus obtained was then substituted in the series and was always found to be very near the desired result. Another slightly different figure was then tried and the correct value calculated by interpolation.

40. STRENGTH RATIO OF STRUTS FOR WHICH $I=I_2-Cx^2$

When b and z are corresponding values at which the series of Art. 38 becomes zero, the critical load is

$$P=\frac{4bEI_2z}{L^2}; \tag{1}$$

since $P = bEC = bEI_2 \dfrac{1 - \dfrac{I_1}{I_2}}{\left(\dfrac{L}{2}\right)^2}$, and $z = 1 - \dfrac{I_1}{I_2}$.

Dividing by the ultimate load for a uniform strut,

$$\text{strength ratio} = \frac{4bEI_2 z}{L^2} \div \frac{\pi^2 EI_2}{L^2} = \frac{4bz}{\pi^2}. \qquad (2)$$

Table XXXVI gives the values of b and of the strength ratio for a series of values of z.

41. RELATIVE EFFICIENCY OF STRUTS FOR WHICH $I=I_2-Cx^2$

When a strut of this type is made with all sections similar, the ratio of the average area to the area at the middle is

$$\frac{A_3}{A_2} = \frac{\sqrt{z}}{2}\left(\sqrt{\frac{1}{z} - 1} + \frac{1}{z}\sin^{-1}\sqrt{z}\right),$$

in which A_3 is the average area of the cross-section, A_2 is the area at the middle, and $z = 1 - \dfrac{I_1}{I_2}$. The ratio of the moment of inertia of the average section to the moment of inertia at the middle is the square of the ratio of the areas. When $z = 0.5$, for instance,

$$\frac{A_3}{A_2} = \frac{1}{2\sqrt{2}}(1 + 2\sin^{-1}\sqrt{0.5}) = \frac{1}{2\sqrt{2}}(1 + \frac{\pi}{2}),$$

$$\frac{I_3}{I_2} = \frac{2.5718^2}{8} = 0.8263.$$

The strength of a uniform strut of the same volume as this tapered strut would, therefore, be 0.8263 of the strength of a uniform strut equal to the section of the tapered strut at the middle. From Table XXXVI the strength ratio of this tapered strut is found to be 0.9259. The relative efficiency is $0.9259 \div 0.8263 = 1.120$. If the slenderness ratio is sufficiently great for the strut to reach its critical load before any part is stressed to the elastic limit, there is a gain of 12 per cent by the use of the tapered strut.

TABLE XXXVI—Strength Ratio and Relative Efficiency of Struts for which $I=I_2-Cx^2$.

$\frac{I_1}{I_2}$	z	b for critical load	$4bz$	Strength ratio	$\frac{I_3}{I_2}$	Relative efficiency
0.	1.	2.0	8.0	0.8106	0.6168	1.314
0.05	0.95	2.148	8.1624	0.8272	0.6431	1.286
0.1	0.9	2.309	8.3124	0.8422	0.6671	1.262
0.2	0.8	2.675	8.5600	0.8673	0.7098	1.222
0.3	0.7	3.132	8.7696	0.8885	0.7504	1.184
0.4	0.6	3.735	8.9640	0.9082	0.7889	1.151
0.5	0.5	4.569	9.1380	0.9259	0.8263	1.120
0.6	0.4	5.815	9.3040	0.9427	0.8622	1.093
0.7	0.3	7.875	9.4500	0.9575	0.8976	1.067
0.8	0.2	12.000	9.6000	0.9727	0.9322	1.043
0.9	0.1	24.344	9.7376	0.9868	0.9643	1.021

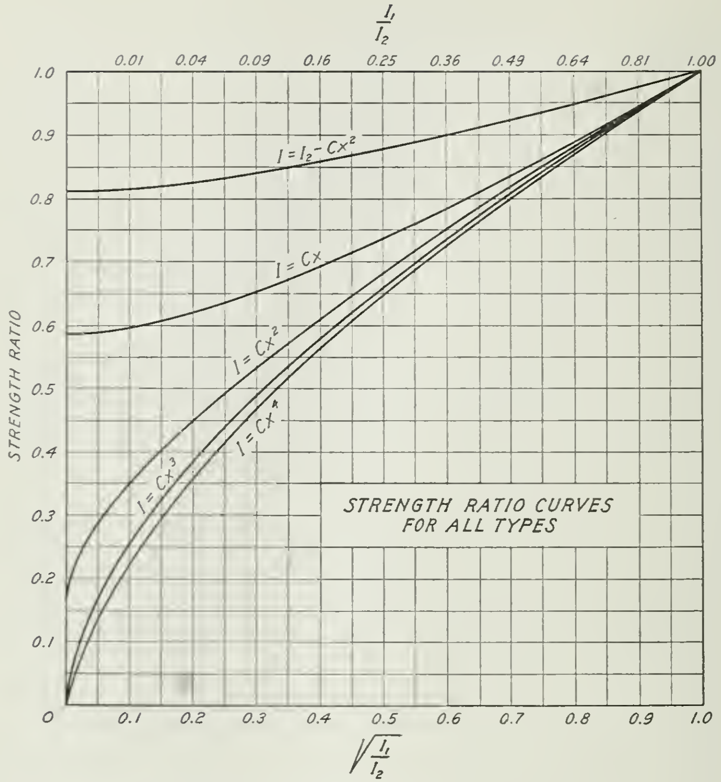

FIG. 22

Strength ratio for five types with abscissas as in Fig. 17.

The sixth column of Table XXXVI gives the ratio $\frac{I_3}{I_2}$ and the last column gives the relative efficiency.

The upper curve of Fig. 22 gives the strength ratio for struts of this type. The abscissas are the square roots of the ratio of the moment of inertia at the end to the moment of inertia at the middle.

42. COMPARATIVE STRENGTH RATIOS

The strength ratios for the first four types have been given as functions of k. In order to compare the different types, it is desirable to give these ratios as functions of the ratio of the moment of inertia at the end to the moment of inertia at the middle. If I_1 is the moment of inertia at the ends, and I_2 is the moment of inertia at the middle,

For Type 1, $I_2 = Cd^4$, $I_1 = C(kd)^4$; $\frac{I_1}{I_2} = k^4$.

For Type 2, $I_2 = Cd^3$, $I_1 = C(kd)^3$; $\frac{I_1}{I_2} = k^3$.

For Type 3, $I_2 = Cd^2$, $I_1 = C(kd)^2$; $\dfrac{I_1}{I_2} = k^2$.

For Type 4, $I_2 = Cd$, $I_1 = Ckd$; $\dfrac{I_1}{I_2} = k$.

For purposes of comparison, the square root of the ratio of I_1 to I_2 has been taken as the independent variable. The curves of Fig. 22 have been plotted with these square roots as abscissas and the strength ratios as ordinates. The data for these curves are given in Table XXXVII for the first four types and in Table XXXVI for the last type.

TABLE XXXVII—The Relation of the Strength Ratio and Relative Efficiency to the Square Root of the Ratio of the Moment of Inertia at the Ends to the Moment of Inertia at the Middle. All Sections Similar.

k	Type 1, $I = Cx^4$			Type 2, $I = Cx^3$		
	$\sqrt{\dfrac{I_1}{I_2}} = k^2$	Strength ratio	Relative effic.	$\sqrt{\dfrac{I_1}{I_2}} = k^{\frac{3}{2}}$	Strength ratio	Relative effic.
0.01	0.0001	0.0004		0.001	0.0142	
0.05	0.0025	0.0090		0.0112	0.0687	
0.1	0.01	0.0326	0.238	0.0316	0.1300	0.662
0.2	0.04	0.1071	0.621	0.0894	0.2419	1.003
0.3	0.09	0.2108	0.981	0.1643	0.3459	1.172
0.4	0.16	0.3067	1.134	0.2530	0.4452	1.242
0.5	0.25	0.4170	1.225	0.3536	0.5416	1.249
0.6	0.36	0.5306	1.243	0.4648	0.6359	1.223
0.7	0.49	0.6463	1.217	0.5857	0.7284	1.177
0.8	0.64	0.7632	1.153	0.7156	0.8194	1.121
0.9	0.81	0.8814	1.080	0.8538	0.9085	1.058

k	Type 3, $I = Cx^2$			Type 4, $I = Cx$		
	$\sqrt{\dfrac{I_1}{I_2}} = k$	Strength ratio	Relative effic.	$\sqrt{\dfrac{I_1}{I_2}} = k^{\frac{1}{2}}$	Strength ratio	Relative effic.
0.01	0.01	0.2176	0.853	0.1	0.5952	1.315
0.05	0.05	0.2841	1.031	0.2236	0.6257	1.299
0.1	0.1	0.3492	1.154	0.3162	0.6570	1.277
0.2	0.2	0.4497	1.249	0.4472	0.7101	1.233
0.3	0.3	0.5341	1.264	0.5477	0.7560	1.193
0.4	0.4	0.6107	1.246	0.6236	0.7974	1.157
0.5	0.5	0.6821	1.213	0.7071	0.8357	1.125
0.6	0.6	0.7480	1.172	0.7741	0.8717	1.095
0.7	0.7	0.8152	1.128	0.8367	0.9058	1.068
0.8	0.8	0.8784	1.084	0.8944	0.9382	1.044
0.9	0.9	0.9399	1.041	0.9486	0.9684	1.020

43. COMPARATIVE RELATIVE EFFICIENCIES

A comparison of relative efficiencies may be made from Tables XXXVI and XXXVII. The results are plotted in Fig. 23, with the square roots of $\dfrac{I_1}{I_2}$ as abscissas. For convenience of reference, the corresponding values of $\dfrac{I_1}{I_2}$ are given at the top. The curves for the first four types practically coincide after the ratio of the moment of inertia at the ends to the moment of inertia at the middle reaches 0.25. When the area at the end is one-half the area at the middle, or the diameter at the end is 0.707 the diameter at the middle, the strength of any one of these types is a little over 21 per cent greater than the strength of a uniform strut of equal volume. A strut of Type 3 has its maximum efficiency when the ratio of the moment of inertia at the end to the moment of inertia at the middle is about 0.09. Types 1 and 2 are the most efficient when the ratio is slightly larger. A strut of any one of the first three types should not have the ratio of the diameter at the ends to the diameter at the middle less than 0.55.

Type 4, $I = Cx$, is the best under all conditions. While Type 5, $I = I_2 - Cx^2$, has greater strength for the same ratio of moment of inertia at the ends to moment of inertia at the middle, its volume is so much greater than the volume of Type IV that its efficiency is always lower. The efficiency of these types is the greatest when the moment of inertia at the ends is the least. The gain in efficiency by making the diameter at the ends less than one-half the diameter at the middle is very little. On account of the danger of exceeding the elastic limit of the material in the reduced area, it is not generally advisable to use a smaller ratio, even when the slenderness ratio is very large.

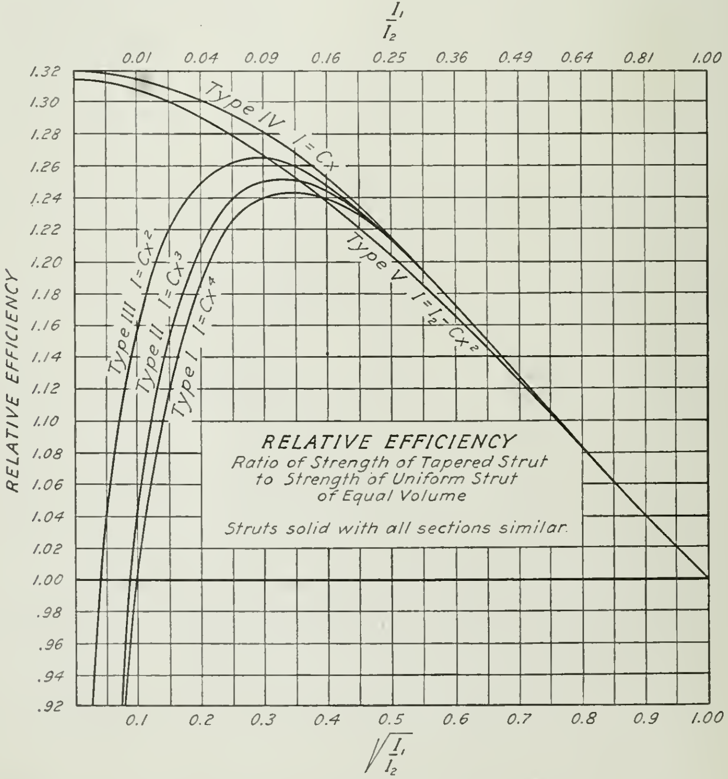

FIG. 23

Relative efficiency for five types.

44. CONCLUSION

In this paper formulas have been derived for five types of tapered struts, covering a range which probably includes all forms which are likely to be used. Tests were made on struts of cold-rolled steel of three of these types. The deflections and ultimate strengths measured in these tests agree so closely with the calculated values as to leave no doubt as to the validity of the formulas. The theory for the other types is comparatively simple and the calculated values of the strength ratios and relative efficiencies fit in closely with those which were tested. It may, therefore, be taken for granted that the formulas for these types also are correct.

For solid struts of similar sections, when the ratio of the moment of inertia at the ends to the moment of inertia at the middle is not less than 0.25, the relative efficiency is the same for any one of the first four types. Any combination of these types will, therefore, have the same efficiency. For Type 5 the efficiency is slightly lower. The combined curve of Fig. 23 may be used with little error for any long, solid strut with similar sections.

Tapered struts built up of structural shapes are usually made of four angles latticed together. When each angle is straight from the end to the middle, the moment of inertia varies approximately as the square of x. The strength ratio for this type is given in Table XXIX and in Figs. 17 and 22. Since the only saving of material is in the lattice bars, the relative efficiency is much smaller than that of a solid strut of this type.

Instead of making derrick booms and masts with uniform taper from ends to middle, as is frequently done, it is probably better to make them of uniform size for the greatest part of their length, and with only enough taper at the ends to make convenient connections. In a large boom, the column action is complicated with the beam action caused by its own weight. For this reason column formulas do not entirely govern the design. For convenience of transportation, booms and masts are frequently made of two or more portions which must be bolted together in the field. It is desirable to have as large section modulus as possible at the planes of connection. Vertical compression members of dirigibles, which are not subjected to transverse forces, should taper at the ends.

There is some considerable misapprehension in regard to tapered struts in the minds of some engineers. This misapprehension is probably caused by comparison with tapered beams. A tapered beam for a particular location of loads may be fully as strong as a uniform beam and be scarcely one-half as heavy. A tapered strut, on the other hand, is always weaker than a uniform strut unless the maximum sections of the tapered strut are larger than the sections of the uniform strut. The bending moment in a beam is practically independent of the deflection. The bending moment in a strut increases as the deflection increases. On account of the increase of bending moment with increase of deflection, tapered struts of the first and second types, which are relatively flexible near the ends, fail at extremely small loads when the ends are small.

The strength ratios and relative efficiencies which are given in this paper apply to struts which reach their critical loads before any part of the material is stressed to its elastic limit. The expressions for the ultimate strength of these tapered struts are comparable with the results of Euler's formula for uniform struts. The formulas for deflection are valid for struts of any length. If the eccentricity is known, the deflection may be computed at any section and the unit stress may be determined by adding the bending stress to the direct compressive stress.

The relative efficiency of a round-end tapered strut of small slenderness ratio can not exceed the values given by the curves of Fig. 23, and may be much smaller. Where there are combined compressive and transverse forces, as in a connecting rod, the gain in efficiency may be greater.

APPENDIX A

DEFLECTION OF ROUND-END STRUT OF UNIFORM SECTION

$$y_{\max} = e \sec \sqrt{\frac{P}{EI}}\,\frac{L}{2} \text{ in radians} = e \sec \theta^{\circ}.$$

$$y = y_{\max} \cos \sqrt{\frac{P}{EI}}\left(\frac{L}{2} - x\right) \text{in radians} = e \sec \theta^{\circ} \cos \varphi^{\circ}.$$

Deflection $= y - e$; deflection at middle $= y_{\max} - e$.

$$P = R\theta^2;\ R = \frac{\text{ultimate load}}{90^2}.$$

$$\text{For } x = \frac{L}{6},\ \varphi = \frac{2\theta}{3}.$$

θ°	Sec θ°	For $x = \frac{L}{6}$: φ°	Cos φ	Sec θ cos φ	Total load $= P = R\theta^2$ For ultimate load of 11250	24300	29600
I	II	III	IV	V	VI	VII	VIII
5°	1.00382	3° 20′	0.99847	1.00229	35	75	91
10°	1.01543	6° 40′	0.99390	1.00923	138	300	365
15°	1.03528	10° 00′	0.98481	1.01955	312	675	822
20°	1.06418	13° 20′	0.97305	1.03550	556	1200	1462
25°	1.10338	16° 40′	0.95799	1.05703	869	1875	2284
30°	1.15470	20° 00′	0.93969	1.08506	1250	2700	3289
32°	1.17918	21° 20′	0.93148	1.09838	1422	3072	3742
34°	1.20622	22° 40′	0.92276	1.11908	1606	3468	4224
36°	1.23607	24° 00′	0.91355	1.12921	1800	3888	4736
38°	1.26902	25° 20′	0.90383	1.14698	2006	4332	5277
40°	1.30541	26° 40′	0.89363	1.16656	2222	4800	5847
42°	1.34563	28° 00′	0.88020	1.18442	2450	5292	6446
44°	1.39016	29° 20′	0.87178	1.21192	2689	5808	7075
46°	1.43956	30° 40′	0.86015	1.23824	2939	6348	7733
48°	1.49448	32° 00′	0.84805	1.26739	3200	6912	8420
50°	1.55572	33° 20′	0.83549	1.29979	3472	7500	9136
52°	1.62427	34° 40′	0.82248	1.33592	3755	8112	9881
54°	1.70130	36° 00′	0.80902	1.37638	4050	8748	10656
56°	1.78829	37° 20′	0.79512	1.42191	4356	9406	11460
58°	1.88708	38° 40′	0.78079	1.47342	4672	10092	12293
60°	2.00000	40° 00′	0.76604	1.53209	5000	10800	13156
62°	2.13005	41° 20′	0.75088	1.59941	5339	11532	14047
64°	2.28117	42° 40′	0.73531	1.67736	5689	12288	14968
66°	2.45859	44° 00′	0.71934	1.76856	6050	13068	15918
68°	2.66947	45° 20′	0.70298	1.87659	6422	13972	16898
70°	2.92380	46° 40′	0.68624	2.00643	6806	14700	17906
72°	3.23607	48° 00′	0.66913	2.16535	7200	15552	18944
74°	3.62976	49° 20′	0.65166	2.36419	7606	16428	20011
76°	4.13357	50° 40′	0.63383	2.61998	8022	17328	21107
78°	4.80973	52° 00′	0.61566	2.96116	8450	18252	22233
80°	5.75877	53° 20′	0.59716	3.43890	8889	19200	23388
82°	7.18530	54° 40′	0.57833	4.15540	9339	20172	24572
84°	9.56677	56° 00′	0.55919	5.34967	9800	21168	25785
86°	14.33559	57° 20′	0.53975	7.73764	102 2	22188	27027
88°	28.65371	58° 40′	0.52002	14.90226	10750	23232	28299

APPENDIX B

CALCULATION OF CONSTANTS FOR $I=Cx^4$

$$\operatorname{Tan}\left(\frac{q}{d}+\alpha\right)=\frac{q}{d} \text{ radians.}$$

$\frac{q}{d}$ degrees	$\frac{q}{d}$ radians	$\frac{q}{d}+\alpha$ degrees	$\operatorname{Sin}\left(\frac{q}{d}+\alpha\right)$	α degrees
I	II	III	IV	V
20°	0.3491	19° 14′.7	0.3296	− 0° 45′.3
30°	0.5236	27° 38′.3	0.4639	− 2° 21′.7
40°	0.6981	34° 55′.2	0.5724	− 5° 04′.8
50°	0.8727	41° 07′.6	0.6577	− 8° 52′.4
60°	1.0472	46° 19′.3	0.7233	− 13° 40′.7
70°	1.2217	50° 41′.9	0.7738	− 19° 18′.1
80°	1.3963	54° 23′.4	0.8130	− 25° 36′.6
90°	1.5708	57° 31′.1	0.8435	− 32° 28′.9
100°	1.7453	60° 11′.3	0.8677	− 39° 48′.7
110°	1.9199	62° 23′.3	0.8861	− 47° 36′.7
120°	2.0944	64° 28′.6	0.9024	− 55° 31′.4
130°	2.2689	66° 12′.9	0.9154	− 63° 47′.1
140°	2.4435	67° 44′.6	0.9254	− 72° 15′.4
150°	2.6180	69° 05′.7	0.9342	− 80° 54′.3
160°	2.7925	70° 17′.8	0.9415	− 89° 42′.2
170°	2.9671	71° 23′.5	0.9477	− 98° 34′.5
180°	3.1416	72° 20′.6	0.9529	−107° 39′.4
190°	3.3161	73° 13′.1	0.9574	−116° 46′.9
200°	3.4907	74° 00′.9	0.9613	−125° 59′.1
210°	3.6652	74° 44′.3	0.9647	−137° 15′.7
220°	3.8397	75° 24′.1	0.9677	−144° 35′.9
230°	4.0143	76° 00′.9	0.9704	−153° 59′.1
240°	4.1888	76° 34′.4	0.9726	−163° 25′.6
250°	4.3633	77° 05′.4	0.9747	−172° 54′.6
260°	4.5379	77° 34′.3	0.9765	−182° 25′.7
270°	4.7124	78° 01′.2	0.9782	−191° 58′.8
280°	4.8869	78° 26′.1	0.9797	−201° 33′.9
290°	5.0615	78° 49′.4	0.9810	−211° 10′.6
300°	5.2360	79° 11′.2	0.9822	−220° 48′.8
310°	5.4105	79° 31′.7	0.9834	−230° 28′.3
320°	5.5851	79° 50′.9	0.9843	−240° 09′.1
330°	5.7596	80° 09′.0	0.9853	−249° 11′.0
340°	5.9341	80° 26′.1	0.9861	−259° 33′.9
350°	6.1087	80° 42′.2	0.9869	−269° 17′.8
360°	6.2832	80° 57′.4	0.9876	−279° 02′.6

APPENDIX C

DEFLECTION AT THE MIDDLE AND AT ONE-SIXTH THE LENGTH FROM THE END FOR STRUT FOR WHICH $I=Cx^4$ AND $k=0.7$

$$y_{\max} = \frac{ed \sin\left(\frac{q}{d}+\alpha\right)}{kd \sin\left(\frac{q}{kd}+\alpha\right)} = \frac{10 \sin\left(\frac{q}{d}+\alpha\right)}{7 \sin\left(\frac{q}{kd}+\alpha\right)} e.$$

$$d = \frac{L}{2(1-0.7)} = \frac{10\,L}{6};\ kd = \frac{7\,L}{6}.$$

At sixth-points, $x = \frac{7\,L}{6} + \frac{L}{6} = \frac{8\,L}{6} = \frac{8d}{10}.$

$$y = \frac{ex \sin\left(\frac{q}{x}+\alpha\right)}{kd \sin\left(\frac{q}{kd}+\alpha\right)} = \frac{8 \sin\left(\frac{q}{x}+\alpha\right)}{7 \sin\left(\frac{q}{kd}+\alpha\right)} e.$$

APPENDIX C—Continued.

$\frac{q}{d}$	$\frac{q}{kd}$	$\frac{q}{kd}+\alpha$	$\text{Sin}\left(\frac{q}{kd}+\alpha\right)$	$7\,\text{Sin}\left(\frac{q}{kd}+\alpha\right)$	$\frac{y_{\max}}{e}$
I	II	III	IV	V	VI
60°	85° 42′.9	72° 02′.2	0.9512	6.6584	1.0863
80°	114° 17′.1	88° 40′.5	0.9997	6.9979	1.1618
100°	142° 51′.4	103° 02′.7	0.9742	6.8194	1.2724
120°	171° 25′.7	115° 54′.3	0.8994	6.2958	1.4335
140°	200° 00′.0	127° 44′.6	0.7908	5.5356	1.6717
160°	228° 34′.4	138° 52′.2	0.6578	4.6046	2.0447
180°	257° 08′.6	149° 29′.2	0.5077	3.5539	2.6813
200°	285° 42′.9	159° 43′.8	0.3465	2.4255	3.9633
220°	314° 17′.1	169° 41′.2	0.1792	1.2444	7.7764
240°	342° 51′.4	179° 25′.8	0.0100	0.0700	138.94
250°	357° 08′.6	184° 14′.0			

Interpolation between 240° and 250° gives $\frac{q}{d} = 241°.187$ when $\frac{q}{kd} + \alpha = 180°$.

A second interpolation between 240° and 245° gives 241°.17.

At one-sixth the length from ends.

$\frac{q}{d}$	$\frac{q}{x}$	$\frac{q}{x}+\alpha$	$8\,\text{Sin}\left(\frac{q}{x}+\alpha\right)$	$\frac{y}{e}$	$\frac{y-e}{y_{\max}-e}$	Total load for tested strut.
I	VII	VIII	IX	X	XI	XII
60°	75°	61° 19′.3	7.0188	1.0540	0.626	462
80°	100°	74° 23′.4	7.7048	1.1010	0.624	821
100°	125°	85° 11′.3	7.9720	1.1690	0.620	1283
120°	150°	94° 28′.6	7.9760	1.2668	0.615	1848
140°	175°	102° 44′.6	7.8024	1.4095	0.609	2515
160°	200°	110° 17′.9	7.5032	1.6295	0.603	3285
180°	225°	117° 20′.6	7.1064	1.9996	0.595	4158
200°	250°	124° 00′.9	6.6312	2.7339	0.585	5133
220°	275°	130° 24′.1	6.0920	4.8674	0.571	6211
240°	300°	136° 34′.4	5.4992	78.5703	0.562	7392
			Maximum load			7464

For $L=36$ inches, $d=60$ inches. The size correction for the tested strut was 1.0182. With $E=30{,}340{,}000$ and $\frac{q}{d}$ in degrees,

$$P = \frac{30{,}340{,}000 \times \pi \times 1.0182}{64 \times 3600 \times 57.296^2}\left(\frac{q}{d}\right)^2 = 0.12833\left(\frac{q}{d}\right)^2.$$

APPENDIX D

FUNCTIONS USED FOR CALCULATING y AND ITS FIRST DERIVATIVE FROM

$$x\frac{d^2y}{dx^2}+by=0 \text{ AND } x^3\frac{d^2y}{dx^2}+by=0.$$

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
0.1	0.09508	0.00731	0.09025	0.01442
0.2	0.18066	0.02848	0.16196	0.05546
0.3	0.25719	0.06241	0.21653	0.11989
0.4	0.32516	0.10806	0.25531	0.20468
0.5	0.38499	0.16442	0.27957	0.30697
0.6	0.43713	0.23054	0.29053	0.42406
0.7	0.48197	0.30548	0.28936	0.55342
0.8	0.51993	0.38837	0.27718	0.69267
0.9	0.55139	0.47837	0.25501	0.83959
1.0	0.57672	0.57468	0.22389	0.99209
1.1	0.59629	0.67654	0.18476	1.14824
1.2	0.61043	0.78322	0.13852	1.30620
1.3	0.61948	0.89402	0.08604	1.46432
1.4	0.62377	1.00828	0.02812	1.62101
1.5	0.62360	1.12539	−0.03445	1.77484
1.6	0.61928	1.24473	−0.10096	1.92449
1.7	0.61108	1.36576	−0.17070	2.06872
1.8	0.59928	1.48794	−0.24304	2.20641
1.9	0.58416	1.61076	−0.31737	2.33656
2.0	0.56596	1.73374	−0.39310	2.45823
2.1	0.54493	1.85644	−0.46969	2.57059
2.2	0.52130	1.97342	−0.54665	2.67289
2.3	0.49531	2.09925	−0.62350	2.76447
2.4	0.46716	2.21869	−0.69979	2.84474
2.5	0.43706	2.33624	−0.77511	2.91319
2.6	0.40521	2.45164	−0.84909	2.96939
2.7	0.37181	2.56456	−0.92135	3.01295
2.8	0.33702	2.67473	−0.99159	3.04361
2.9	0.30104	2.78188	−1.05949	3.06108
3.0	0.26401	2.88576	−1.12478	3.06519
3.1	0.22610	2.98615	−1.18720	3.05581
3.2	0.18746	3.08284	−1.24652	3.03287
3.3	0.14824	3.17564	−1.30253	2.99632
3.4	0.10857	3.26438	−1.35505	2.94622
3.5	0.06857	3.34890	−1.40390	2.88259
3.6	0.02839	3.42906	−1.44894	2.80556
3.7	−0.01188	3.50473	−1.49004	2.71527
3.8	−0.05212	3.57579	−1.52707	2.61190
3.9	−0.09222	3.64216	−1.55995	2.49566
4.0	−0.13209	3.70375	−1.58860	2.36680
4.1	−0.17162	3.76048	−1.61295	2.22560
4.2	−0.21074	3.81230	−1.63294	2.07237
4.3	−0.24935	3.85915	−1.64855	1.90743
4.4	−0.28739	3.90101	−1.65974	1.73114
4.5	−0.32477	3.93783	−1.66651	1.54386
4.6	−0.36143	3.96962	−1.66885	1.34601
4.7	−0.39731	3.99635	−1.66679	1.13800
4.8	−0.43234	4.01805	−1.66033	0.92027
4.9	−0.46647	4.03470	−1.64950	0.69324
5.0	−0.49965	4.04635	−1.63438	0.45738

APPENDIX D—Continued.

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
5.1	—0.53183	4.05301	—1.61497	0.21318
5.2	—0.56297	4.05472	—1.59136	—0.03889
5.3	—0.59303	4.05153	—1.56361	—0.29833
5.4	—0.62196	4.04348	—1.53179	—0.56466
5.5	—0.64975	4.03064	—1.49598	—0.83734
5.6	—0.67635	4.01305	—1.45627	—1.11589
5.7	—0.70175	3.99081	—1.41275	—1.39977
5.8	—0.72592	3.96397	—1.36552	—1.68846
5.9	—0.74883	3.93261	—1.31468	—1.98147
6.0	—0.77048	3.89683	—1.26036	—2.27824
6.1	—0.79084	3.85670	—1.20264	—2.57827
6.2	—0.80990	3.81233	—1.14166	—2.88100
6.3	—0.82766	3.76380	—1.07753	—3.18596
6.4	—0.84411	3.71122	—1.01057	—3.49260
6.5	—0.85924	3.65469	—0.94032	—3.80040
6.6	—0.87304	3.59432	—0.86751	—4.10887
6.7	—0.88552	3.53022	—0.79205	—4.41748
6.8	—0.89669	3.46249	—0.71409	—4.72576
6.9	—0.90653	3.39126	—0.63376	—5.03318
7.0	—0.91506	3.31664	—0.55120	—5.33928
7.1	—0.92228	3.23875	—0.46654	—5.64355
7.2	—0.92820	3.15771	—0.37993	—5.94553
7.3	—0.93284	3.07364	—0.29150	—6.24477
7.4	—0.93619	2.98066	—0.20140	—6.54078
7.5	—0.93828	2.89690	—0.10975	—6.83315
7.6	—0.93912	2.80448	—0.01671	—7.12141
7.7	—0.93873	2.70954	0.07759	—7.40515
7.8	—0.93712	2.61218	0.17299	—7.68393
7.9	—0.93430	2.51255	0.26940	—7.95737
8.0	—0.93030	2.41077	0.36664	—8.22506
8.1	—0.92514	2.30697	0.46458	—8.48661
8.2	—0.91884	2.20127	0.56309	—8.74166
8.3	—0.91141	2.09380	0.66204	—8.98985
8.4	—0.90289	1.98469	0.76128	—9.23080
8.5	—0.89330	1.87406	0.86070	—9.46419
8.6	—0.88265	1.76205	0.96015	—9.68971
8.7	—0.87097	1.64876	1.05951	—9.90703
8.8	—0.85830	1.53434	1.15866	—10.11585
8.9	—0.84465	1.41890	1.25747	—10.31588
9.0	—0.83005	1.30256	1.35581	—10.50685
9.1	—0.81453	1.18545	1.45357	—10.68850
9.2	—0.79811	1.06769	1.55062	—10.86057
9.3	—0.78083	0.94939	1.64686	—11.02283
9.4	—0.76271	0.83068	1.74216	—11.17505
9.5	—0.74377	0.71166	1.83643	—11.31702
9.6	—0.72406	0.59246	1.92955	—11.44856
9.7	—0.70358	0.47319	2.02141	—11.56945
9.8	—0.68239	0.35395	2.11191	—11.67955
9.9	—0.66049	0.23487	2.20095	—11.77867
10.0	—0.63793	0.11604	2.28844	—11.86670

$$f_1(x) = x - \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 2 \lfloor 3} - \frac{x^4}{\lfloor 3 \lfloor 4} + \frac{x^5}{\lfloor 4 \lfloor 5} - \frac{x^6}{\lfloor 6 \lfloor 7} + \text{etc.}$$

$$f_2(x) = \frac{x^2}{\lfloor 2}\left(\frac{1}{1} + \frac{1}{2}\right) - \frac{x^3}{\lfloor 2 \lfloor 3}\left(\frac{1}{1} + \frac{2}{2} + \frac{1}{3}\right) + \frac{x^4}{\lfloor 3 \lfloor 4}$$

$$\left(\frac{1}{1} + \frac{2}{2} + \frac{2}{3} + \frac{1}{4}\right) - \frac{x^5}{\lfloor 4 \lfloor 5}\left(\frac{1}{1} + \frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \frac{1}{5}\right) + \text{etc.}$$

APPENDIX D—Continued.

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
10.1	—0.61473	—0.00243	2.37426	—11.94348
10.2	—0.59093	—0.12043	2.45836	—12.00889
10.3	—0.56654	—0.23787	2.54063	—12.06282
10.4	—0 54160	—0.35463	2.62098	—12.10518
10.5	—0.51615	—0.47063	2.69933	—12.13589
10.6	—0.49020	—0.58576	2.77560	—12.15486
10.7	—0.46379	—0.69993	2.84971	—12.16204
10.8	—0.43694	—0.81306	2.92160	—12.15738
10.9	—0.40969	—0.92504	2.99118	—12.14085
11.0	—0.38207	—1.03579	3.05840	—12.11242
11.1	—0.35409	—1.14524	3.12319	—12.07207
11 2	—0.32580	—1.25328	3.18548	—12.01982
11.3	—0.29722	—1.35985	3.24521	—11.95565
11.4	—0.26837	—1.46486	3.30234	—11.87960
11.5	—0.23929	—1.56824	3.35681	—11.79169
11.6	—0 21000	—1.66991	3.40856	—11.69196
11.7	—0.18053	—1.76980	3.45756	—11.58047
11.8	—0.15091	—1.86784	3.50376	—11.45727
11.9	—0.12116	—1.96397	3.54711	—11.32244
12.0	—0.09130	—2.05811	3 58759	—11.17606
12.1	—0.06137	—2.15021	3.62516	—11.01822
12.2	—0.03139	—2.24021	3.65978	—10.84903
12.3	—0.00138	—2.32805	3.69142	—10.66858
12.4	0.02862	—2.41367	3.72007	—10.47700
12.5	0.05861	—2.49702	3.74569	—10.27442
12.6	0.08855	—2.57804	3.76827	—10.06096
12.7	0.11841	—2.65670	3.78779	— 9.83678
12.8	0 14819	—2.73294	3.80423	— 9.60202
12 9	0.17785	—2.80671	3.81759	— 9.35684
13.0	0.20737	—2.87799	3.82785	— 9.10141
13.1	0.23673	—2.94672	3.83500	— 8.83591
13.2	0.26591	—3.01288	3.83905	— 8.56052
13.3	0.29489	—3.07642	3.83999	— 8.27541
13.4	0.32365	—3.13731	3.83782	— 7 98080
13.5	0.35217	—3.19552	3.83254	— 7.67687
13.6	0.38042	—3.25103	3.82417	— 7.36383
13 7	0.40840	—3.30381	3.81270	— 7.04191
13.8	0.43608	—3.35382	3.79815	— 6.71130
13.9	0.46344	—3.40106	3.78053	— 6.37225
14.0	0.49047	—3 44550	3.75986	— 6.02498
14.1	0 51714	—3.48713	3.73616	— 5.66972
14.2	0.54346	—3.52592	3.70944	— 5.30671
14.3	0 56938	—3.56187	3.67972	— 4.93620
14.4	0.59492	—3.59496	3.64703	— 4.55842
14.5	0.62003	—3.62518	3.61140	— 4.17365
14.6	0.64472	—3.65252	3.57285	— 3.78212
14.7	0.66897	—3.67698	3.53141	— 3.38409
14.8	0.69276	—3.69856	3.48711	— 2.97984
14.9	0.71609	—3.71725	3.43999	— 2.56962
15.0	0.73893	—3 73305	3.39008	— 2.15370

$$f_8(x) = x - x^2 + \frac{x^3}{\lfloor 2 \lfloor 2} - \frac{x^4}{\lfloor 3 \lfloor 3} + \frac{x^5}{\lfloor 4 \lfloor 4} - \text{etc.}$$

$$f_4(x) = x^2\left(\frac{1}{1} + \frac{1}{2}\right) - \frac{x^3}{\lfloor 2 \lfloor 2}\left(\frac{1}{1} + \frac{2}{2} + \frac{1}{3}\right) +$$

$$\frac{x^4}{\lfloor 3 \lfloor 3}\left(\frac{1}{1} + \frac{2}{2} + \frac{2}{3} + \frac{1}{4}\right) - \text{etc.}$$

APPENDIX D—Continued.

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
15.1	0.76129	—3.74597	3.33742	—1.73237
15.2	0.78314	—3.75600	3.28204	—1.30587
15.3	0.80447	—3.76315	3.22399	—0.87454
15.4	0.82528	—3.76743	3.16330	—0.43851
15.5	0.84555	—3.76885	3.10003	0.00175
15.6	0.86527	—3.76742	3.03421	0.44607
15.7	0.88444	—3.76314	2.96589	0.89415
15.8	0.90305	—3.75603	2.89512	1.34569
15.9	0.92109	—3.74611	2.82194	1.80043
16.0	0.93855	—3.73339	2.74641	2.25806
16.1	0.95542	—3.71789	2.66858	2.71831
16.2	0.97169	—3.69963	2.58850	3.18088
16.3	0.98737	—3.67863	2.50622	3.64549
16.4	1.00244	—3.65490	2.42179	4.11185
16.5	1.01690	—3.62849	2.33528	4.57967
16.6	1.03075	—3.59940	2.24674	5.04868
16.7	1.04397	—3.56767	2.15622	5.51857
16.8	1.05657	—3.53332	2.06379	5.98906
16.9	1.06854	—3.49638	1.96949	6.45987
17.0	1.07988	—3.45688	1.87341	6.93072
17.1	1.09058	—3.41485	1.77558	7.40132
17.2	1.10064	—3.37033	1.67608	7.87138
17.3	1.11007	—3.32334	1.57496	8.34064
17.4	1.11885	—3.27392	1.47229	8.80880
17.5	1.12699	—3.22210	1.36814	9.27559
17.6	1.13448	—3.16792	1.26255	9.74074
17.7	1.14134	—3.11142	1.15561	10.20397
17.8	1.14754	—3.05263	1.04736	10.66501
17.9	1.15310	—2.99160	0.93789	11.12360
18.0	1.15802	—2.92836	0.82724	11.57946
18.1	1.16230	—2.86295	0.71550	12.03233
18.2	1.16593	—2.79542	0.60271	12.48195
18.3	1.16892	—2.72580	0.48896	12.92805
18.4	1.17127	—2.65415	0.37429	13.37040
18.5	1.17299	—2.58049	0.25879	13.80872
18.6	1.17407	—2.50488	0.14252	14.24278
18.7	1.17452	—2.42736	0.02553	14.67232
18.8	1.17435	—2.34797	—0.09209	15.09711
18.9	1.17354	—2.26676	—0.21029	15.51690
19.0	1.17212	—2.18378	—0.32900	15.93145
19.1	1.17008	—2.09908	—0.44816	16.34054
19.2	1.16743	—2.01269	—0.56769	16.74393
19.3	1.16417	—1.92468	—0.68754	17.14140
19.4	1.16031	—1.83508	—0.80763	17.53274
19.5	1.15585	—1.74394	—0.92791	17.91771
19.6	1.15079	—1.65132	—1.04830	18.29611
19.7	1.14515	—1.55727	—1.16875	18.66774
19.8	1.13893	—1.46182	—1.28918	19.03237
19.9	1.13213	—1.36504	—1.40953	19.38983
20.0	1.12476	—1.26697	—1.52974	19.73990

APPENDIX E

COMPUTATION OF DEFLECTION OF STRUT FOR WHICH $I=Cx^3$

Let $\dfrac{b}{d}=u$, and $\dfrac{b}{kd}=v$.

$$\frac{A}{B}+\log d=\frac{f_1(u)+f_4(u)-f_2(u)+1}{f_3(u)-f_1(u)}=\frac{S_1}{S_2}$$

$$y_{\max}=\frac{e}{k}\,\frac{\left(\dfrac{A}{B}+\log d\right)f_1(u)-f_2(u)+1}{\left(\dfrac{A}{B}+\log kd\right)f_1(v)-f_2(v)+1}=\frac{eS_4}{kS_3}$$

$\dfrac{b}{d}=u$	5.4	6.0	6.6	7.2	7.8
$f_1(u)$	−0.62196	−0.77048	−0.87304	−0.92820	−0.93712
$f_4(u)$	−0.56466	−2 27824	−4.10887	−5.94553	−7.68393
$-f_2(u)$	−4.04348	−3.89683	−3.59432	−3.15771	−2.61218
Sum $+\ 1=S_1$	−4.23010	−5.94555	−7.57623	−9.03144	−10.23323
$f_3(u)$	−1.53179	−1.26036	−0.86750	−0.37933	0.17299
$-f_1(u)$	0.62196	0.77048	0.87304	0.92820	0.93712
Sum $=S_2$	−0.90983	0.48988	0.00554	0.54827	1.11011
$\dfrac{A}{B}+\log d$	4.64933	12.13675	−1367.55054	−16.47261	−9.89198
$\dfrac{S_1}{S_2}f_1(u)$	−2.89170	−9.35112	1193.92632	15.28988	8.63857
$-f_2(u)$	−4.04348	−3 89683	−3.59432	−3.15771	−2.61218
Sum $+\ 1=S_4$	−5.93518	−12.24795	1191.33200	13.13216	7.02639

For $k=0.6,\ \dfrac{A}{B}+\log kd=\dfrac{A}{B}+\log d-0.51083$

$\dfrac{b}{kd}=v$	9	10	11	12	13
$\dfrac{A}{B}+\log kd$	4.13850	11.62592	−1368.06137	−16.98345	−9.72904
$f_1(v)$	−0.83005	−0.63793	−0.38207	−0.09130	0.20737
Product	−3.43516	−7.41652	522.69521	1.55059	−2.01751
$-f_2(v)$	−1.30256	−0.11604	1 03579	2.05811	2.97799
Sum $+\ 1=S_3$	−3.73772	−6.53256	524.73100	4.60870	1.86048
$\dfrac{S_4}{S_3}$	1.58791	1.87490	2.27036	2.84943	3.77655
$\dfrac{y_{\max}}{e}$	2.6465	3.1248	3 7839	4.7490	6.2944
$\dfrac{S_3}{S_4}$	0 6298	0.5334	0.4405	0.3505	0.2648
Load	8301	9223	10146	11068	11990

Loads are calculated for a strut 36 inches long, 1.25 inches wide, and 1 inch thick at the middle, with $E=30{,}880{,}000$, and size correction factor 0.9677.

APPENDIX F

COMPUTATION OF THE ULTIMATE LOAD FOR STRUTS FOR WHICH $I = Cx^2$

$$\operatorname{Tan} B = -\sqrt{4b-1}\ ;\quad B + \sqrt{4b-1}\,\log k^{\frac{1}{2}} = 0$$

$$\frac{57.296}{2}\log k = G \text{ (for convenience)}$$

k	G	$\sqrt{4b-1}$	B	$G\sqrt{4b-1}$	$B + G\sqrt{4b-1}$	$\sqrt{4b-1}$
I	II	III	IV	V	VI	VII
0.05	—85.82130	1.451	124°.5735	—124°.5367	468	
		1.452	124°.5539	—124°.6125	—586	1.45145
0.1	—65.96446	1.804	119°.0006	—118°.9999	7	
		1.80401	119°.0006	—119°.0005	1	1.80401
0.2	—46.10718	2.436	112°.3185	—112°.3171	14	
		2.4361	112°.3175	—112°.3217	42	2.43602
0.3	—34.49142	3.124	107°.7500	—107°.7512	— 12	
		3.1239	107°.7503	—107°.7477	26	3.12397
0.4	—26.24990	3.967	104°.1484	—104°.1333	151	
		3.9675	104°.1469	—104°.1464	5	3.96752
0.5	—19.85728	5.0918	101°.1112	—101°.1093	19	
		5.0919	101°.1110	—101°.1113	— 3	5.09189
0.6	—14.63414	6.728	98°.4540	— 98°.4584	— 44	
		6.7277	98°.4546	— 98°.4541	5	6.72773
0.7	—10.21803	9.4021	96°.0716	— 96°.0709	7	
		9.4022	96°.0710	— 96°.0720	— 10	9.40214
0.8	— 6.39262	14.6880	93°.8949	— 93°.8948	1	
		14.6881	93°.8949	— 93°.8954	— 5	14.68802
0.9	— 3.01835	30.440	91° 8816	— 91°.8786	30	
		30.441	91°.8815	— 91°.8816	— 1	30.44097

APPENDIX G

COMPUTATION OF DEFLECTION OF STRUT FOR WHICH I=Cx

$$\frac{A}{B} + \log d = -\frac{f_1(bd) + f_4(bd)}{f_3(bd)} = -\frac{S_1}{S_2}$$

$$y_{\max} = \frac{\left(\frac{A}{B} + \log d\right) f_1(bd) + f_2(bd) - 1}{\left(\frac{A}{B} + \log kd\right) f_1(bkd) + f_2(bkd) - 1} \qquad e = \frac{S_4 e}{S_3}$$

bd	3.5	4.0	4.5	5.0	5.5
$f_1(bd)$	0.06857	−0.13209	−0.32477	−0.49965	−0.64975
$f_4(bd)$	2.88259	2.36680	1.54386	0.45738	−0.83734
Sum $= S_1$	2.95116	2.23471	1.21909	−0.04227	−1.48709
$f_3(bd)$	−1.40390	−1.58860	−1.66651	−1.63438	−1.49598
$\frac{A}{B} + \log d$	2.10211	1.40672	0.73152	−0.02586	−0.99406
$\frac{S_1}{S_2} f_1(bd)$	0.14414	−0.18581	−0.23758	0.01292	0.64589
$f_2(bd)$	3.34890	3.70375	3.93783	4 04635	4 03064
Sum $- 1 = S_4$	2.49304	2.51794	2.70025	3.05927	3.67653

For $k = 0.4$, $\frac{A}{B} + \log kd = \frac{A}{B} + \log d - 0.91630$

bkd	1.4	1.6	1.8	2.0	2.2
$\frac{A}{B} + \log kd$	1.18581	0.49042	−0.18478	−0.94216	−1.91036
$f_1(bkd)$	0.62377	0.61928	0.59288	0.56596	0.52130
Product	0.73967	0.30171	−0.11073	−0.53322	−0.99587
$f_2(bkd)$	1.00828	1.24473	1.48789	1.73374	1.97842
Sum $- 1 = S_3$	0.74795	0.54844	0.37721	0.20052	−0.01745
$\frac{S_4}{S_3} = \frac{y_{\max}}{e}$	3.3332	4.5911	7.1585	15.2567	
$\frac{S_3}{S_4}$	0.30001	0 21781	0.13968	0.06654	−0.00475
	Maximum bd = 5.4658				

For strut 1.5 inches wide, 1 inch thick, 36 inches long, $k = 0.4$, and $E = 31,090,000$

load	15113	17272	19431	21590	
				Ultimate	23600

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